EAS270, "The Atmosphere" Take-home Assignment 1

Professor: J.D. Wilson Value: 15% Due 26 Oct., 2005

Instructions: A maximum of one page per question will be marked. Your assignment need not be typed, but, tidiness and legibility will factor into your score. You are permitted to work with others to perform this assignment, however, you are expected to hand in work that reflects your own reasoning, ordering and writing. As best you can, strive to make your responses creative, clear and interesting.

NOTE: The below solutions represent only one of many possible paths to arrive at the correct solution. You are encouraged to use them as a guideline only and to develop you own mathematical and writing styles.

1. An aircraft is flying at $z_1 = 9,144$ m ASL[1], at which level the pressure and temperature are $p_1, T_1 =$ (300 mb, -57^oC). The pilot receives an instruction to descend to $z_2 = 8,839.2$ m . Estimate the atmospheric pressure and temperature at the new flight level, noting any uncertainties and/or any assumptions you make in your reasoning. (4%)

   Solution: When the plane descends we would expect the pressure to increase as there will be more air above it and the weight of the atmosphere is what is responsible for pressure. Since the change in height is not that significant we *assume* that the density at the initial height is equal to the density at the final height, $\rho_i = \rho_f$.

   Step 1: Find the density at $z_i = 9144$ metres using the ideal gas law.

$$
\begin{aligned}
p_i &= \rho_i R T_i \\
\rho_i &= \frac{p_i}{RT_i} \\
&= \frac{30000 \text{ Pa}}{(287 \text{ J kg}^{-1}\text{ K}^{-1})(216 \text{ K})} \\
&= 0.484 \text{ kg m}^{-3}
\end{aligned}
$$

   Step 2: Use the hydrostatic equation to find p_f under the assumption that the density is constant $\rho_f \approx \rho_i = \rho$.

$$
\begin{aligned}
\Delta p &= \rho g \Delta z \\
p_f &= p_i + \rho g \Delta z \\
&= (30000 \text{ Pa} + (0.484 \text{ kg m}^{-3})(9.81 \text{ m s}^{-2})(9144 \text{ m} - 8834.2 \text{ m}) \\
&= 31471 \text{ Pa} \\
&\approx 315 \text{ mB}
\end{aligned}
$$

[1]Above Sea Level. For historical reasons, pilots usually cite altitude in feet ASL; then, $z_1 = 30,000$ feet and $z_2 = 29,000$ feet.

$\boxed{\text{Therefore the pressure at the new height is } p_f = 315 \text{ mB.}}$

Step 3: Use the ideal gas law again to find the temperature.

$$\begin{aligned} p_f &= \rho R T_f \\ T_f &= \frac{p_f}{\rho R} \\ &= \frac{31471 \text{ Pa}}{(0.484 \text{ kg m}^{-3})(287 \text{ J kg}^{-1}\text{ K}^{-1})} \\ &= 227 \text{ K} \\ &= -46^\circ \text{ C} \end{aligned}$$

$\boxed{\text{Therefore the temperature new height is } T_f = -46^\circ \text{ C.}}$

2. Suppose the radiative energy output of a certain star in its "solar band" is $Q = 10^{28}$ W. Calculate the "solar constant" S_0 for a planet whose orbit is a circle of radius $R = 10^{13}$ m. Assuming this planet is a perfect black body (albedo $a = 0$, emissivity $\epsilon = 1$), that it is isothermal, has no atmosphere, and has no internal energy sources, calculate its equilibrium temperature T_{eq}. (4%)

   Solution: The key concepts to understand here is that the planet will absorb radiation as a disk and emit it as a sphere and at equilibrium the incoming radiation equals the outgoing radiation.

   Step 1: Find the solar constant at the orbit of the planet. The radiation of the star is emitted in all directions making a sphere. Thus at the planet the solar constant is

$$\begin{aligned} S_0 &= \frac{Q_*}{4\pi R^2} \\ &= \frac{10^{28} \text{ W}}{4\pi(10^{13} \text{ m})^2} \\ &= 7.96 \text{ W m}^{-2} \end{aligned}$$

   $\boxed{\text{Therefore the solar constant for this star is } S_0 = 7.96 \text{ W m}^{-2}.}$

   Step 2: Find the equilibrium temperature by doing a radiation balance and using the Stefan-Boltzmann Law, $S = \sigma T^4$. Since it is not given we will assume that the planet has a radius of R_P.

$$\begin{aligned} Q_{in} &= Q_{out} \\ S_0 A_{disk} &= S_{out} A_{sphere} \\ \pi R_P^2 S_0 &= 4\pi R_P^2 \sigma T^4 \\ S_0 &= 4\sigma T^4 \end{aligned}$$

which gives a temperature of

$$
\begin{aligned}
T &= \left(\frac{S_0}{4\sigma}\right)^{\frac{1}{4}} \\
&= \left(\frac{7.96\ \mathrm{W\,m^{-2}}}{4(5.67\times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}})}\right)^{\frac{1}{4}} \\
&\approx 77\ \mathrm{K}
\end{aligned}
$$

$\boxed{\text{The equilibrium temperature of this planet is } T_{eq} = 77 \text{ K.}}$

3. Write up to one page, explaining in your own words the diurnal temperature cycle in the lowest 10 m above ground. Assume a fair-weather day (no cloud, no fronts), and identify the processes "driving" the diurnal cycle, and the significant flows of energy. Comment on the meteorological situation an observer would note in relation to time of day (eg. winds diminishing or rising overnight, etc.), and explain in what way the diurnal cycle would differ between two fair days sharing the same cycles in $Q*$, but differing in that day 2 has much stronger winds than day 1. (7%)

   Essential elements are:

   - clear statement of the surface energy budget, defining its individual terms and establishing the sign convention either verbally or with a diagram; consciousness of the necessity for a choice of averaging duration, and recognition that in this context we want (say) order 15-60 mins, not (say) 24 hours or a week
   - intelligent, consistent use of symbols to abbreviate the discussion and make it unambiguous
   - recognition that the sign of the heat flux Q_H goes through a diurnal cycle, and that when $Q_H > 0$ (or the opposite if they pick the other sign convention) mean temperature decreases with increasing height; unstable stratification, strong mixing, direction of heat flow is away from ground; all the better if a diagram of T versus z is given
   - recognition that if $Q_H > 0$ (in the normal convention) sensible heat is flowing to the atmosphere, so air temperature will be increasing... so Q_H "drives" the daily (fairweather) cycle in near ground temperature
   - link the above to sign of the net radiation Q^* by virtue of some explicit and rational assumption in regard to Q_E, Q_G, Q_S. For example one might declare the surface to be a dry (set $Q_E \sim 0$), bare soil (storage $Q_S \sim 0$) and, to boot, a poor conductor ($Q_G \sim 0$); in this case $Q_H = Q^*$... then air temperature will keep increasing until $Q^* \equiv K^* + L^*$ turns negative late in the afternoon ($K^* > 0$ getting small as sun sinks; $L^* = L\downarrow - L\uparrow$ maximally negative in late afternoon because the surface temperature T_{gnd} is warm, maximising $L\uparrow = \epsilon\sigma T_{gnd}^4$)
   - statement contrasting observable meteorological situation at several key times: eg.
     - near dawn, pre-sunrise: nocturnal inversion, stable stratification, shallow friction layer, vertical mixing suppressed, weak winds; low temperature-dewpoint spread, possible fog and or frost

- early afternoon: deep mixing layer, unstable stratification, strong mixing, strongest winds (momentum mixed down from aloft), low relative humidity (ie. $T - T_d$ large)
- late afternoon: maximum temperature, minimum relative humidity
- early evening: cooling temperatures, diminishing winds, inversion re-forming

On the windier day, heat exchange would occur with a deeper layer of the atmosphere; in consequence the diurnal range in temperature would be smaller on the windy day (see Sec. 3-4 of textbook).

Bonus elements:

- recognition of that, in view of the vast geographic range of types of surface and moisture status, and the plethora of latitudinal and seasonal influences, there are wide variations on this basic picture.
- clarifying employment of the Bowen ratio in the discussion
- recognition that T represents a short term average, say 15 mins

Notes: In (1,2) show your reasoning and working. Make whatever unit conversions are necessary, and always show units in association with numbers. Use diagrams if appropriate and where they clarify your work. Orderly and comprehensible presentation of your answer is important; brevity and directness are also important, to which purpose you may use point form with abbreviations. In (3) you should define and use symbols (eg. Q_H [W m^{-2}], the vertical flux of sensible heat) and use a diagram if helpful. You will have to find a compromise between an exhaustive treatment (impossible in one tidy, legible page) and an overview.