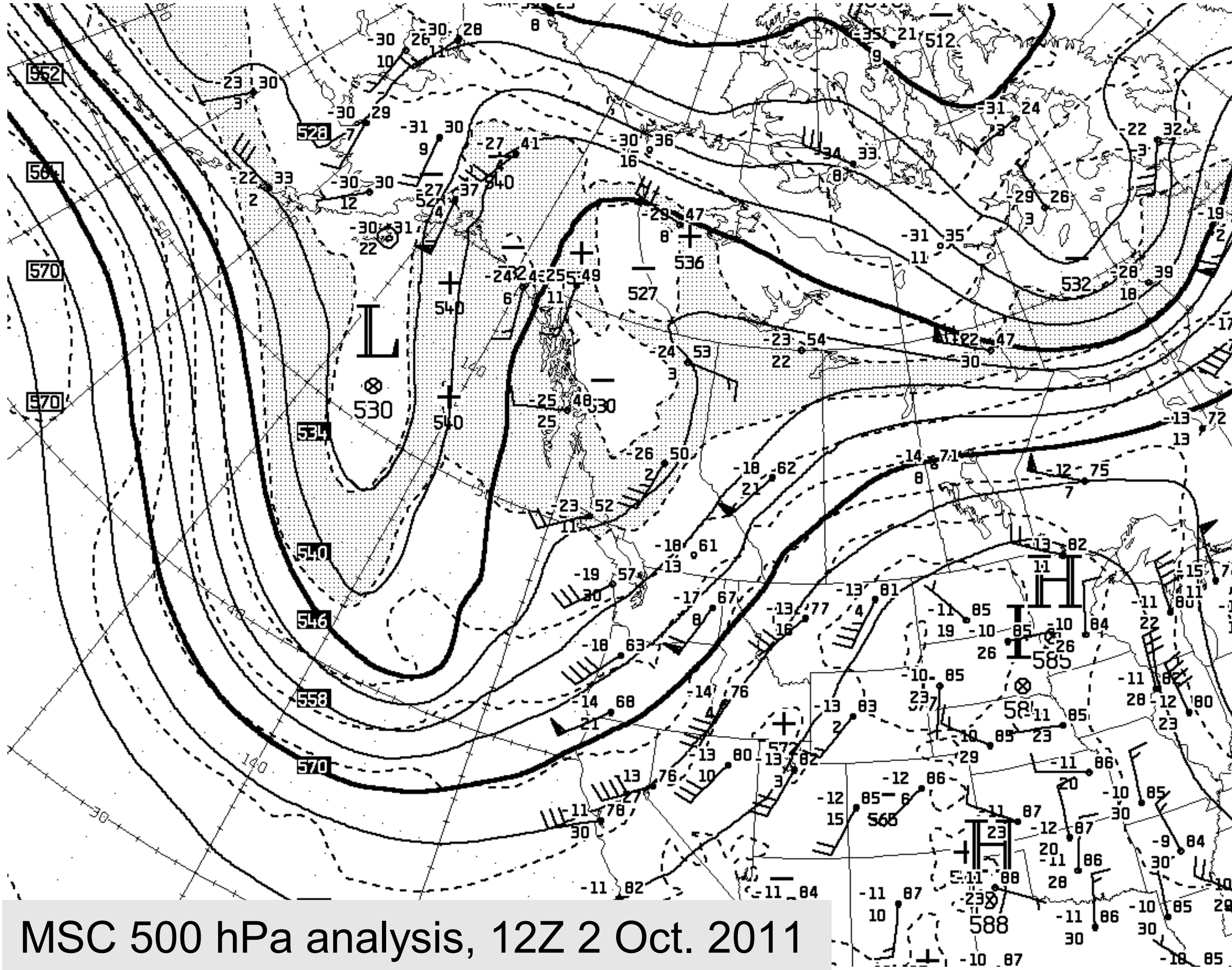
Goals: 3 Oct. 2011

By considering the horizontal force balance, we obtain a law stating:

- that the closer together the isobars or height contours, the faster the wind speed

- that the wind blows parallel to the height contours

This is the basis for what is known as “Buys-Ballot’s law” – that if in the N. hemisphere you stand with your back to the winds aloft (whose direction you can infer from cloud motion), lower pressure (height) lies to your left



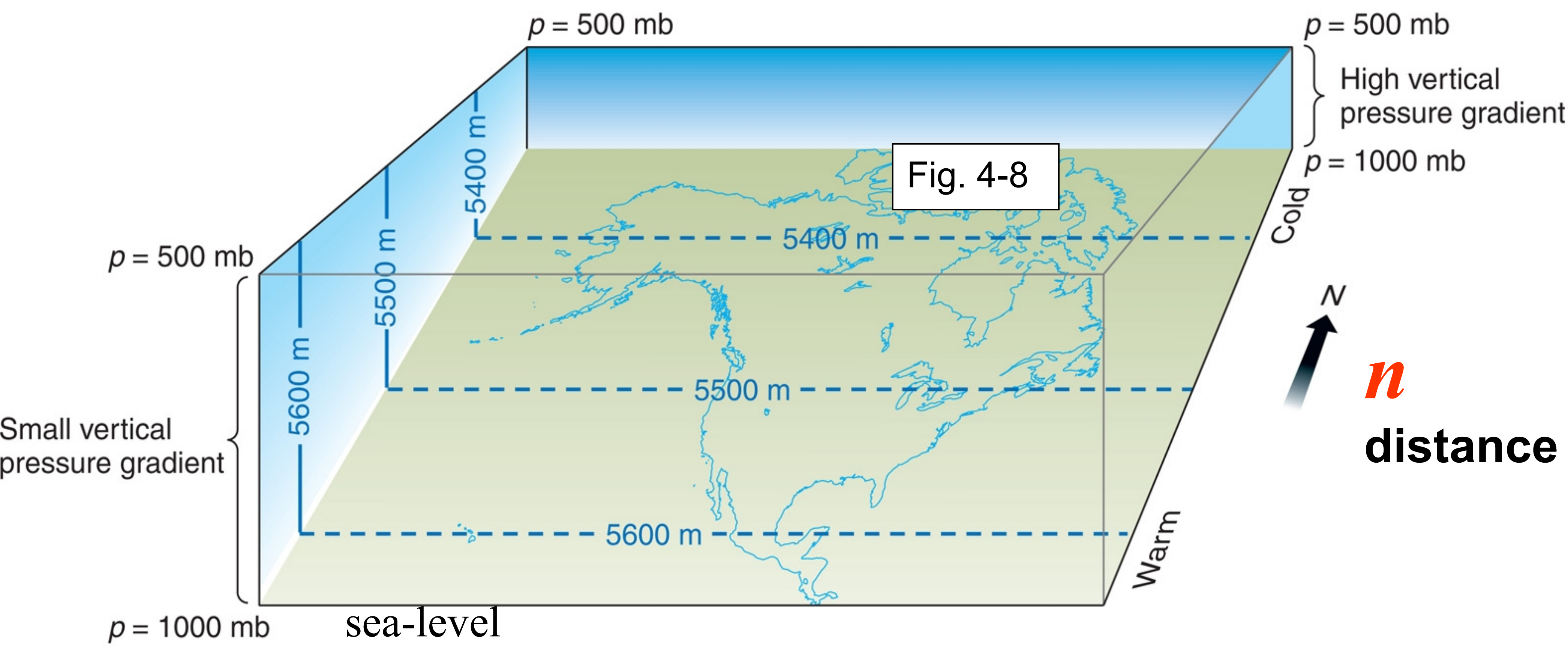
MSC 500 hPa analysis, 12Z 2 Oct. 2011

Forces affecting the wind:

- pressure-gradient force** (PGF), i.e. difference in pressure per unit of distance
- Coriolis force (CF) – result of referring motion to rotating axes
- Friction force (FF) – only in the friction layer
- Gravity/buoyancy force – influences vertical wind only

** we shall actually be using the "force per unit mass," which (from Newton's Law) is the same as acceleration

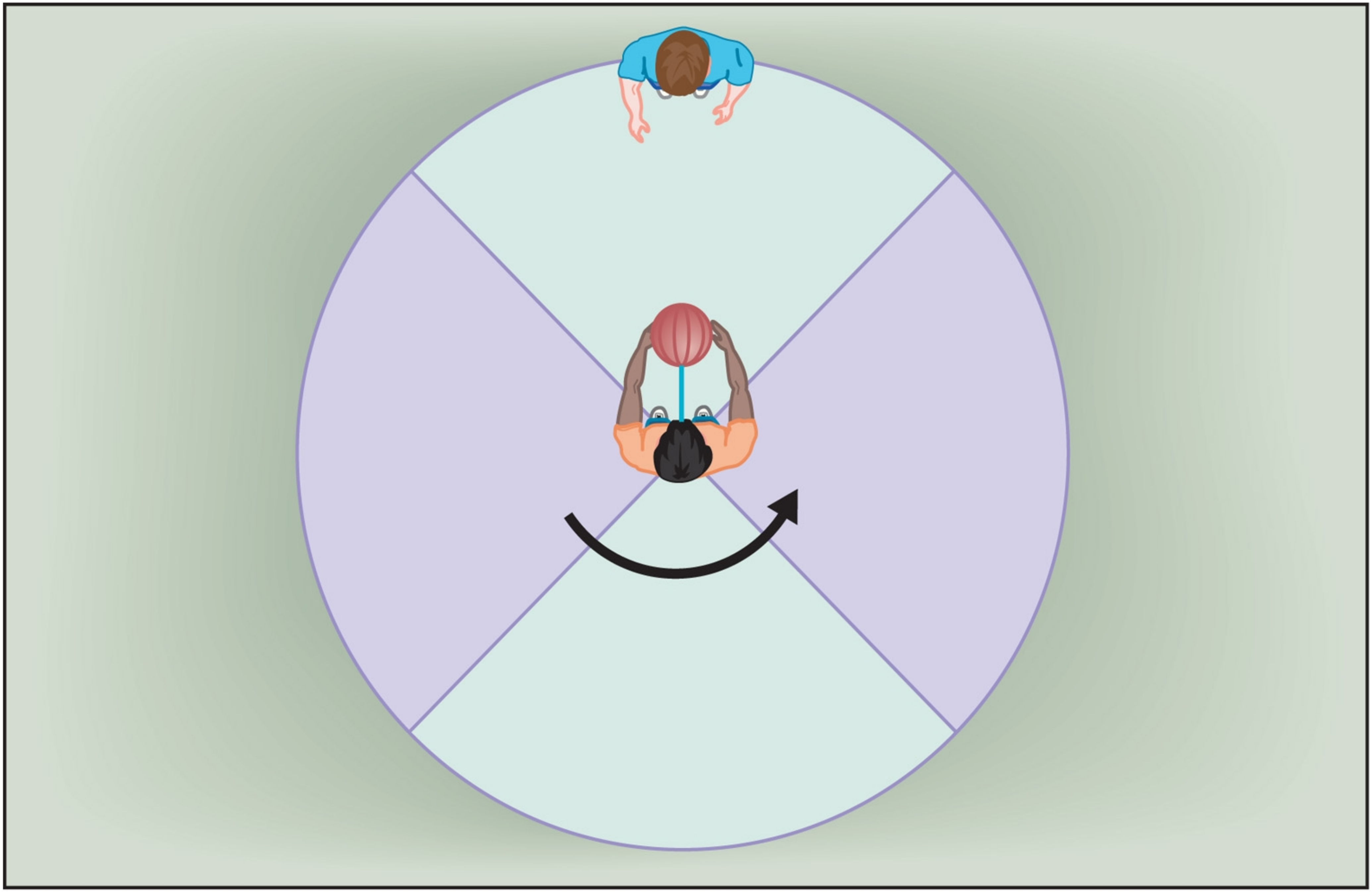
Idealized sloping 500 hPa surface: height (h) is lower in the colder, poleward air



The magnitude of the "pressure gradient force per unit mass" (PGF or F_{PG}) can be written as:

$$F_{\mathrm{PG}} = \frac{1}{\rho}\,\frac{\Delta p}{\Delta n} = g\,\frac{\overbrace{\Delta h}^{\text{height}}}{\underbrace{\Delta n}_{\text{distance}}} \quad [\mathrm{m\ s^{-2}}]$$

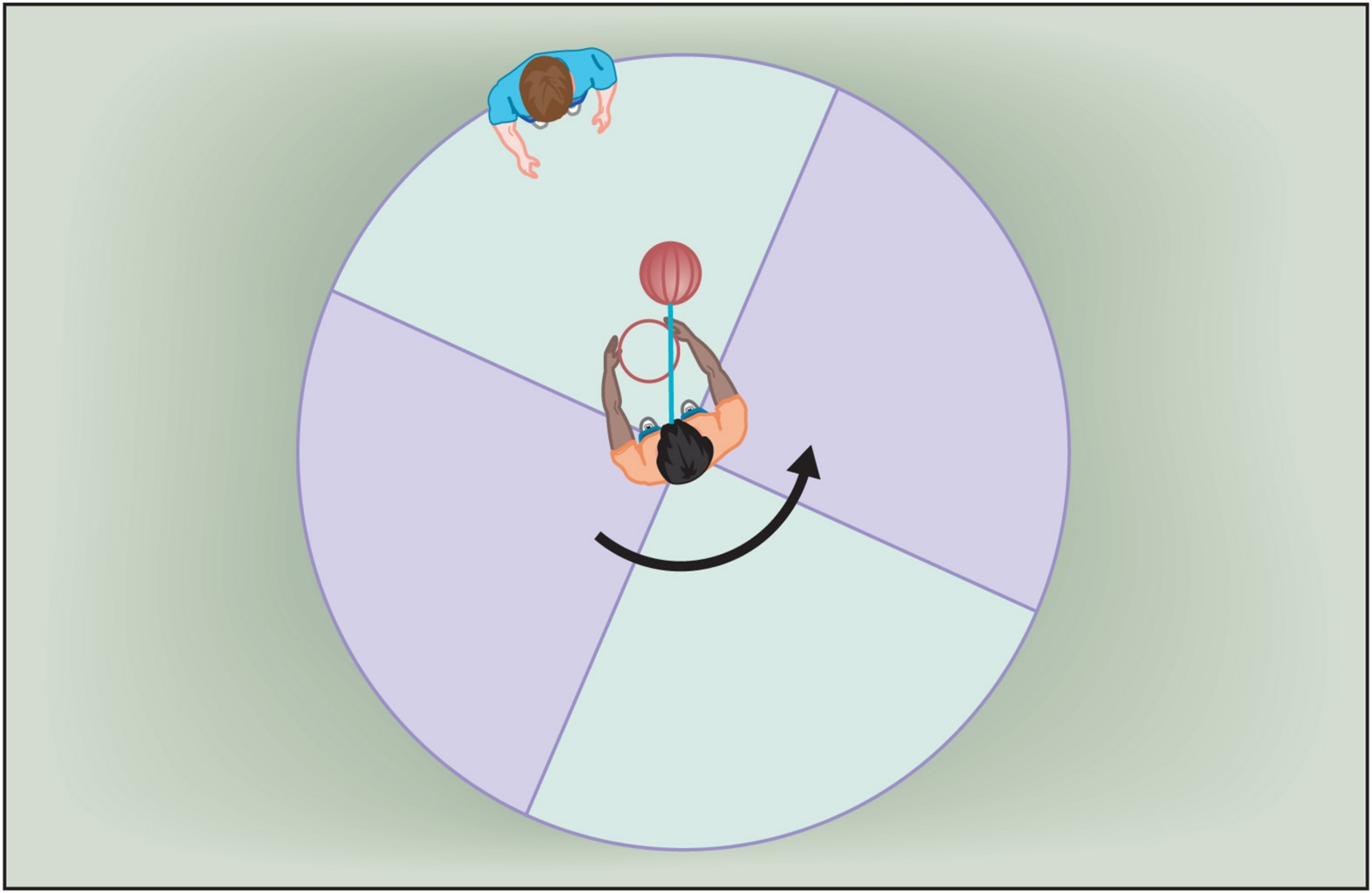
Consequence of referring motion to a moving frame



(a)

© 2010 Pearson Education, Inc.

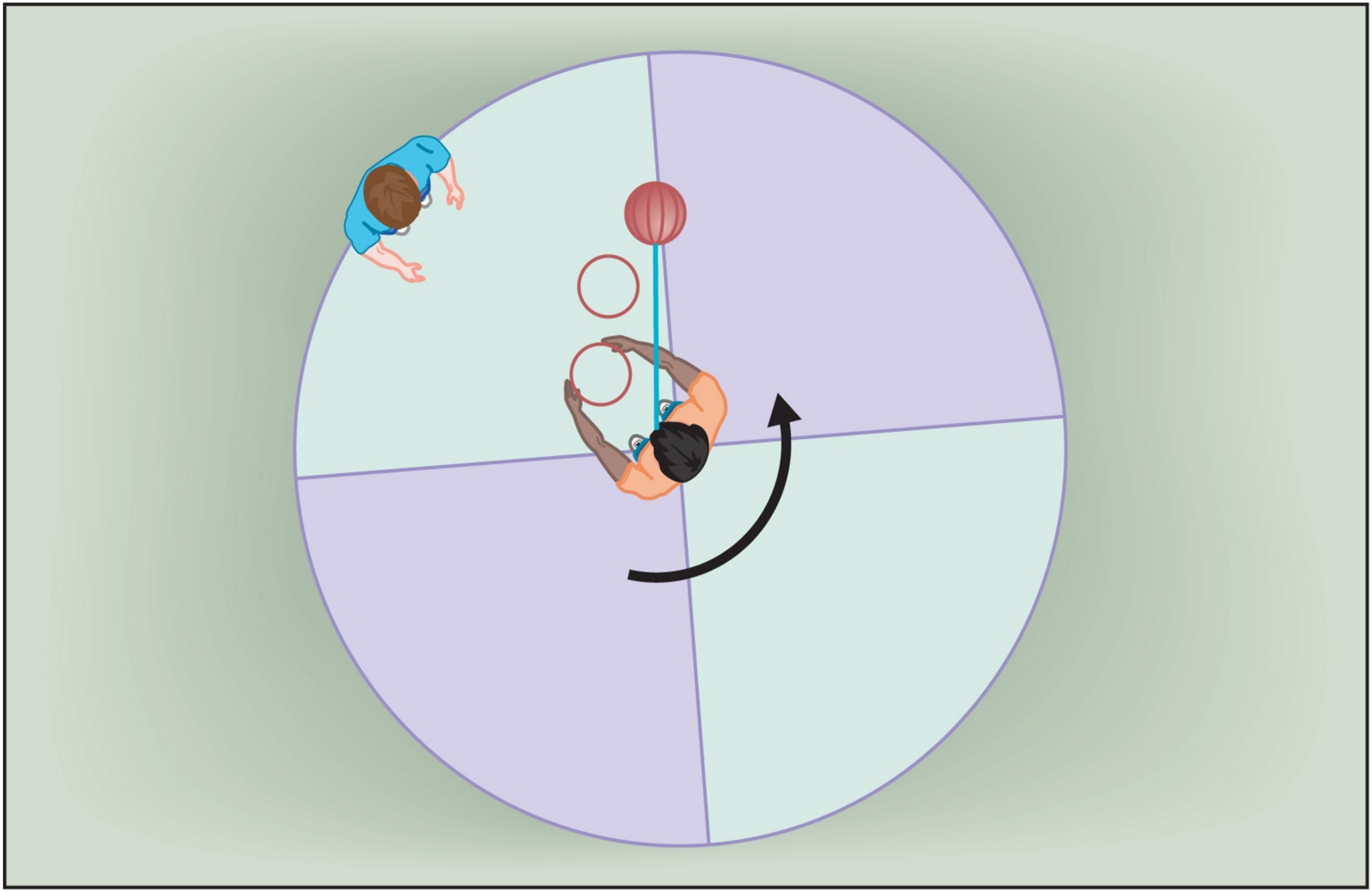
Consequence of referring motion to a moving frame



(b)

© 2010 Pearson Education, Inc.

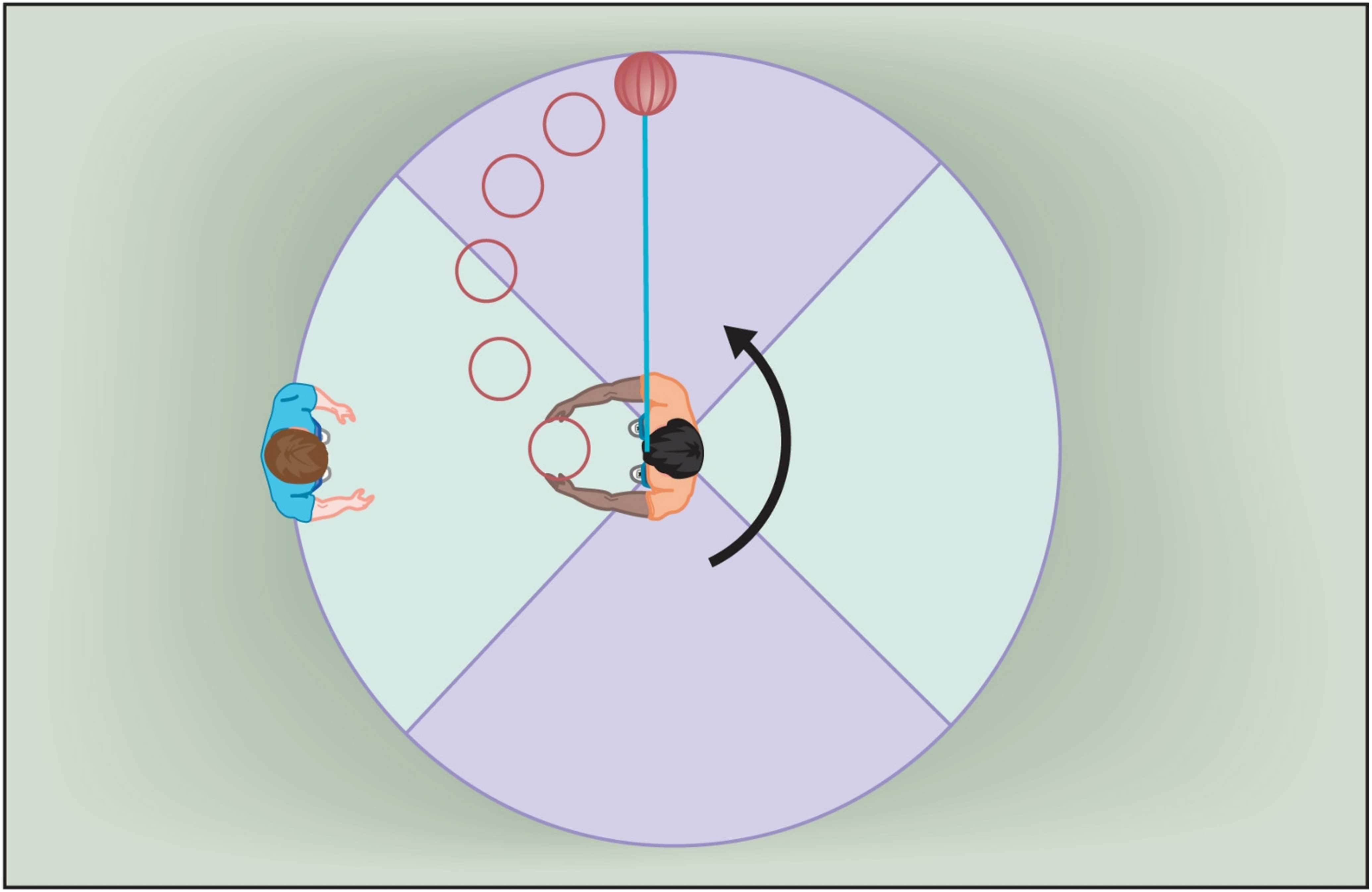
Consequence of referring motion to a moving frame



(c)

© 2010 Pearson Education, Inc.

Consequence of referring motion to a moving frame



(d)

© 2010 Pearson Education, Inc.

The Coriolis Effect (Sec. 4-4)

• “In describing wind… we take the surface as reference frame”

• but (except at the equator) the surface is rotating about the local vertical (at one revolution, or 2π radians) per day. Angular rotation rate:

$$\Omega \approx 2\pi \, / \, (24 \times 3600) \text{ radians s}^{-1} = 7.27 \times 10^{-5} \text{ s}^{-1}$$

• thus “we are describing motions relative to a rotating reference frame” and “an object moving in a straight line with respect to the stars <u>appears</u> to follow a curved path” (it <u>*does*</u> follow a curved path) relative to the coordinates fixed on the earth’s surface

• as a result we may say there is an extra force (or acceleration) - the Coriolis force (acceleration) - which is fictitious - a “book-keeping necessity” because of our choice of a rotating frame of reference

Magnitude and orientation of the Coriolis force

- always acts perpendicular to the motion (so does no work**)
- deflects all moving objects, regardless of direction of their motion
- deflection is to the right in the Northern Hemisphere
- vanishes at equator, increases with latitude φ, maximal at the poles
- increases in proportion to the speed V of the object or air parcel
- magnitude is: $F_C = 2\Omega\, V \sin\phi = f\ V \quad [\mathrm{m\ s^{-2}}]$

where Coriolis parameter $f = 2\Omega \sin\phi$

f ~ $10^{-4}\ \mathrm{s}^{-1}$ at mid latitudes (its reciprocal, the "Coriolis timescale, has a magnitude of several *hours*)

an aside:

---

rate of working by vector force **F on object with velocity **U** is: $P = \mathbf{F} \cdot \mathbf{U}$

The Equation of (horizontal) Motion (Sec. 4-5)

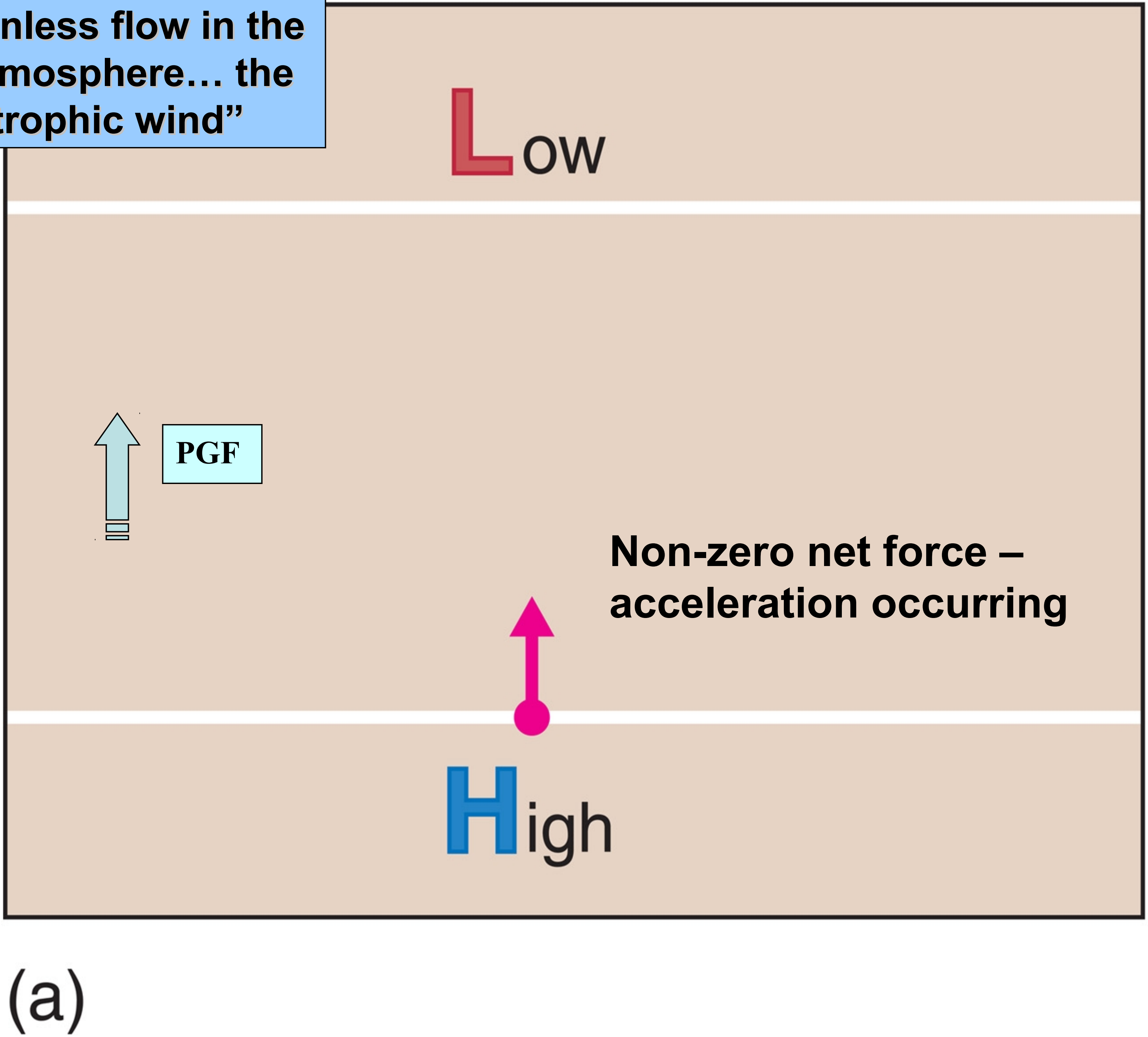
$$\frac{\Delta \vec{V}}{\Delta t} = \underbrace{F_{PG} + F_{C} + F_{f}}$$

net force per unit mass equals the acceleration, and is the vector sum of all forces [N kg^{-1}] acting
(actually each force is a vector)

$\vec{V}$ is the velocity vector (whose magnitude is the "speed" V),
and the l.h.s. is the acceleration vector

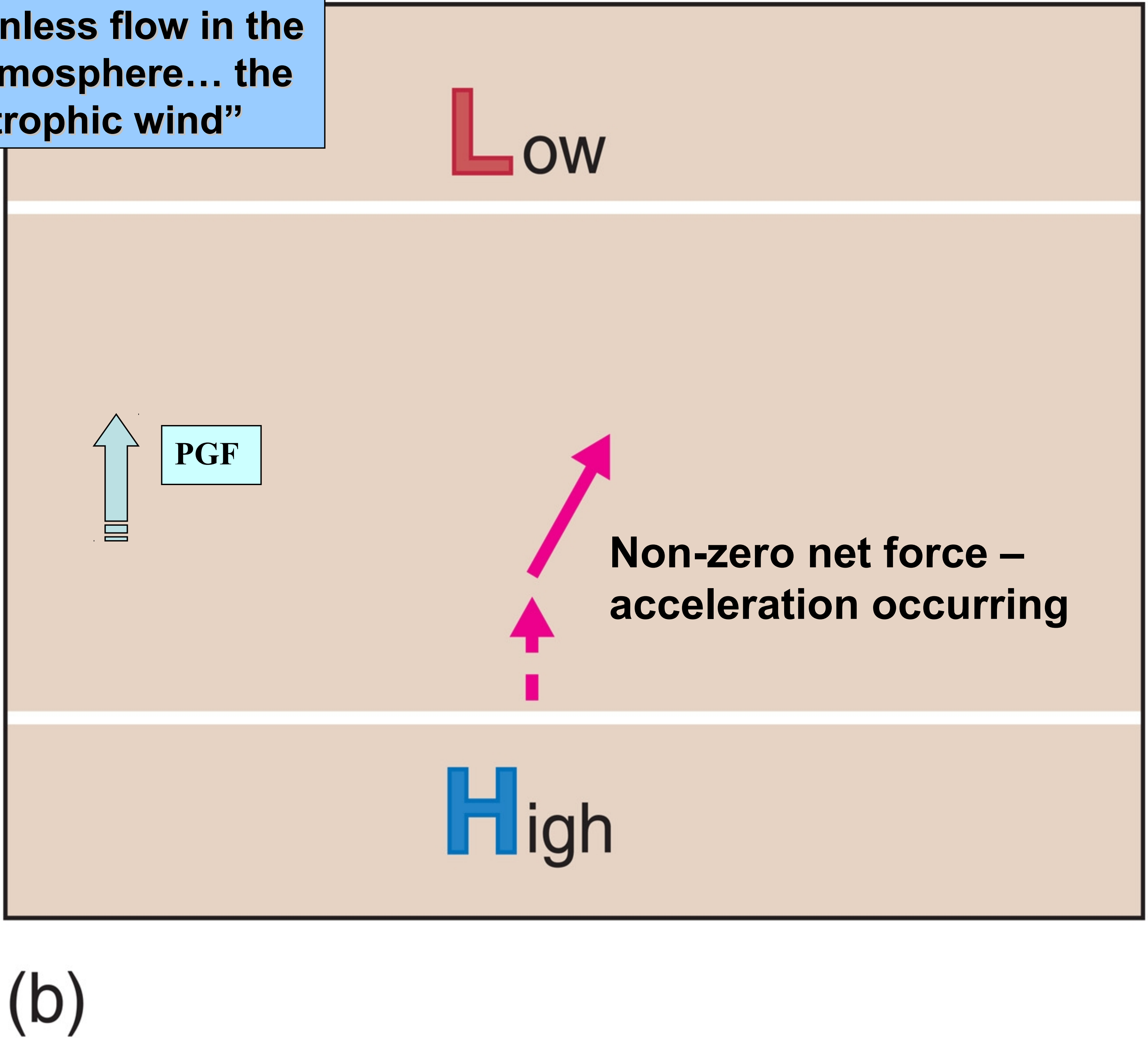
- "often the individual terms in the eqn of motion nearly cancel one another"
- even if the acceln is zero, this does not mean there is no wind

Frictionless flow in the free atmosphere… the "geostrophic wind"



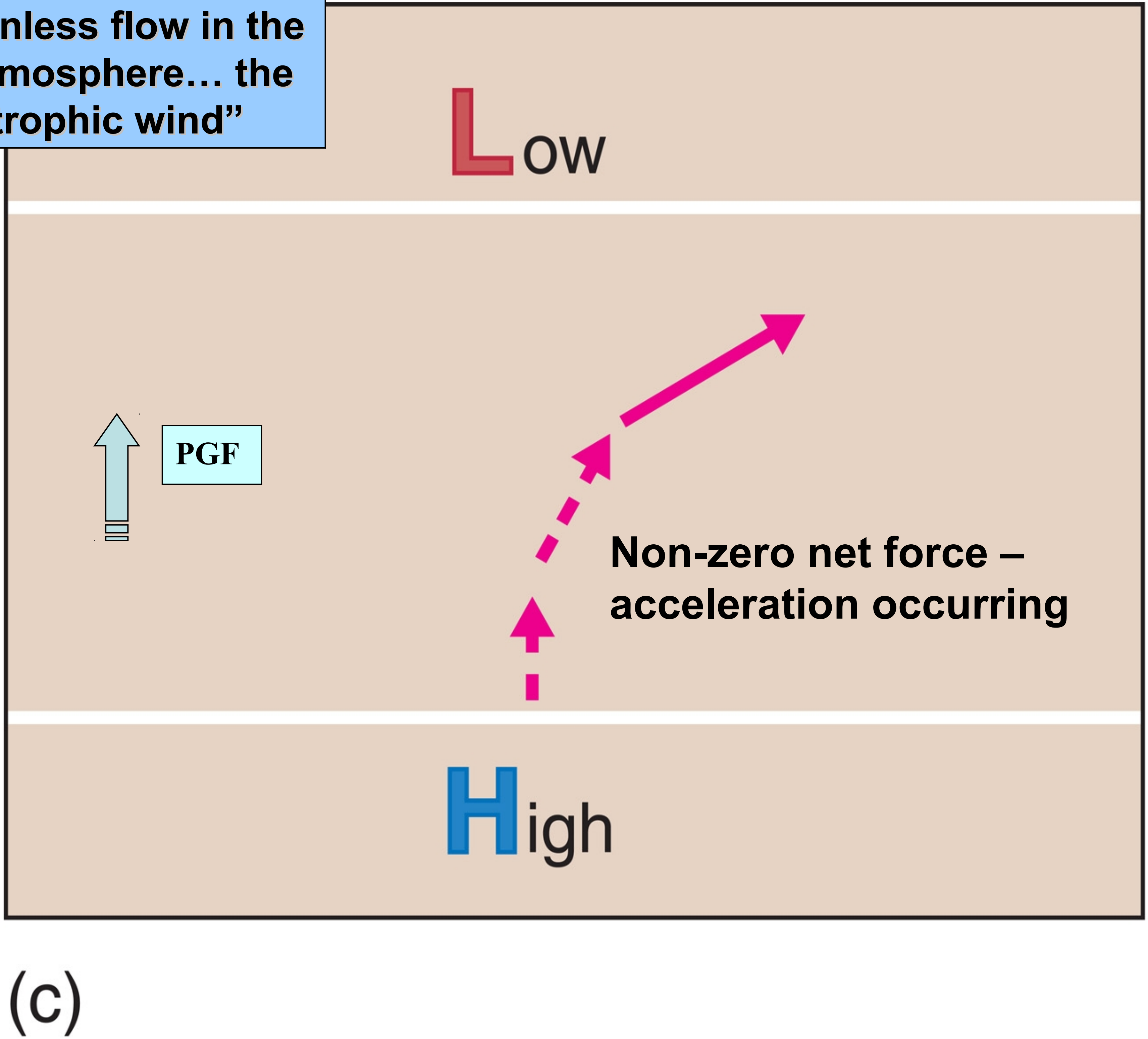
© 2010 Pearson Education, Inc.

Frictionless flow in the free atmosphere… the "geostrophic wind"



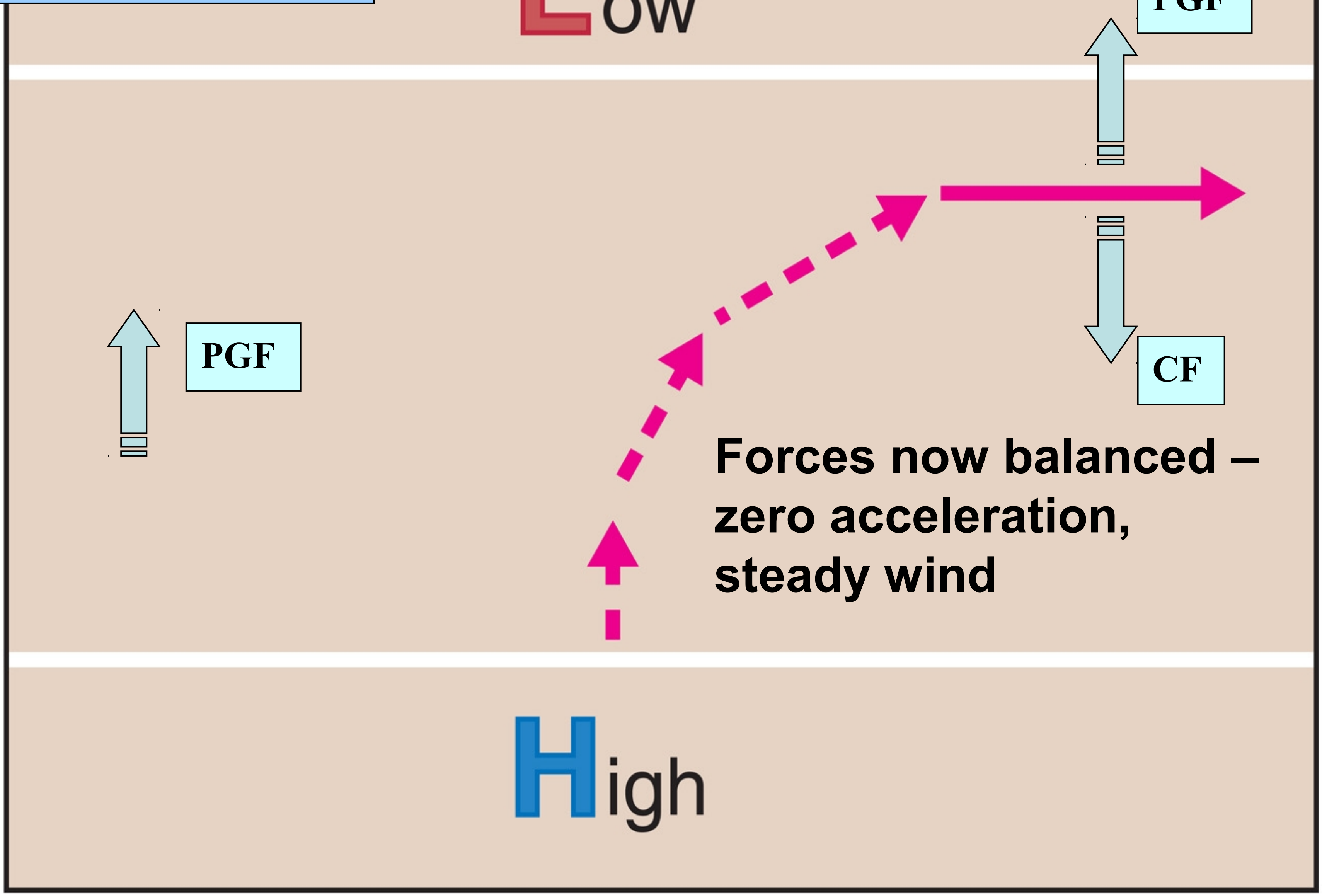
© 2010 Pearson Education, Inc.

Frictionless flow in the free atmosphere… the "geostrophic wind"



© 2010 Pearson Education, Inc.

Frictionless flow in the free atmosphere… the "geostrophic wind"



(d)

© 2010 Pearson Education, Inc.

Frictionless flow in the free atmosphere… the "geostrophic wind"

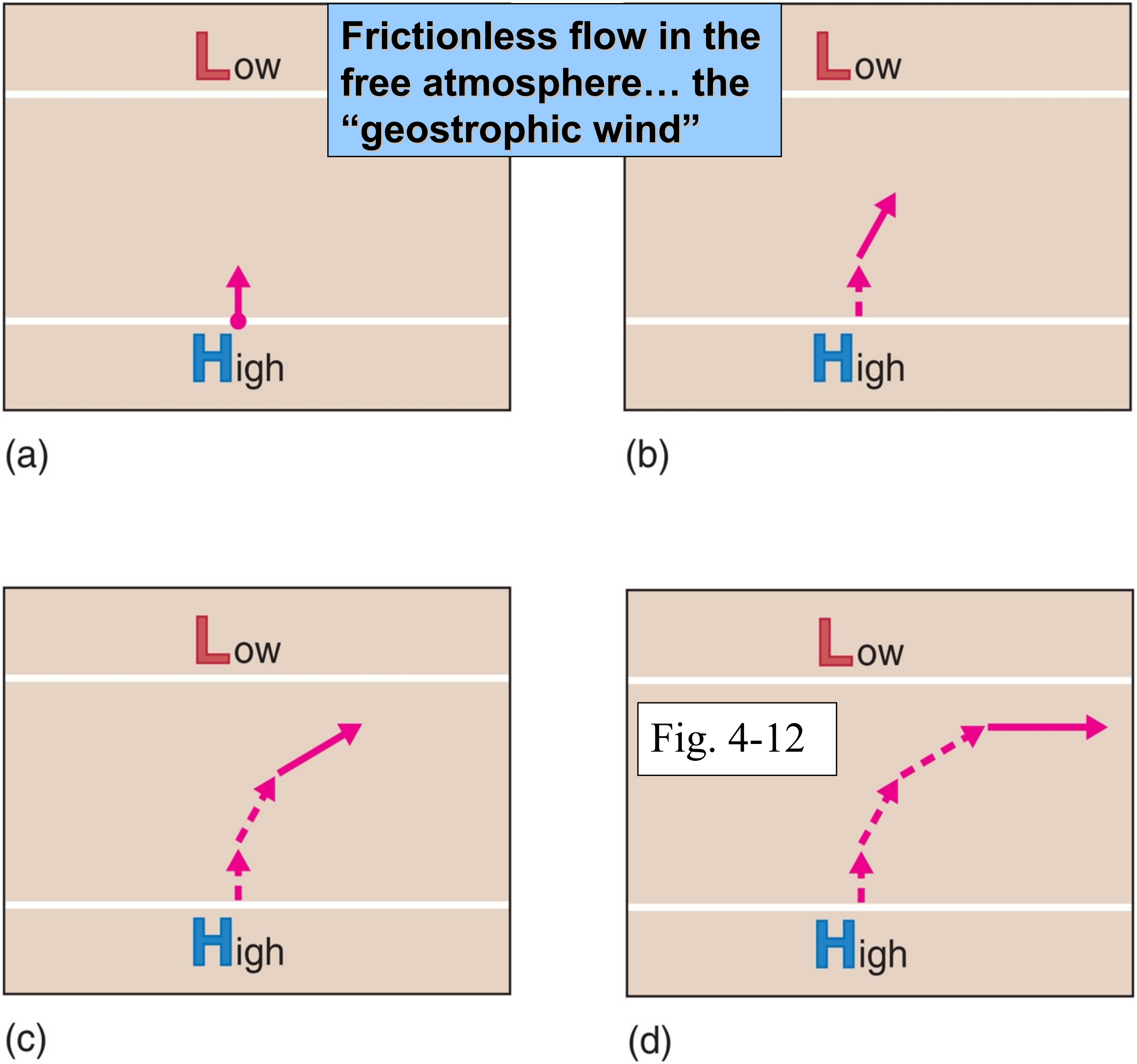


Fig. 4-12

The Geostrophic wind equation

Valid for balanced motion in the "free atmosphere" (no friction), and expresses the balance between:

$$V = \frac{g}{f}\,\frac{\Delta h}{\Delta n}$$

- Coriolis force

$$F_C = 2\Omega\, V \sin\phi \;, \qquad = f\, V$$

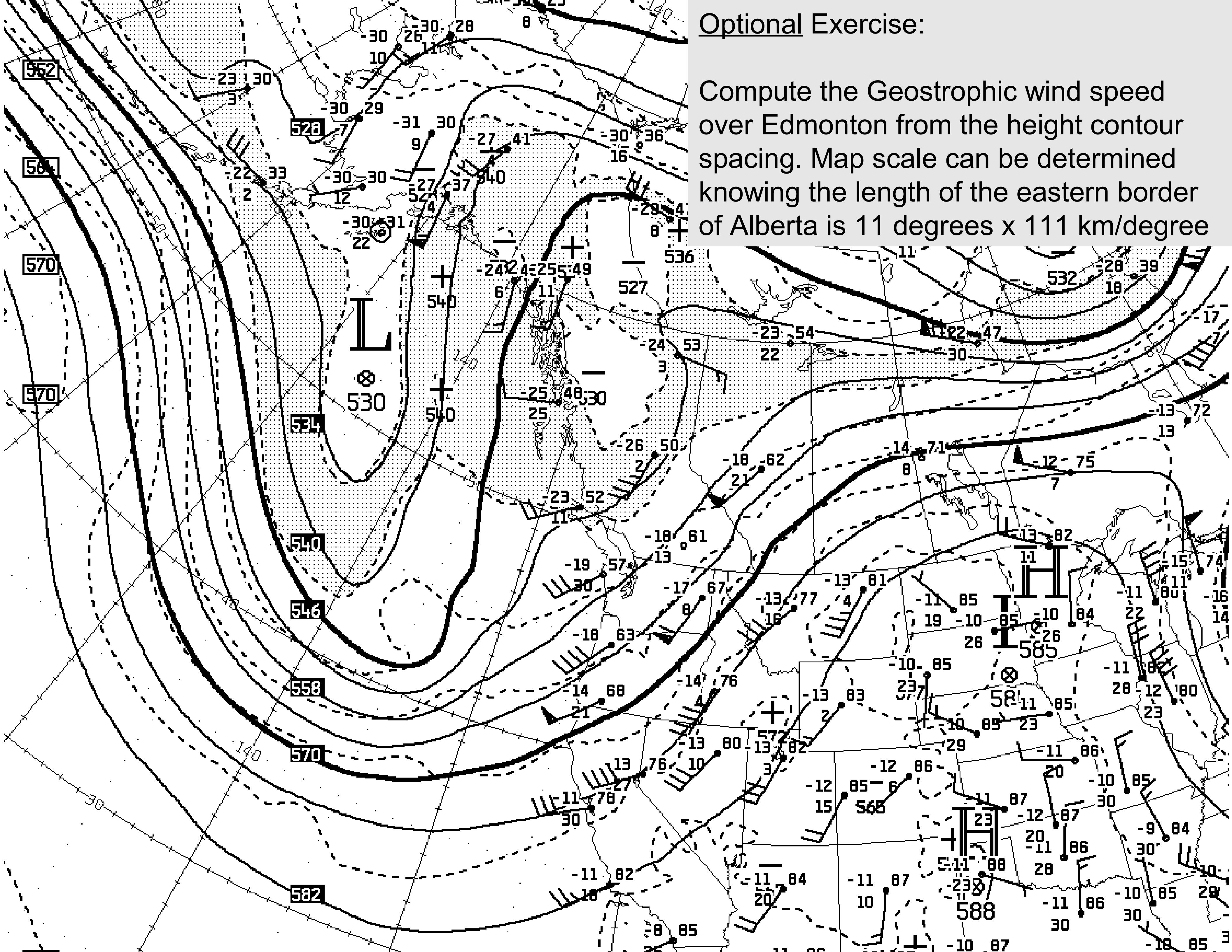
$$\left(f = 2\Omega \sin\phi \right)$$

- Pressure-gradient force

$$F_{PG} = \frac{1}{\rho}\,\frac{\Delta p}{\Delta n} = g\,\frac{\Delta h}{\Delta n}$$

height

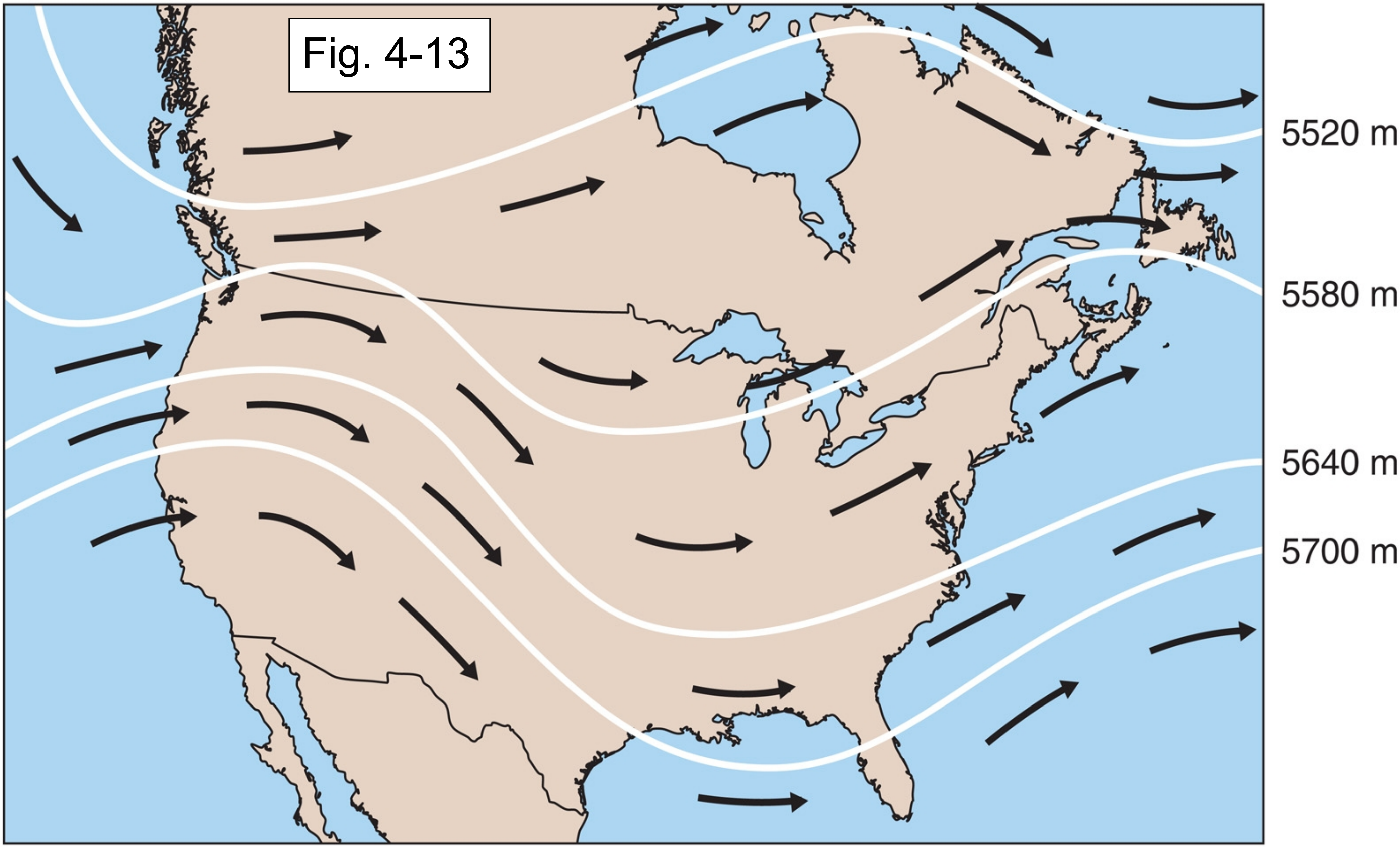
distance



<u>Optional</u> Exercise:

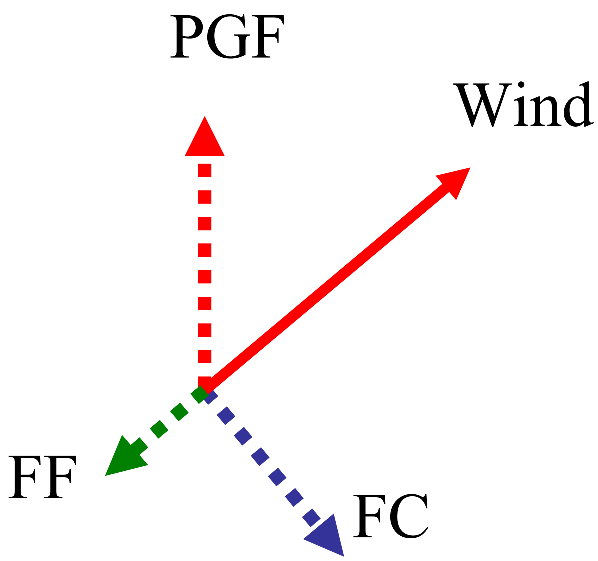
Compute the Geostrophic wind speed over Edmonton from the height contour spacing. Map scale can be determined knowing the length of the eastern border of Alberta is 11 degrees x 111 km/degree

Gradient wind in the free atmosphere. Slight imbalance between PGF and CF results in the accelerations that assure wind blows along the height contours (i.e. perpendicular to the PGF); in practise, Geostrophic model usually a very good estimator of the speed even along curved contours.



Influence of friction in the atmospheric boundary layer (ABL)

- reduces wind speed
- therefore reduces the Coriolis force
- which therefore cannot balance the PGF, so there is a component of motion *down* the pressure gradient



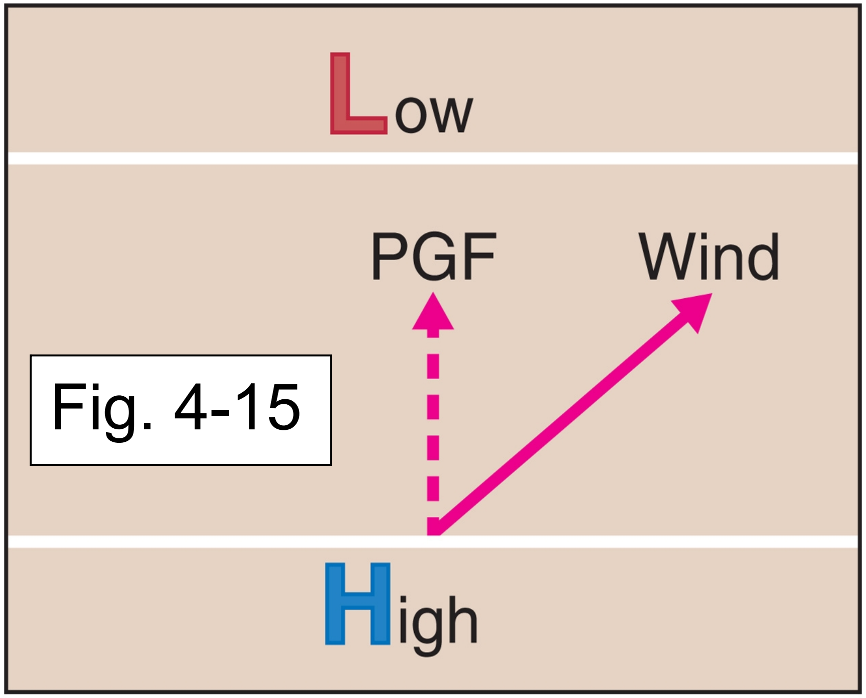


Fig. 4-15

- the resultant of FC and FF (Coriolis + friction) exactly balances PGF

In the N.H. free atmosphere, wind spirals anticlockwise about a centre of low pressure and parallel to contours. Within the ABL, due to friction a component across the isobars results: air “leaks” down the pressure gradient, and has “nowhere to go but up” (p122)

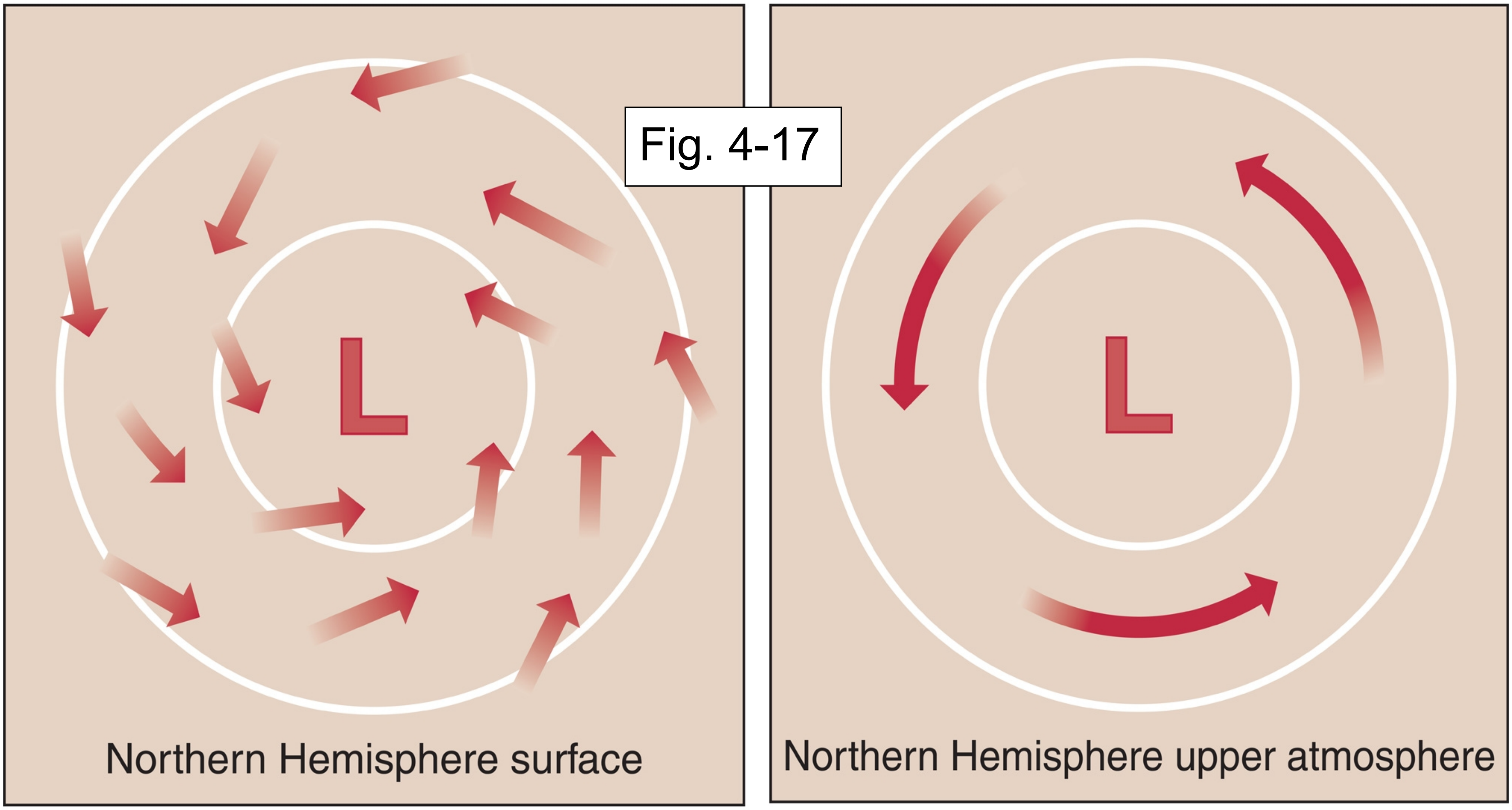


Fig. 4-17

In the N.H. free atmosphere, wind spirals clockwise about a centre of high pressure and parallel to contours. Within the ABL, due to friction a component across the isobars results: air “leaks” down the pressure gradient, and “is replaced by sinking air” (p122)

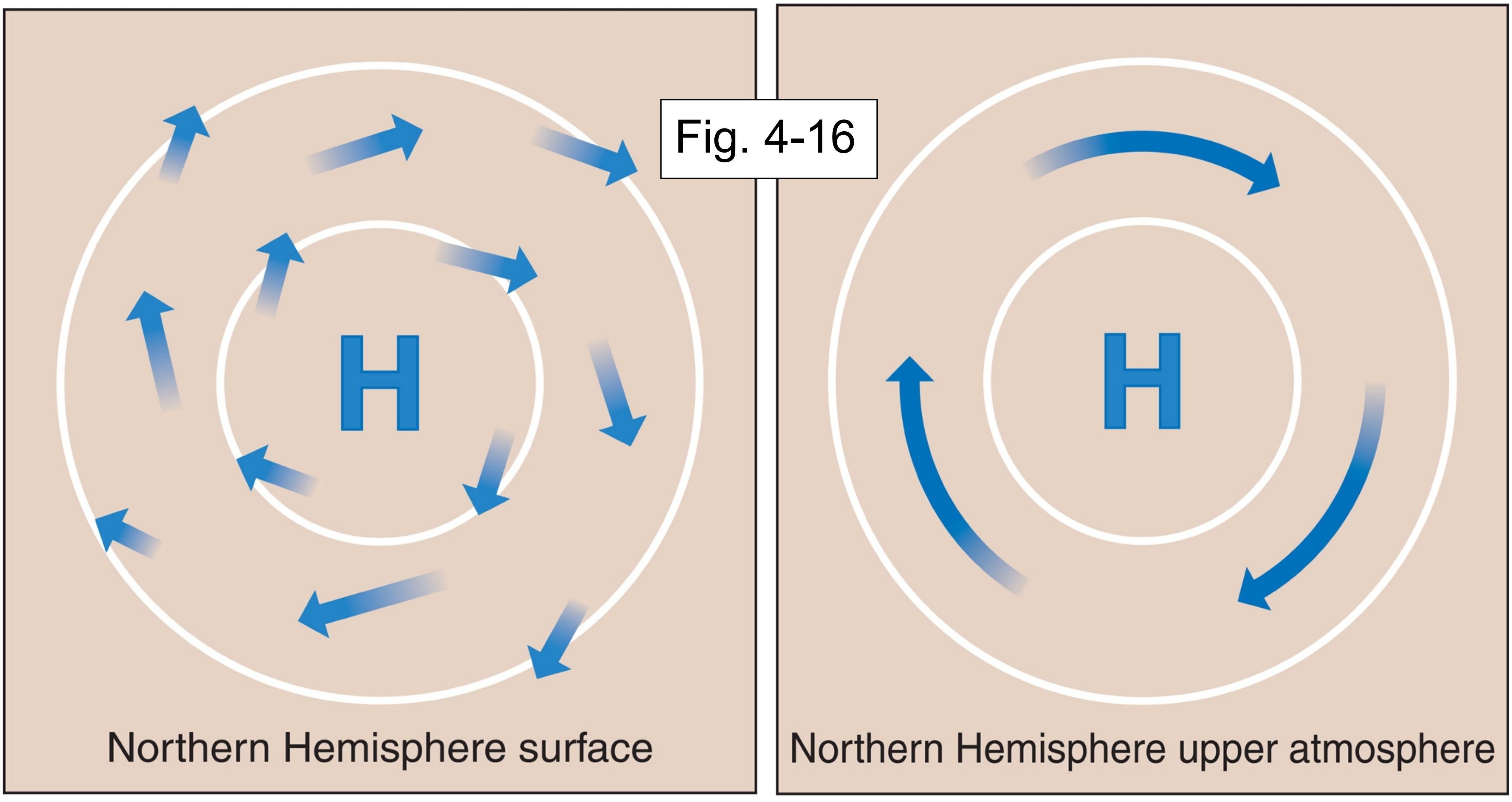


Fig. 4-16

Reminder: force balance in the vertical direction reduces to a "hydrostatic balance" (valid except in "sub-synoptic" scales of motion)

• pressure (p) decreases with increasing height (z)

• vertical pressure gradient force is (minus $1/\rho$ times) $\Delta p/\Delta z$ and <u>large</u>…

• why doesn't that PGF cause large vertical accelerations?

• because it is almost perfectly balanced by the downward force of gravity… this is "hydrostatic balance" and is expressed by the "hydrostatic equation",

$$\frac{\Delta p}{\Delta z} = -\rho g$$

• but in some smaller scale circulations (or "motion systems"), for example cumulonimbus clouds, the vertical acceleration must be accounted