Chapter 10. Precipitation.

- two droplet growth mechanisms operate, nominally responsible for precip from "warm" and "cold" clouds

- precip type at ground is controlled by temperature profile beneath the cloud

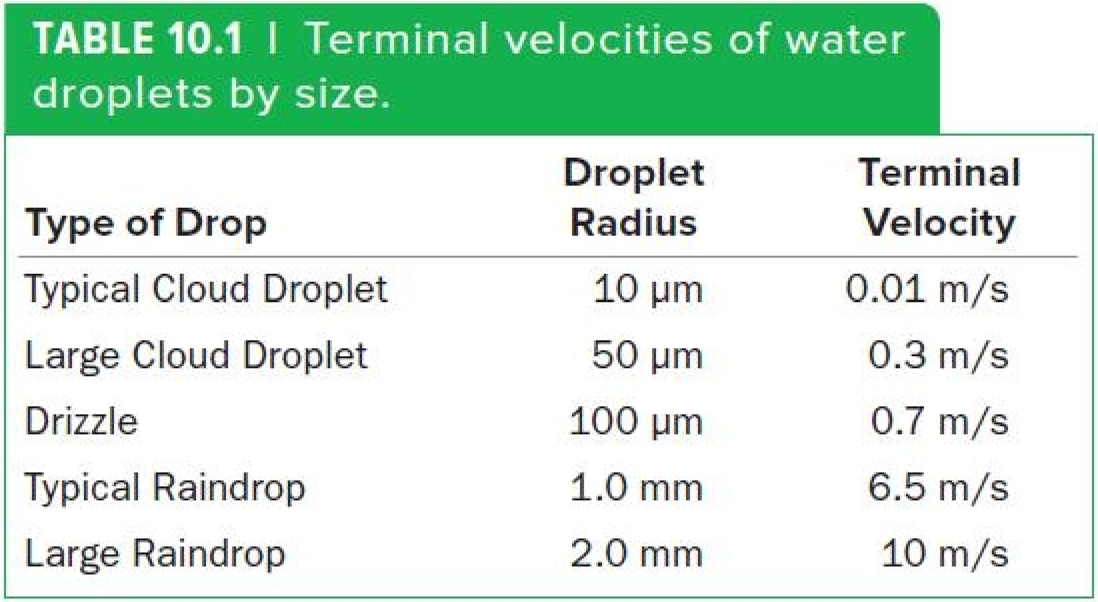
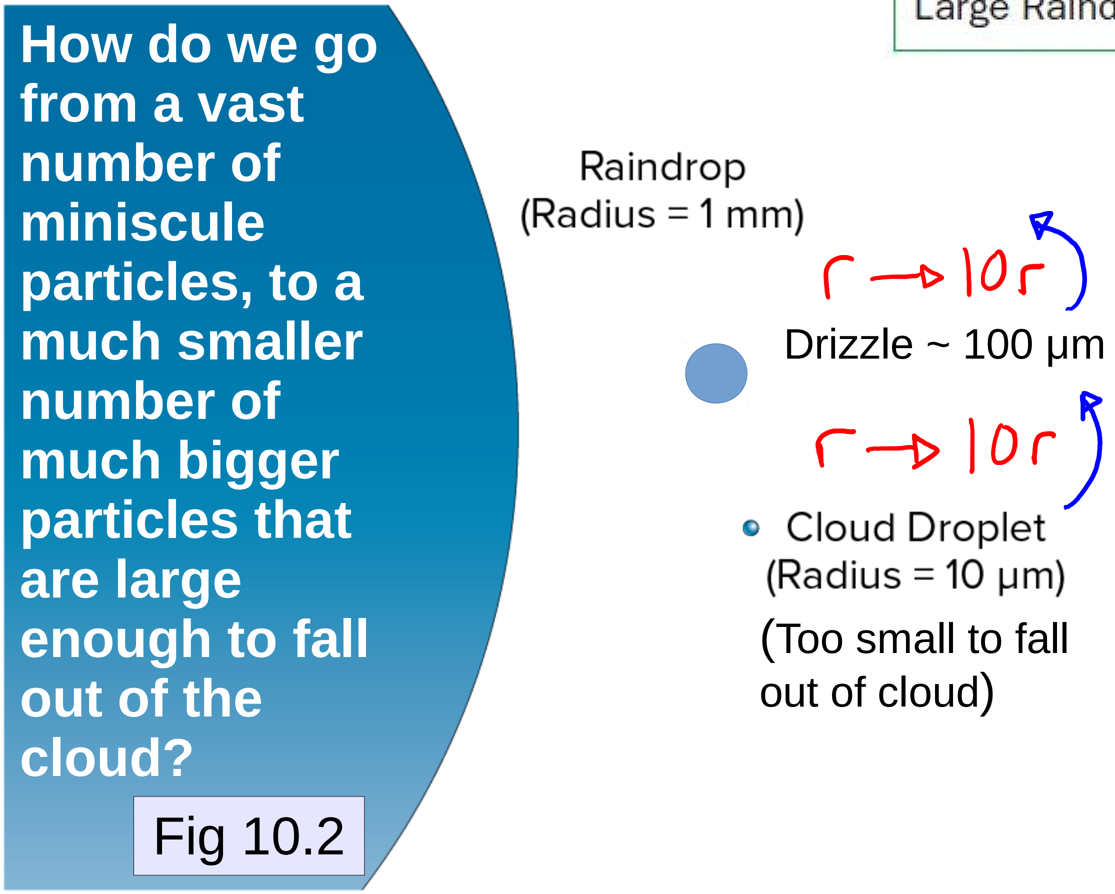


TABLE 10.1 | Terminal velocities of water droplets by size.

Type of Drop	Droplet Radius	Terminal Velocity
Typical Cloud Droplet	10 μm	0.01 m/s
Large Cloud Droplet	50 μm	0.3 m/s
Drizzle	100 μm	0.7 m/s
Typical Raindrop	1.0 mm	6.5 m/s
Large Raindrop	2.0 mm	10 m/s



How do we go from a vast number of miniscule particles, to a much smaller number of much bigger particles that are large enough to fall out of the cloud?

Fig 10.2

Raindrop
(Radius = 1 mm)

r → 10r

Drizzle ~ 100 μm

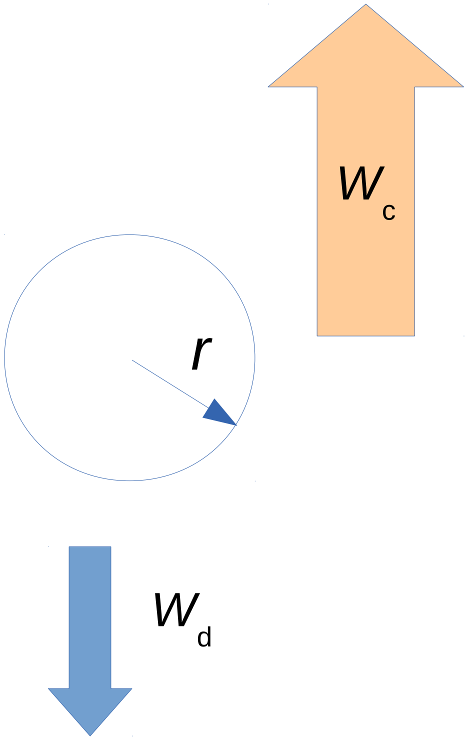
r → 10r

Cloud Droplet
(Radius = 10 μm)

(Too small to fall out of cloud)

Recall that vapour diffusion and deposition onto CCN tends to result in a population of small, equi-sized cloud droplets that compete for vapour and (collectively) deplete the cloud air of vapour – implying a limit to droplet growth by that mechanism

Ch10. "Terminal" droplet velocity from force balance on spherical droplet



$$\frac{dW_d}{dt} \propto \overbrace{\underbrace{\left(\frac{4}{3}\pi r^3\right)}_{\text{vol}} \rho_w}^{\text{mass}} g - \frac{1}{2} c_d \underbrace{\pi r^2}_{\text{frontal area}} \overbrace{\rho_a \left(W_c - W_d\right)^2}^{\text{pressure unit}}$$

For small slip velocities ($W_c - W_d$) the airflow past the droplet is viscous (i.e. laminar), and Stokes' law** applies,

$$c_d = \frac{12\nu}{\left(W_c - W_d\right) r}$$

(no units)

where ν [$m^2 s^{-1}$] is the "kinematic viscosity" of air.

Set the left hand side to zero (i.e. assume the two forces balance) and rearrange to get the terminal velocity (steady state relative velocity of the particle and the cloud air)

$$W_c - W_d = \frac{2}{9}\frac{r^2 \rho_w g}{\nu \rho_a}$$

Compute the fall speed for a droplet with 10 μm radius (assume $\nu \approx 1.3 \times 10^{-5}$ $m^2 s^{-1}$)

***The equations that express Newton's laws for "Newtonian fluids" (such as air, water) are named the "Navier-Stokes equations"*

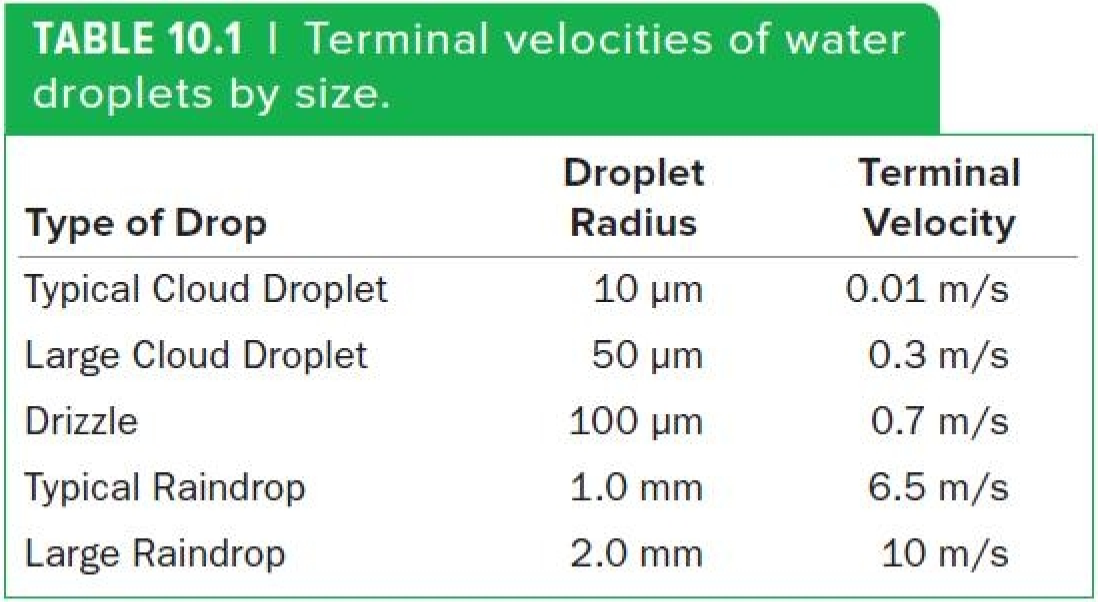
For larger drops (with large relative velocity) a more complex formulation is needed

Ch10. Typical updraft velocities vs. typical terminal velocities

Characteristic cloud updraft speeds:

- stratiform cloud $W_c \lesssim 0.1\ m\,s^{-1}$
- light cumulus $W_c \sim 1\ m\,s^{-1}$
- cumulonimbus $W_c \gtrsim 10\ m\,s^{-1}$
  (up to $40\ m\,s^{-1}$)

For precip to reach ground it must not only overcome the cloud updraft and escape – must survive sub-cloud evaporation enroute to ground



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The stronger the cloud updraft, the larger the droplets that may remain suspended in the cloud to undergo further growth –

– and this implies bigger droplets come from deeper clouds

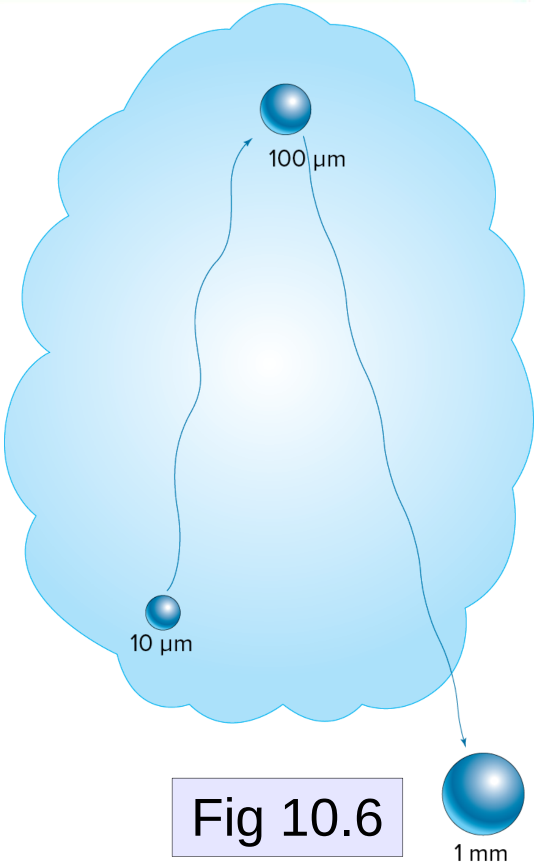
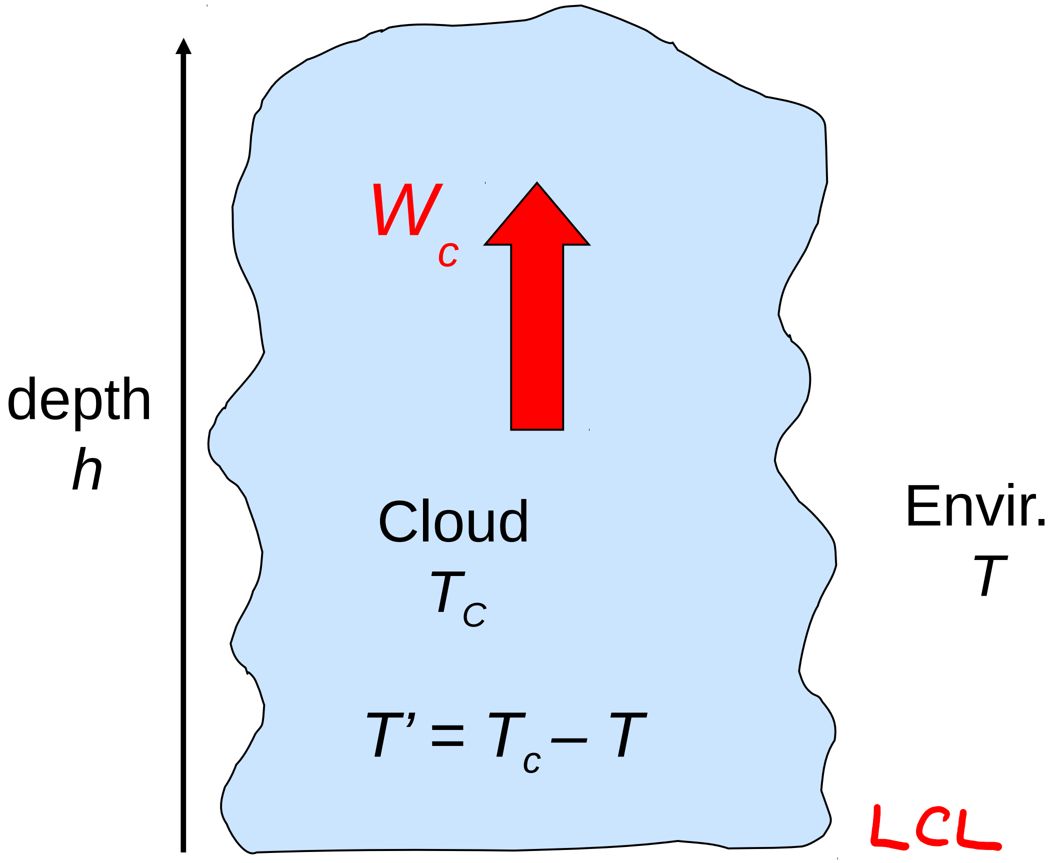


Fig 10.6

Ch10. Vertical velocity in a deep convective cloud – back-of-envelope calculation

- note: $T_C(z)$ follows moist adiabat
- accel'n: $A = g\,\dfrac{T_c - T}{T}$ "reduced gravity"

  in Kelvin
- rise time: $t = \dfrac{h}{W_c}$
- accel'n x time: $W_c = A\,t = g\,\dfrac{T_c - T}{T}\,\dfrac{h}{W_c}$
- rearrange: $W_c^2 = A\,t\,W_c = g\,\dfrac{T_c - T}{T}\,h$



Compute W_c for a cloud of depth h=10 km, assuming cloud air is 5K warmer than the environment

For particles smaller than (nominally) 20 µm, relative velocities are so small that collisions are rare

(if smaller particle is much smaller than the large one, it will follow the airstream and not coalesce)

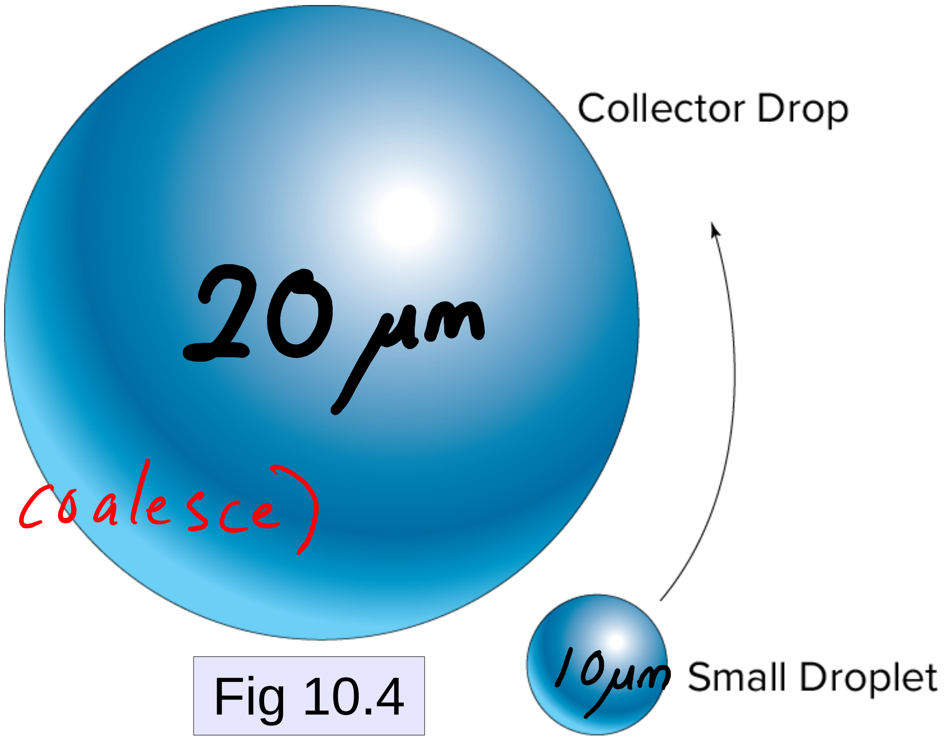


Fig 10.4

But provided *some* particles do grow to 20 µm, collisions allow them to collect (and coalesce with) other droplets

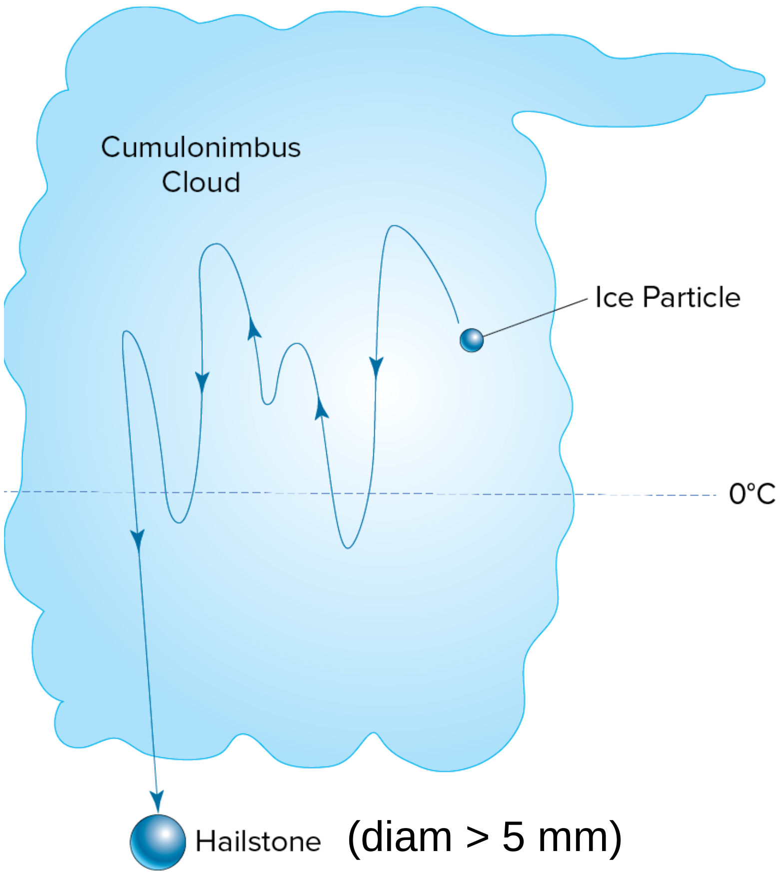
Deeper clouds with stronger updrafts (carrying this population of droplets aloft) permit the c&c process to continue for longer, producing bigger raindrops

<u>Steps in warm cloud precip process</u>:

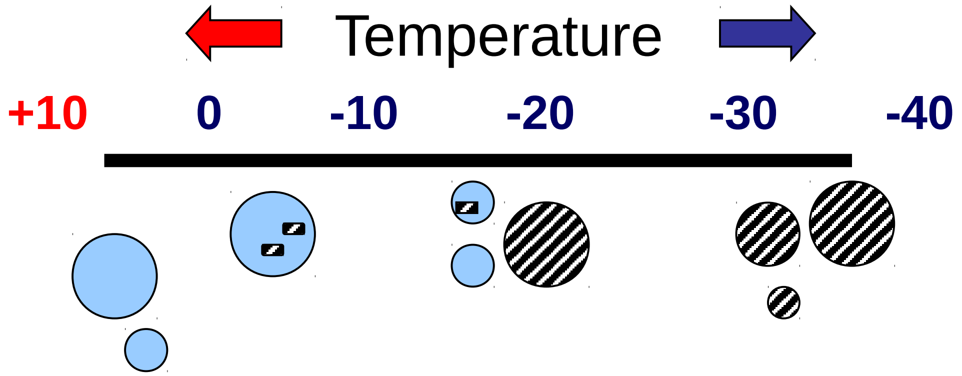
1. Cooling (ascent)
2. Nucleation by CCN
3. Diffusion to activated droplets
4. Collision & coalescence
5. Fallout (*perhaps* to ground)

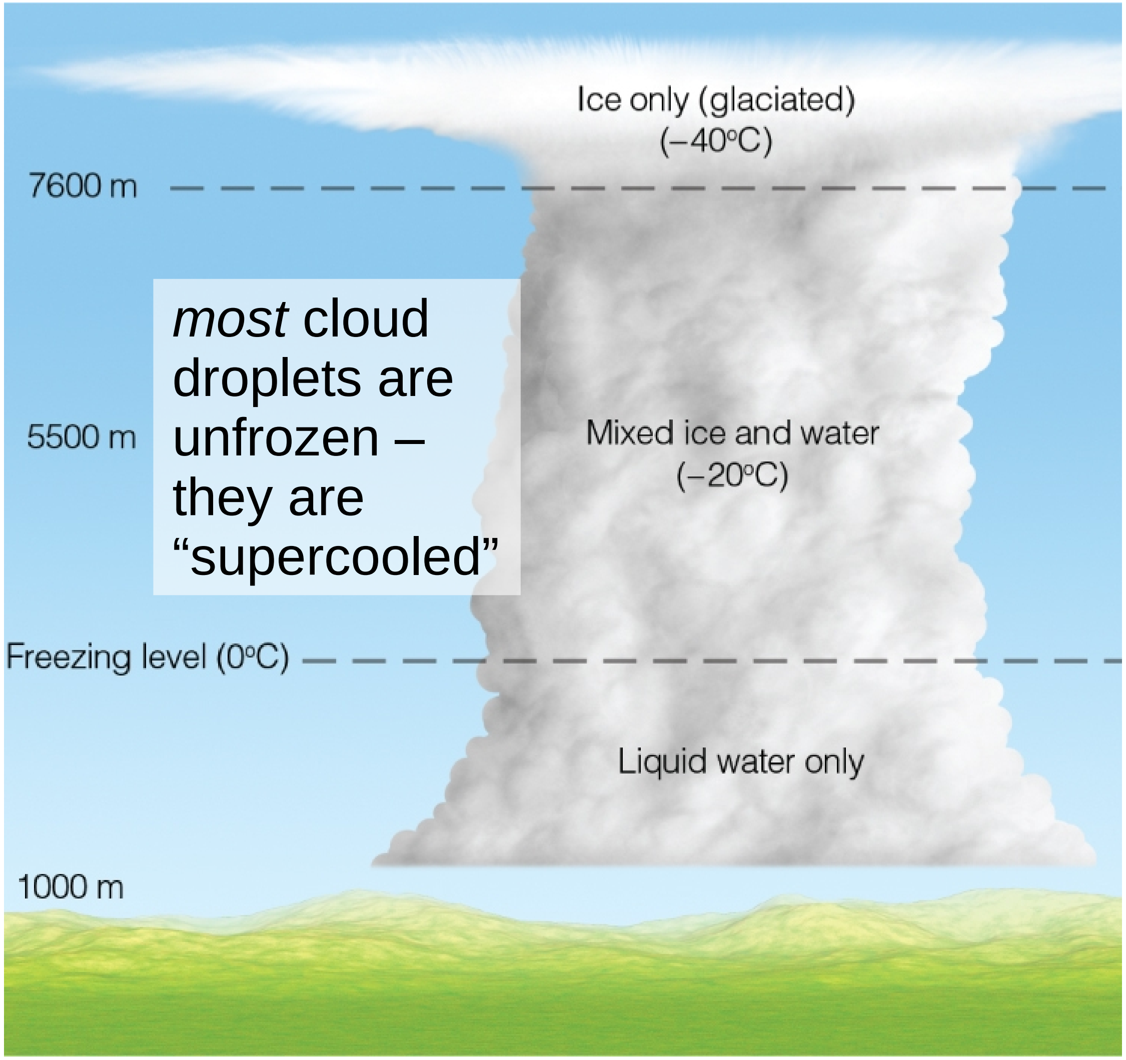
Ch10. The cold cloud (Bergeron-Findeisen/Ice crystal) process

T << 0°C at higher levels



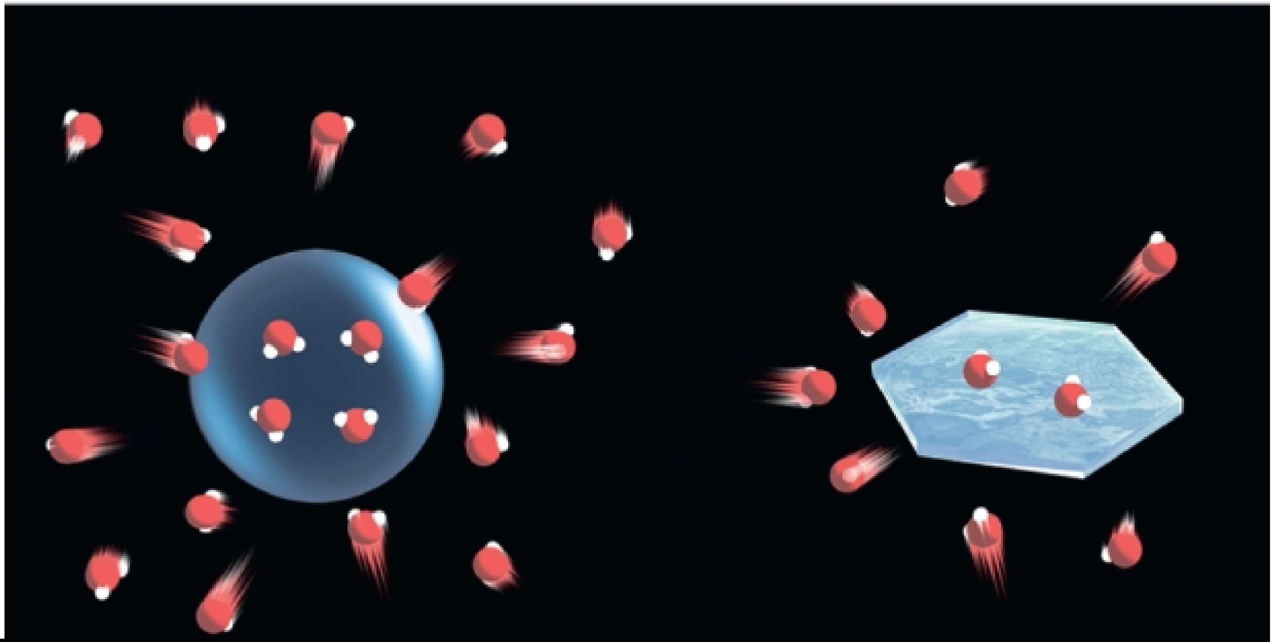
Hail – only produced by Cb clouds; long residence in violent updraft; growth by "accretion"; layering reflects passage through drier/wetter and warmer/colder regions of the cloud





most cloud droplets are unfrozen – they are "supercooled"

Ch10. Implication of co-existing liquid & ice crystals



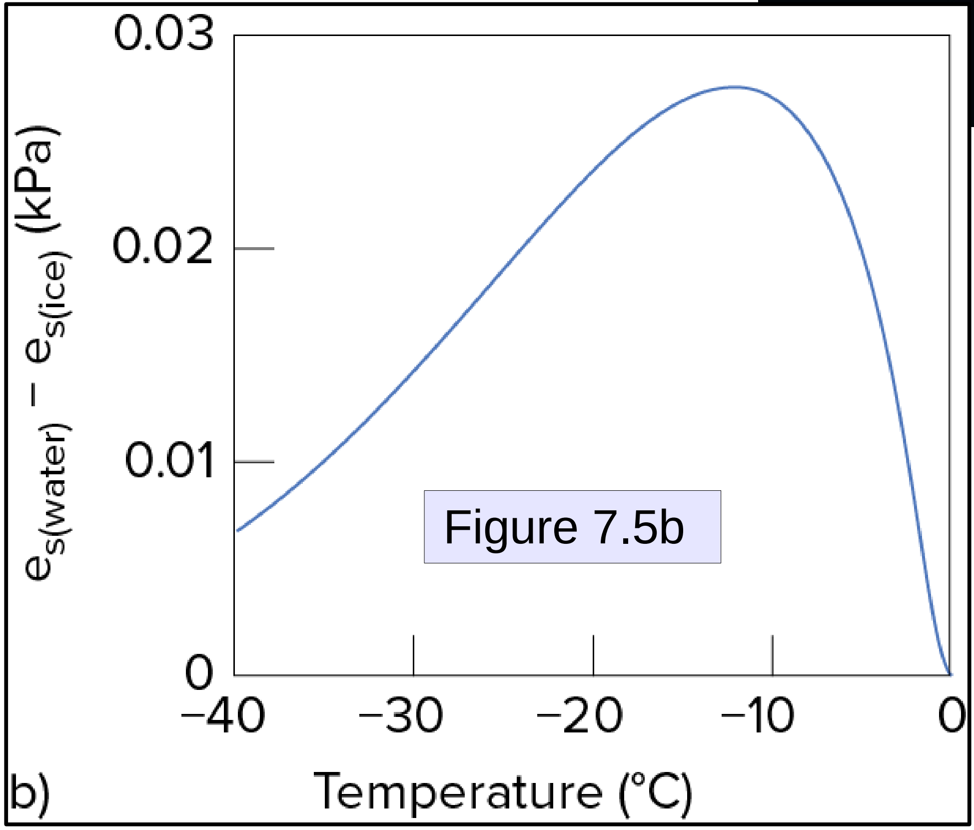
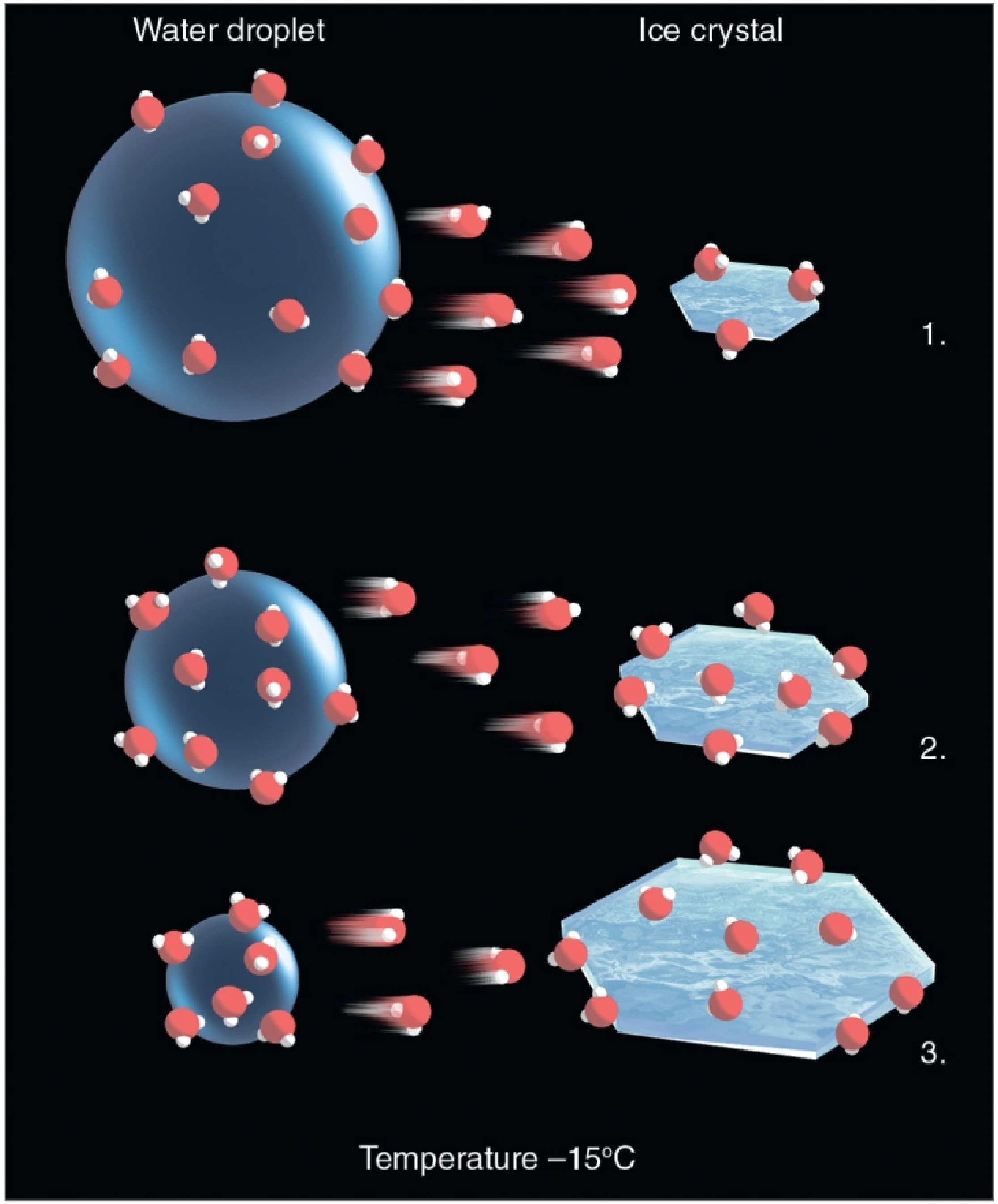


Figure 7.5b

Equilib. vap. pressure over water is higher than it is over ice

Water will migrate from supercooled drops to ice crystals.



- if vapour pressure is such that droplets are at equilibrium, result is a net flux of water vapour molecules onto the ice crystals

- this reduces the vapour pressure (i.e. dries the air)

- therefore water will evaporate off the droplets, driving the vapour pressure back up (rehumidifying the air)

- crystals grow at the expense of droplets, which shrink

- Accretion – ice crystals collide with and collect supercooled droplets that then freeze

- Aggregation – ice crystals collide with and collect other ice crystals, producing snow

Accretion – also known as "riming" – results in graupel pellets or hail

Am. Meteo. Soc. Glossary:

graupel: diam. < 5 mm

hail: diam. > 5 mm

Ch10. Precip types from cold clouds

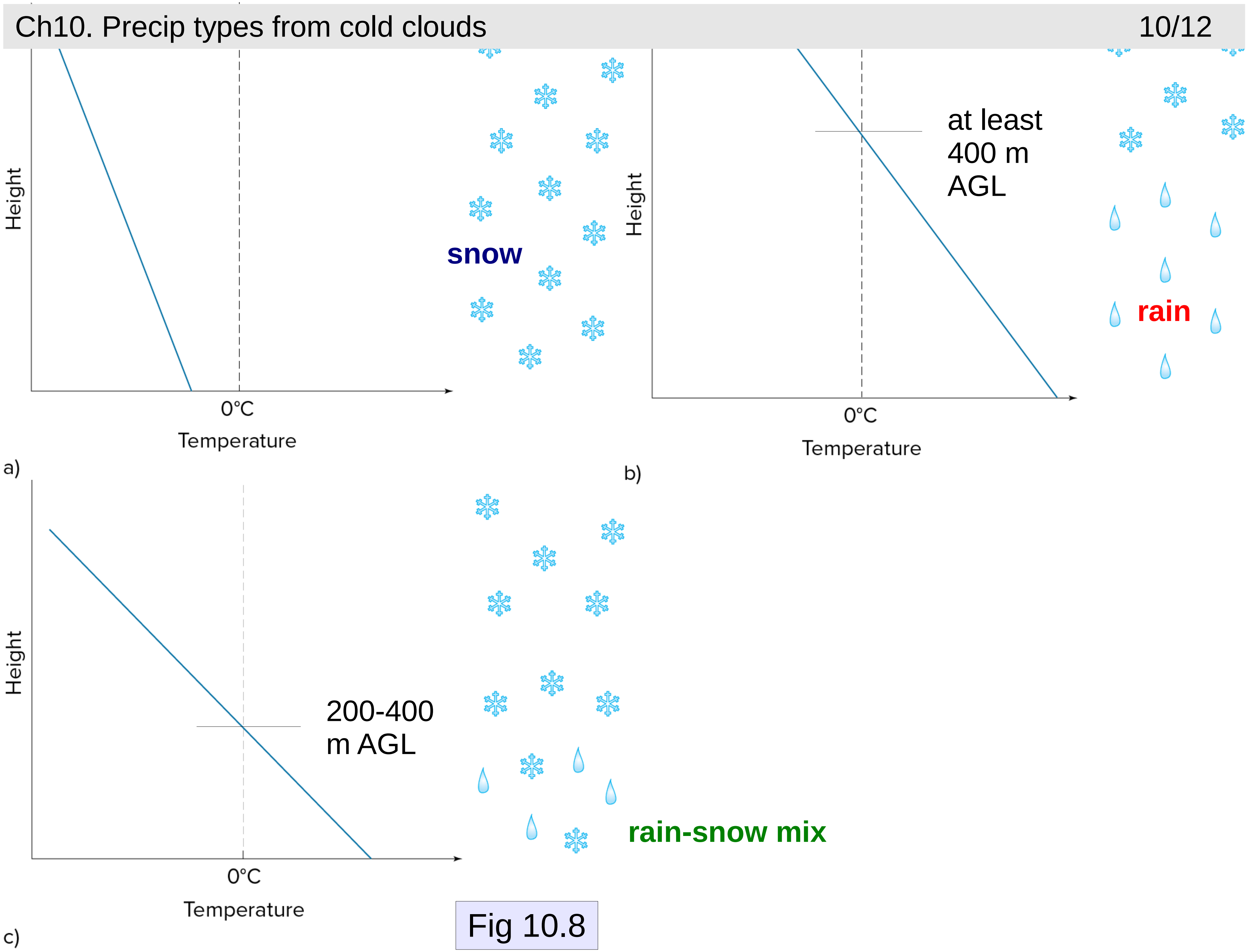


Fig 10.8

Ch10. Precip types from cold clouds: sleet vs. freezing rain

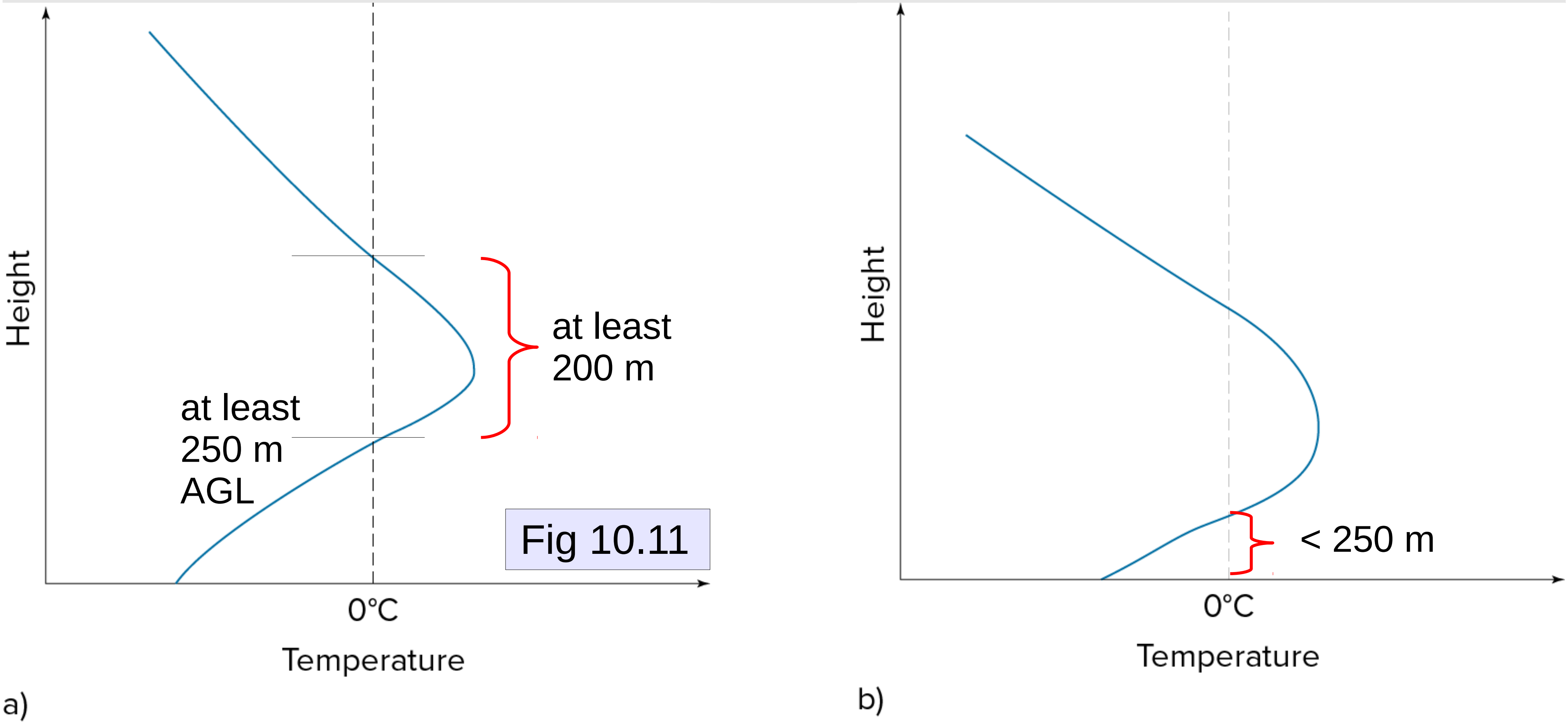


Fig 10.11

Sleet: precip melts in the warm layer then refreezes into **ice pellets** while falling through a sufficently cold and deep surface layer

Freezing rain: precip melts in the warm layer but does not refreeze until contacting the surface

Ch10. Factors controlling survival of a falling raindrop

Suppose a snowflake descends from a cloud and melts, forming a raindrop of mass *m* and

size $$r \sim \left[m / \left(\rho_w \frac{4}{3} \pi \right) \right]^{1/3}$$

falling at relative velocity *W* with respect to the air.

What factors affect whether this drop will survive to reach ground?

Simplified energy balance (steady state, neglect radiative heat exchange):

$$0 = Q_H + \boxed{Q_E}$$

$L_v E$

$$Q_H \propto W(r)\ [T_a - T_{\mathrm{sfc}}]$$

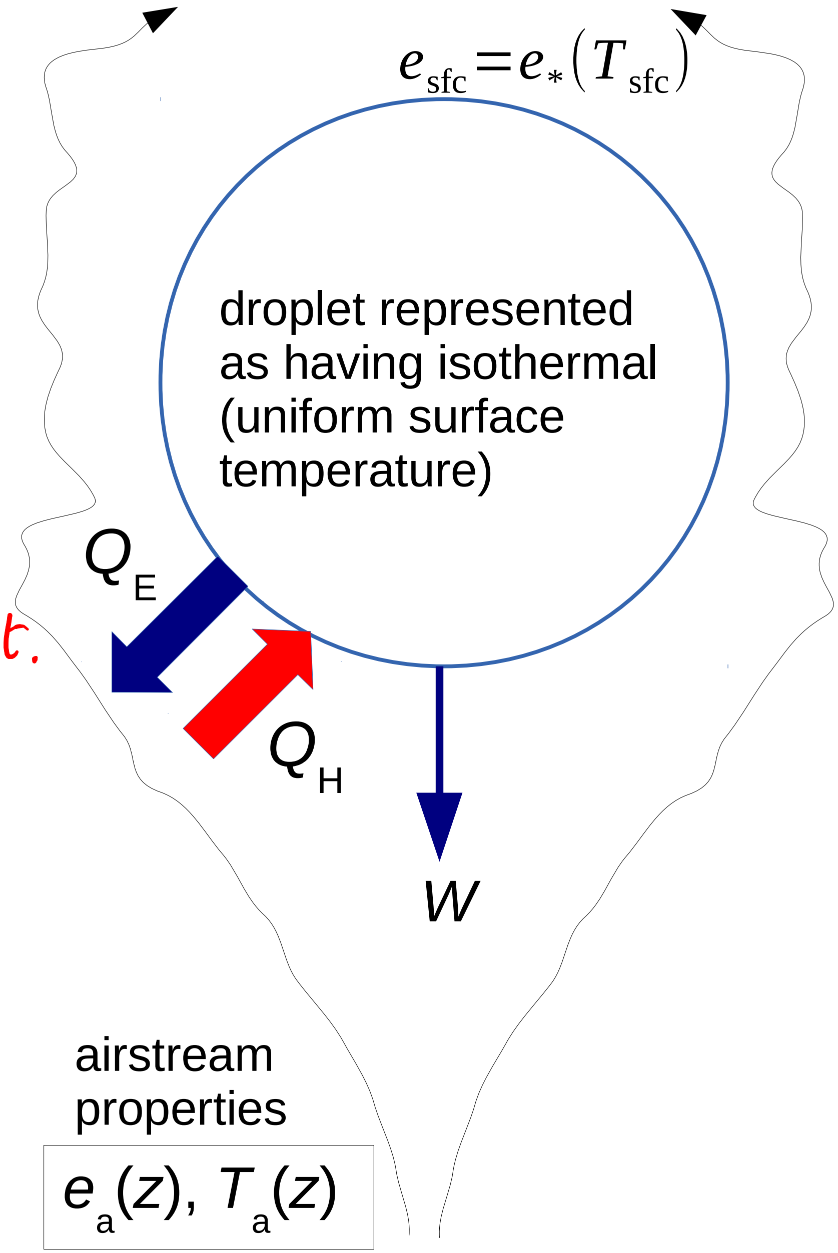
prop. const.

$\times c_p \rho_a$

$$Q_E \propto L_v W(r)\ [e_a - e_*(T_{\mathrm{sfc}})]$$

$$r = r(t), W = W(t), T_{\mathrm{sfc}} = T_{\mathrm{sfc}}(t), \ldots$$

Factors: starting distance above ground, starting radius, profiles of temperature and humidity...



$e_{\mathrm{sfc}} = e_*(T_{\mathrm{sfc}})$

droplet represented as having isothermal (uniform surface temperature)

airstream properties

$e_a(z)$, $T_a(z)$

Topics/concepts covered

- relative sizes of cloud droplets, drizzle and raindrops
- order of magnitude of updraft versus cloud type
- definition of terminal velocity and its relation to the vertical force budget on droplet
- the warm- and cold-cloud processes that lead to droplets of a size sufficient to precipitate from the cloud
- energy budget of a falling raindrop
- temperature profile near ground in relation to precip types from cold clouds