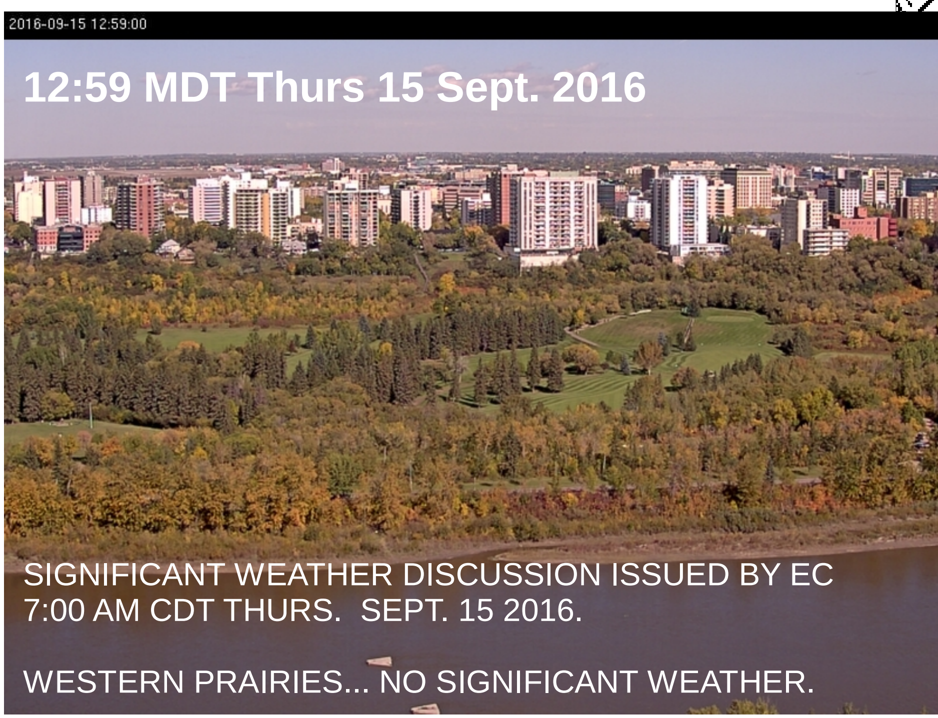
EAS 270, 16 Sept. Chapter 3, Behaviour of the Atmosphere

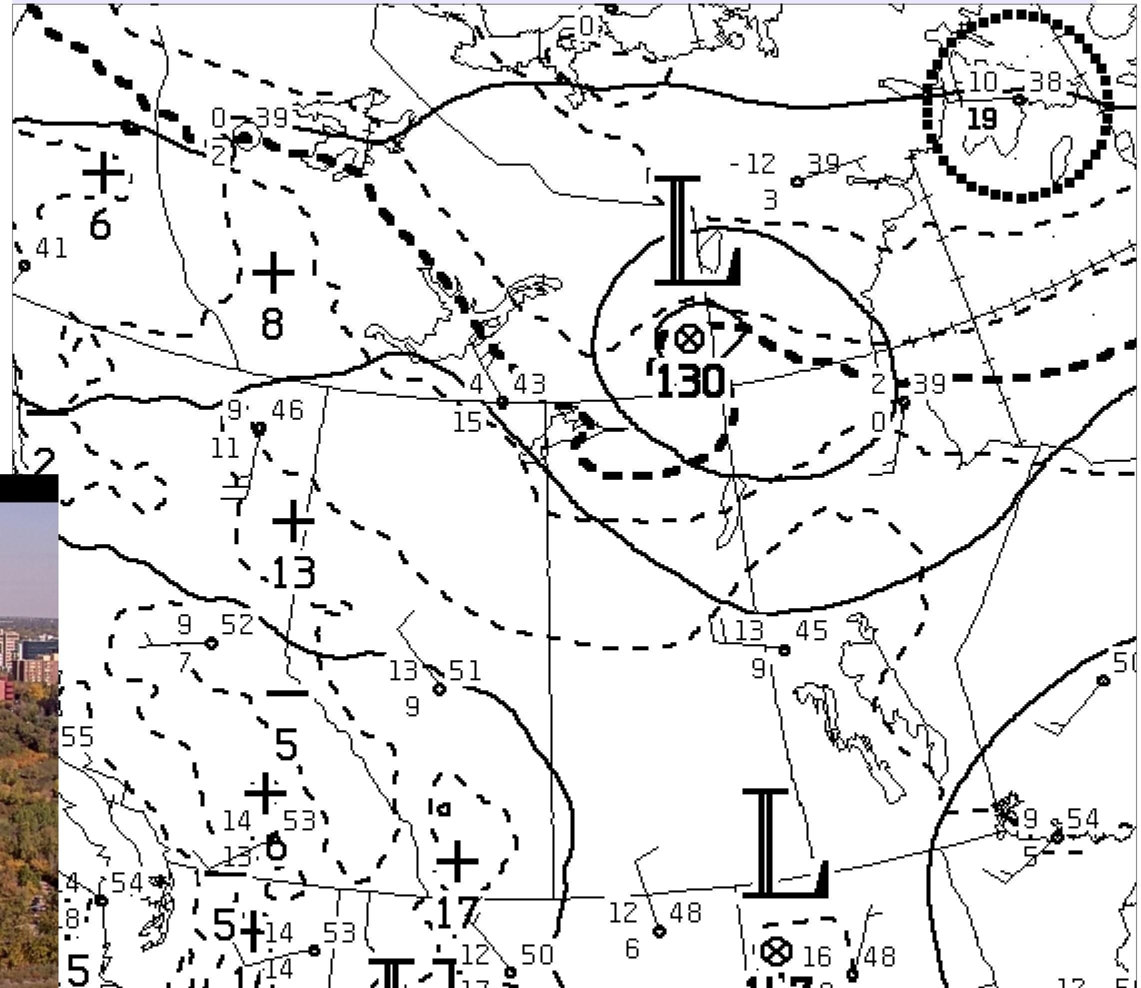
EAS270_Ch3_BehaviourAtmos_A.odp
JDW, EAS UAlberta, last mod. 16 Sept. 2016

Focus of Chapter 3 is the *physical* state of that mixture of gases we call "air" (more specifically the thermodynamic state, excluding variables such as velocity (u,v,w) which in a broader sense are also state variables).

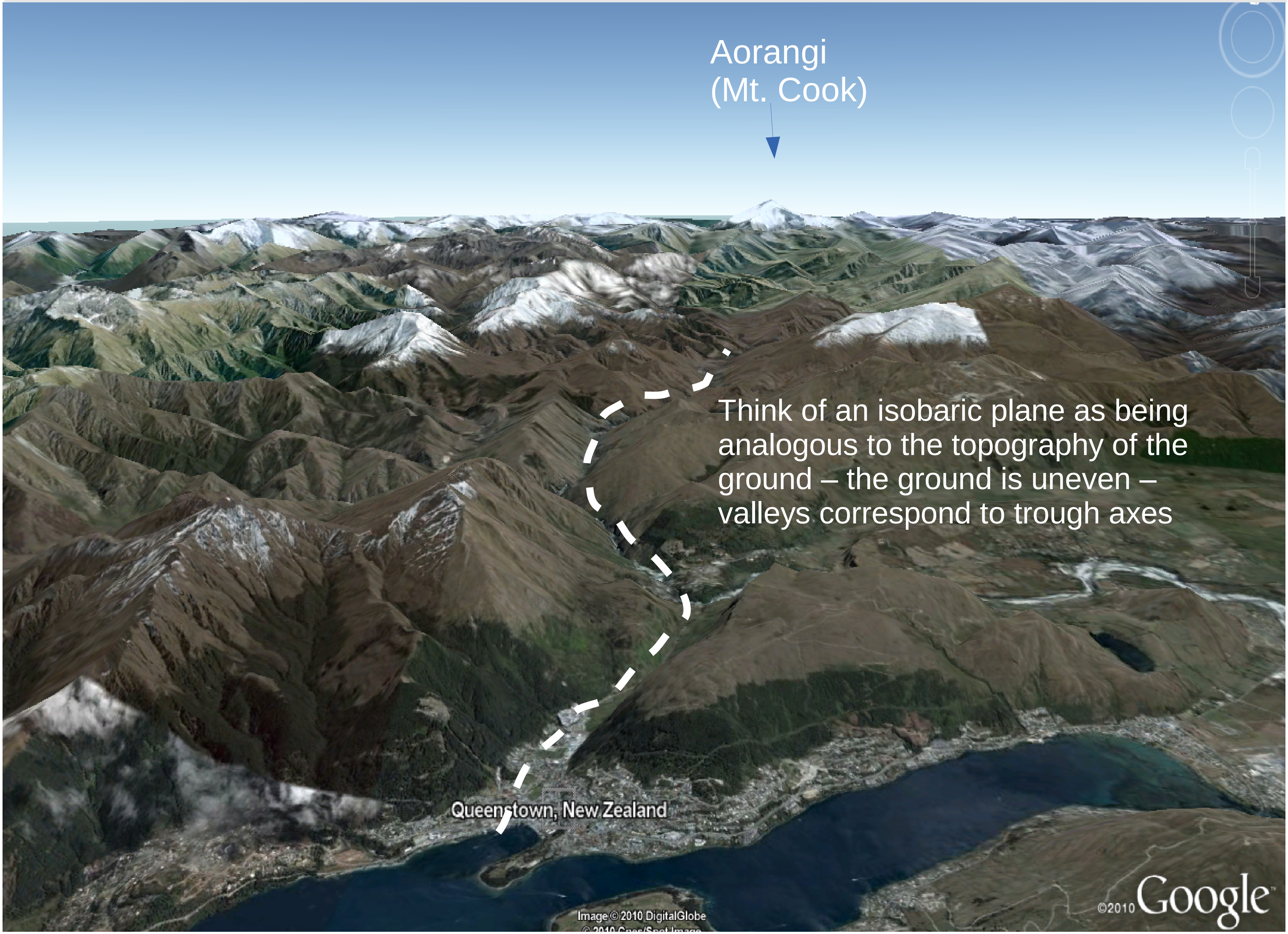
- low in NWT pulling cool air into Saskatchewan
- too far from C. Alberta to have had much effect here. Wind over Edmonton ("YEG") is a ?? at ?? m/s
- T_{850}=13^o, $(T-T_d)_{850}$=9^o at YEG



CMC 850 hPa analysis 12Z Thurs 15 Sept. 2016



- at Edmonton, 850 hPa surface is ?? m above sea level (ASL) and ?? m AGL


Aorangi
(Mt. Cook)
Think of an isobaric plane as being analogous to the topography of the ground – the ground is uneven – valleys correspond to trough axes
Queenstown, New Zealand
Image © 2010 DigitalGlobe
©2010 Google

- for a given gas, and for air as a whole, there are three useful state variables. These are connected by the ideal gas law, so only two are "independent" (i.e. given any pair, you can compute the third)

- the ideal gas law is "ideal" in the sense that (in principle) it applies to a gas of point objects (having mass, but no volume) that interact only by elastic collisions

- but it is a good approximation for the atmospheric gases** under natural conditions, for which the mean free path between collisions is vast compared to molecular diameter

(**N_2, O_2 are diatomic gases)

Universal form of the ideal gas law:

$$P\ V = n\ R^*\ T$$

where $R^* = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$

is the universal gas constant and *n* is the number of moles of the gas. Then for a mixture of gases

$$P = \frac{n}{V}\ R^*\ T = \frac{m}{V}\ \frac{R^*}{m/n}\ T$$

where *m* [kg] is the mass of gas in *V*.

Defining density $\rho = m/V$ then

$$P = \rho \frac{R^*}{\cancel{MW}}\ T$$

specific gas constant

MM

where *MM* [kg mol^{-1}] is the molar mass.

$$P = \rho \frac{R^*}{MM} T$$

specific gas constant R

Taking air as 78% N_2 and 21% O_2 and 1% Ar,

$$MM = \frac{0.78\times(14\times2)+0.21\times(16\times2)+0.01\times40}{1000}$$

$$= 0.02896 \quad \mathrm{kg\ mol^{-1}}$$

Ideal gas law for dry air:

$$\boxed{P = \rho \; R_d \; T}$$

$$R_d = \frac{R^*}{MM} = 287 \; \frac{\mathrm{J}}{\mathrm{kg\ K}} \sim \frac{8.314}{.02896}$$

- when water vapour is added to air at constant pressure, the density of the mixture decreases (*MM* of water is 18 g/mol, smaller than that for dry "air" which we've evaluated as being about 30 g/mol)

- thus adding water vapour has an effect on density that is analogous to raising temperature
- thus define "virtual temperature"

$$T_V = T \; (1+0.61 \; r)$$

where *r* <<1 is the mixing ratio of water vapour (gram/gram or kg/kg) and temperatures are Kelvin.

Ideal gas law for moist air:

$$\boxed{P = \rho \; R_d \; T_V}$$

$p = \rho R_d T$

Example 3.1: What is the density of a parcel of dry air that has a pressure of 98 kPa and a temperature of 32°C?

$T = 273.15 + 32 = 305.15$

$R_d = 287$

$\rho =$

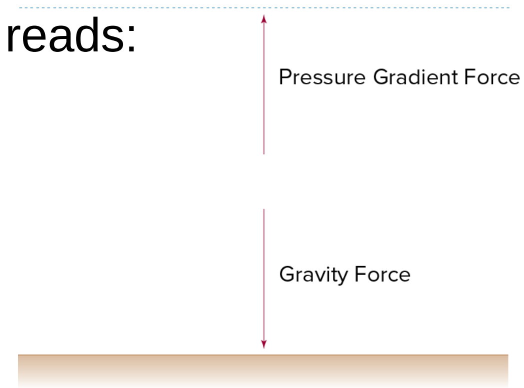
What is pressure of a parcel of dry air that has a density of 0.5 kg m^{-3} and a temperature of -20°C?

What is the density of a parcel of dry air whose pressure is 85 kPa and whose temperature is -10°C?

In verbal form, Newton's law for vertical acceleration of an air "parcel" reads:

$$\text{vertical acceleration} = \frac{dw}{dt} = \frac{\text{net force}}{\text{unit mass}}$$

$$= \text{Pressure Force} + \text{Gravity Force} + \text{smaller terms}$$



With *dw*/*dt*=0 ("static") and neglecting the small terms, PF and GF are equal and opposite...

Net upward force

common factor

$$0 = (P+\Delta P)\cancel{A} - P\cancel{A} - g\rho \cancel{A}|\Delta z|$$

$$0 = \Delta P - g\rho|\Delta z|$$

$$\frac{\Delta P}{|\Delta z|} = \rho g$$

$$\boxed{\frac{\Delta P}{\Delta z} = -\rho g}$$

positive Δz means increase in elevation

$$M = \rho V = \rho A|\Delta z|$$

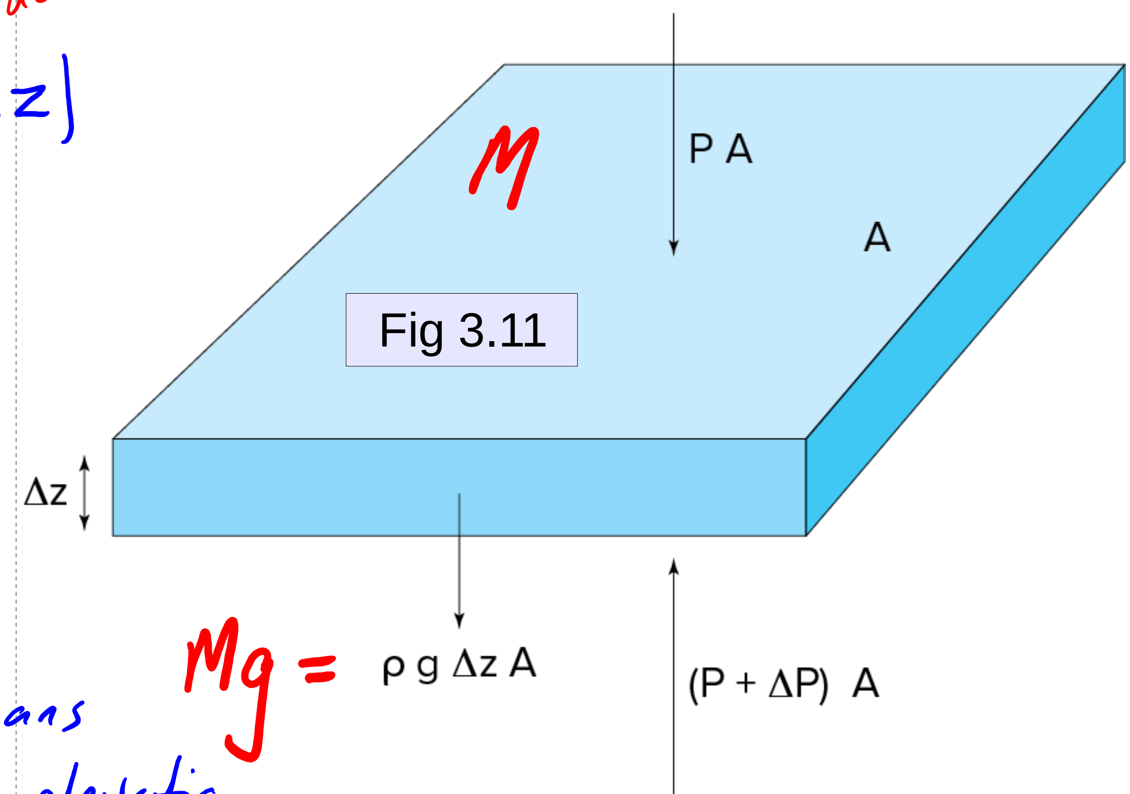


Fig 3.11

Distance from the northern border (60°N) to the southern border (49°N) of the prairie provinces is: 11° x 111 km per degree = 1221 km

Reminder: Appendix (p479) decodes symbols on the weather charts – and a file on eClass lists those you must be able to recognise and decode

- from end to end of the red arrow, distance is of the order of 1000 km
- from end to end of the red arrow, pressure changes by

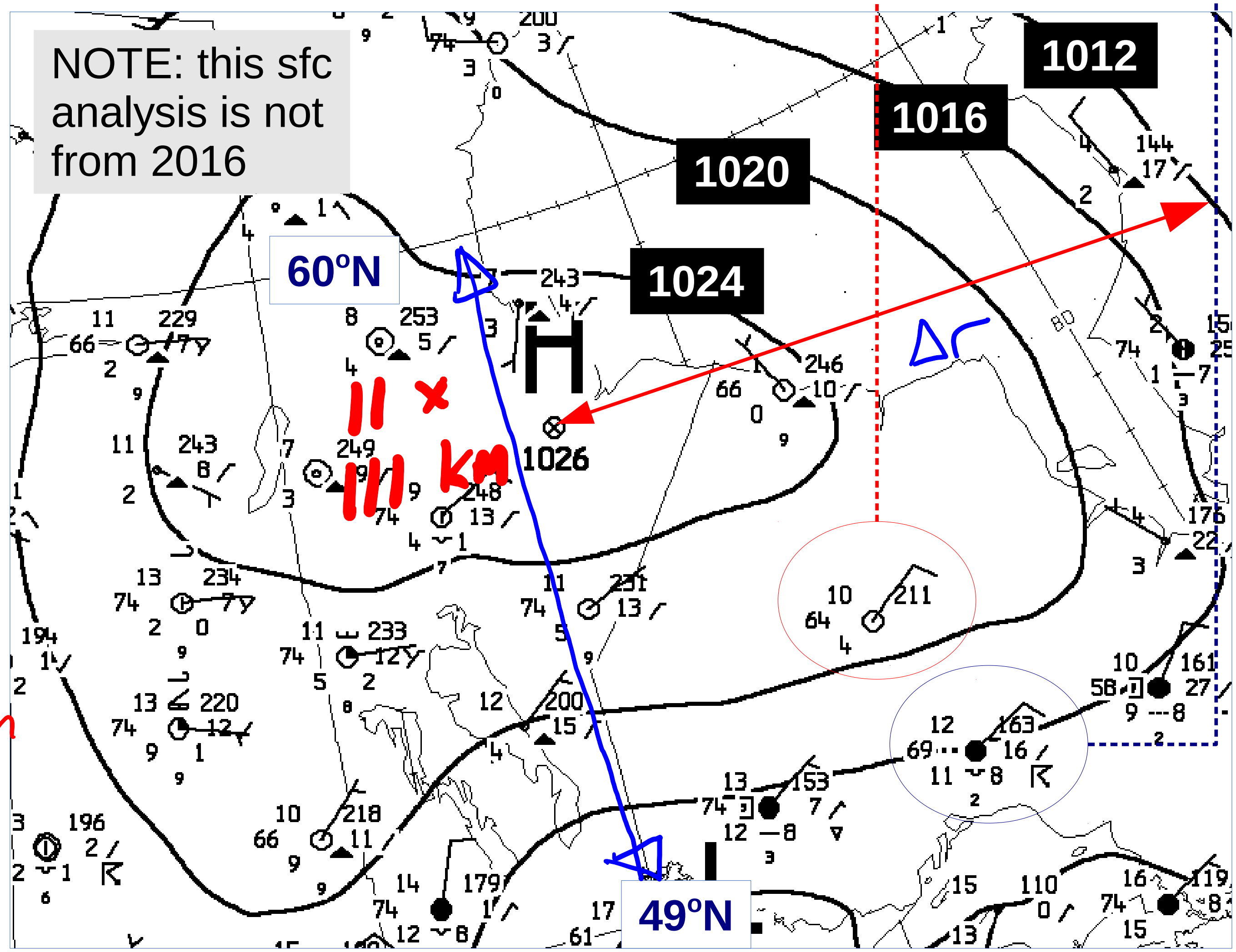
$\Delta P = 14 \text{ hPa} = 1400 \text{ Pa}$

$\Delta r \sim 1000 \text{ km}$

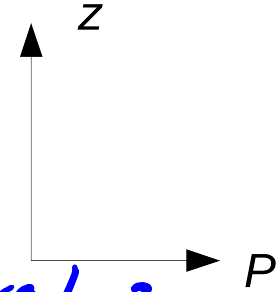
$= 10^6 \text{ m}$

$\frac{\Delta P}{\Delta r} \sim 1.4 \times 10^{-3} \text{ Pa/m}$

- now let's compare with the vertical gradient



To evaluate ΔP/Δz using $\dfrac{\Delta P}{\Delta z} = -\rho\, g$



Clear sky
SSW wind, 2.5 m/s
T=10°C
T_d=7°C
P_0=1010.1 hPa

we'll need the air density. Neglecting vapour,

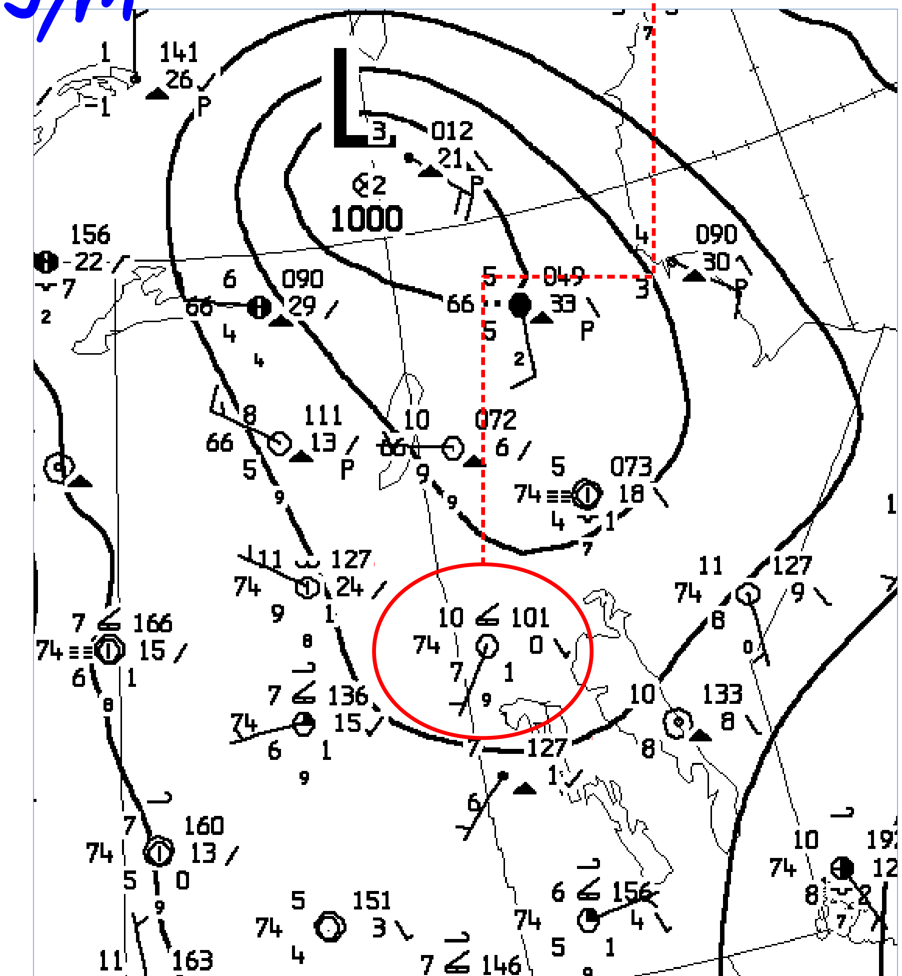
$$\rho = \frac{P}{R_d\ T} = \frac{1010.1 \times 10^2}{287\ (273.15+10)} = 1.34\ \mathrm{kg/m^3}$$

(c.f. example 3.1). Actually the density at ground level will be a bit lower, as 1010.1 is the sea-level corrected value – but it's close enough for our purposes... let's take 1 kg m^{-3}

$$\frac{\Delta P}{\Delta z} \approx -10\ \mathrm{Pa\ m^{-1}}$$

This is four orders of magnitude (powers of ten) stronger than the horizontal gradient

Therefore we "correct" the surface pressure to MSLP (a theoretical value at sea level) using the hypsometric eqn (covered later, Sec 3.5)



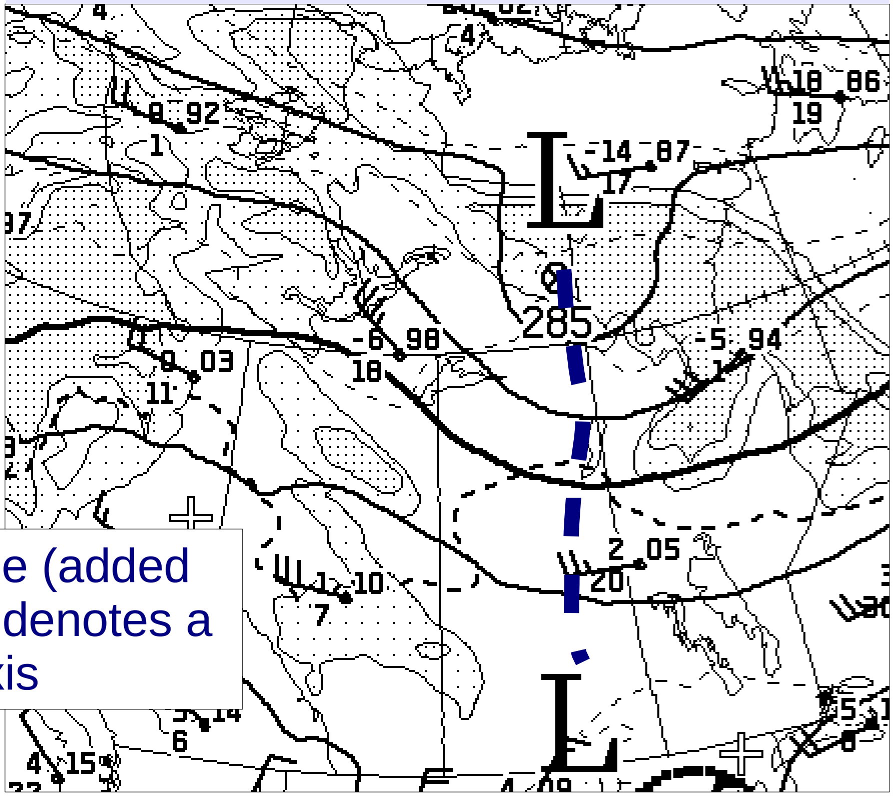
CMC sfc analysis 12Z Thurs 15 Sept. 2016

Lecture of 16 Sept.

- Thursday's fair weather
- reiterating the nature of an "isobaric chart" such as the 700 hPa analysis
- the ideal gas law, and what it's good for
- derivation and meaning of the hydrostatic eqn
- very disparate magnitudes of vertical and horizontal pressure gradients

- next class, we combine the ideal gas law and the hydrostatic law to get the "hypsometric equation"

CMC 700 hPa analysis 12Z Thurs 15 Sept.



Heavy line (added by JDW) denotes a trough axis