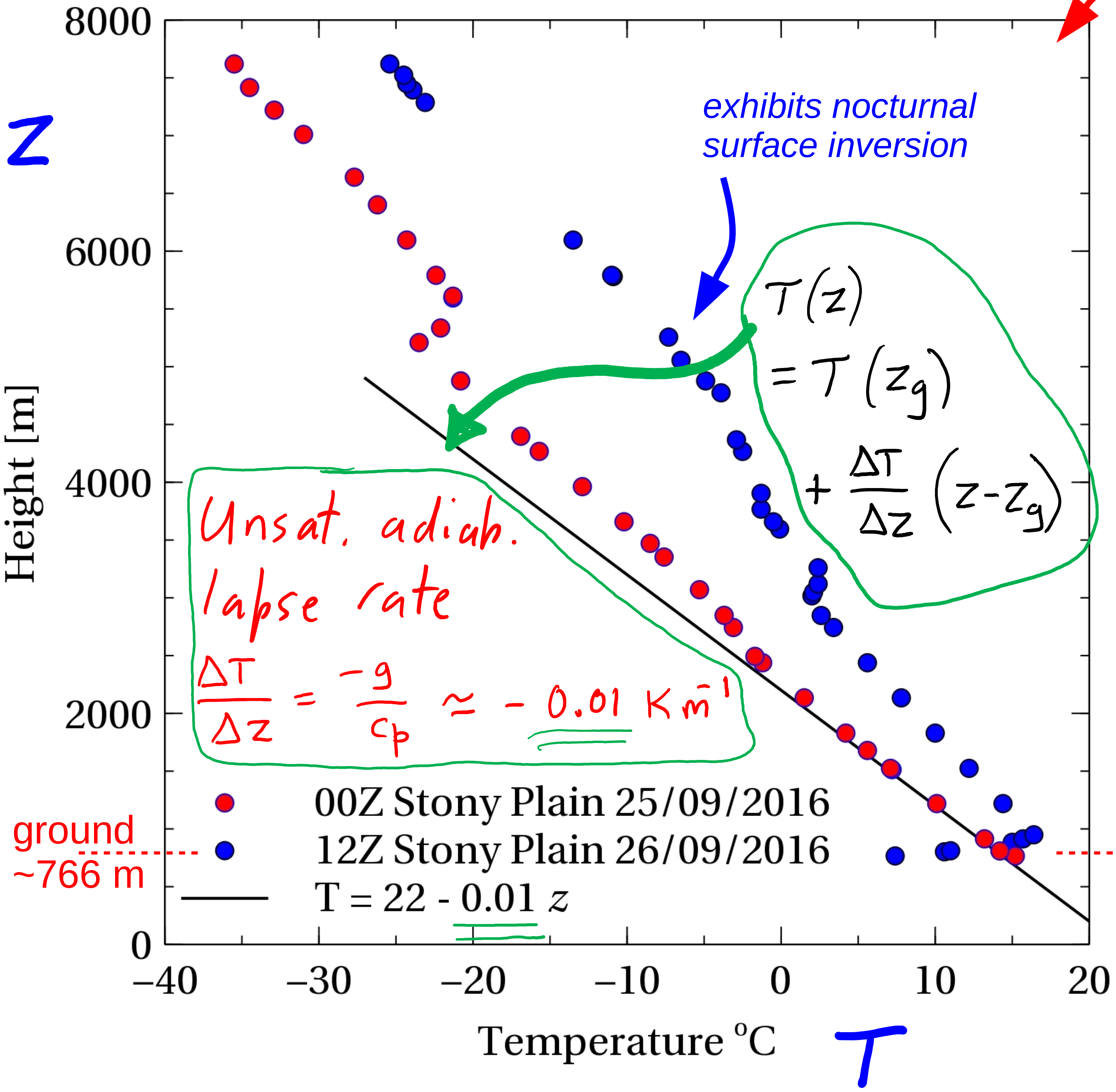
Ch 4 (ctd.)

Aside: an overbar means an average of some kind, e.g. $\overline{T}$

Demonstration of the practical relevance of the adiabatic lapse rate



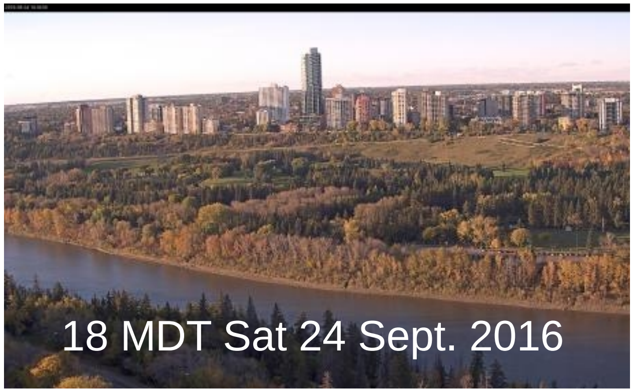
exhibits nocturnal surface inversion

$$T(z) = T(z_g) + \frac{\Delta T}{\Delta z}(z - z_g)$$

Unsat. adiab. lapse rate

$$\frac{\Delta T}{\Delta z} = \frac{-g}{c_p} \simeq -0.01\ \mathrm{K\,m^{-1}}$$

ground
~766 m



18 MDT Sat 24 Sept. 2016

Afternoon sounding

Shallow surface layer has lapse rate stronger than the adiabatic rate

This is due to surface heating

Above the surface layer, lapse rate is adiabatic to about 2500 m AGL

ground level
~766 m

say "Z_g"

Poisson's eqn for an adiabatic process

$$\Delta q = 0 = c_p \, \Delta T - \frac{\Delta P}{\rho}$$

$$P = \rho \, R_d \, T$$

$$\begin{tikzcd} T_1, P_1 \arrow[r, "\text{adiabatic}", "\text{process}"'] & T, P \end{tikzcd}$$

$$\frac{1}{\rho} = \frac{R_d T}{P}$$

$$c_p \, \Delta T = R_d \, T \frac{\Delta P}{P}$$

$$\frac{\Delta T}{T} = \frac{R_d}{c_p} \frac{\Delta P}{P}$$

fractional change

$$\Delta \ln T = \frac{R_d}{c_p} \Delta \ln P$$

$$\frac{\Delta \ln T}{\Delta \ln P} = \frac{R_d}{c_p}$$

Upon integration from state (P_1,T_1) to (P,T) one obtains

$$\boxed{\frac{T}{T_1} = \left[\frac{P}{P_1} \right]^{\frac{R_d}{c_p}}}$$

where $\frac{R_d}{c_p} = \frac{2}{7} = 0.286$

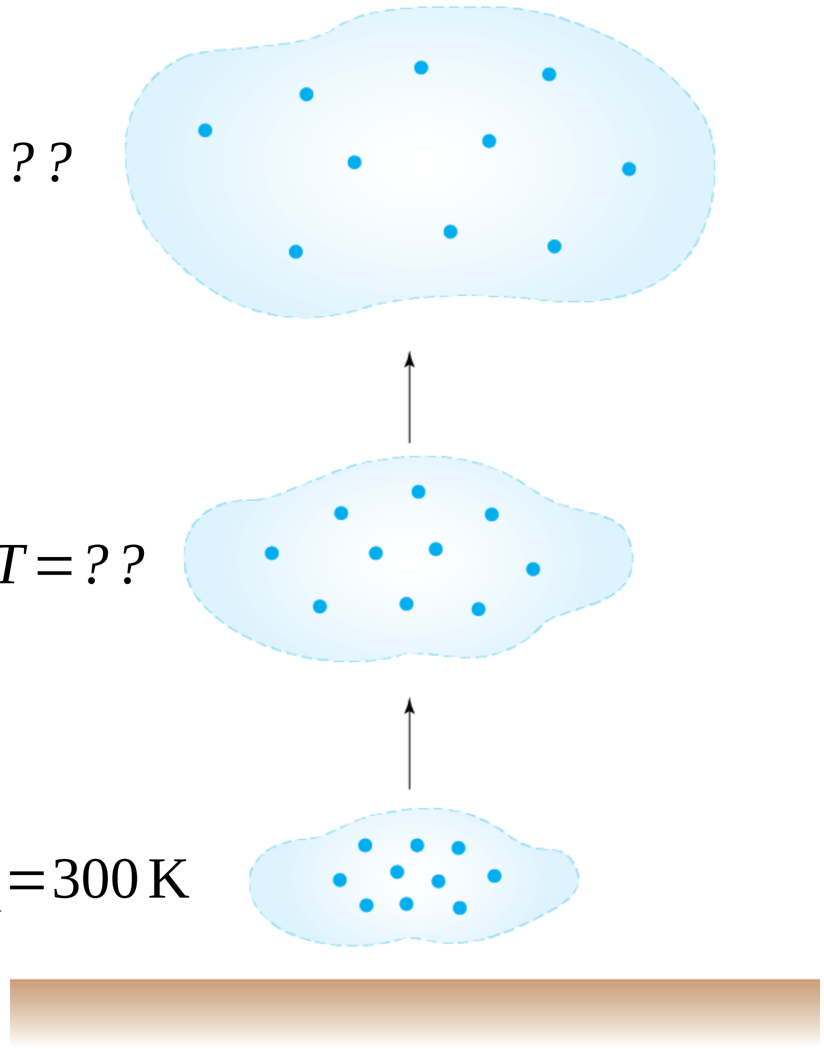
Applying Poisson's eqn

$T/T_1 = (P/P_1)^{0.286}$

Poisson's equation is useful if we want to compute the height variation of temperature based on ascent/descent across specified pressure levels (rather than specified elevations). Suppose the surface parcel rises adiabatically: compute its temperatures at 900 hPa and at 850 hPa...

$$T = T_1 \left(\frac{P}{P_1}\right)^{.286} = 300 \left(\frac{850}{920}\right)^{0.286}$$

$$= 293.29$$



$P = 850\,\text{hPa}\ ,\ T = ??$

$P = 900\,\text{hPa}\ ,\ T = ??$

$P_1 = 920\,\text{hPa}\ ,\ T_1 = 300\,\text{K}$

Precursor to Sec 4.4 – important definitions NOT (explicitly) given in textbook

Quantity of sensible heat per unit volume**, say "*h*" [J m^{-3}], is given by:

$$h = \rho \; c_p \; T$$

air density

specific heat capacity of air ~ 1000 J kg^{-1} K^{-1}

temperature, K

Let ρ_v [kg/m^3] be amount of vapour per unit vol.

** Alternative name: "volumetric (sensible) heat content". This is the appropriate measure in the context of heat transfer occurring at constant pressure, recalling

$$\Delta q = c_p \; \Delta T - \alpha \; \Delta P$$

Quantity of *latent* heat per unit volume, say "h_L" [J m^{-3}], is given by:

$$h_L = \rho_v \; L_v =$$ absolute humidity of the air [kg m^{-3}] x latent heat of vapourization ~ 2.5 x 10^6 J kg^{-1}

molecular level processes

A. Conduction** /Diffusion

- occurs in solids, liquids & gases
- spatial difference in temperature (mean kinetic energy of material particles) implies energetically vibrating matter in contact with less energetically vibrating material
- vibration energy is transferred
- football crowd analogy

- Fourier's law of conduction $Q_x = -k \frac{dT}{dx}$

Flux of sensible heat (Q_x, W m^{-2}) along direction *x* equals (minus) the conductivity (*k*) times the spatial gradient of temperature in the *x* direction.

Vector generalisation:

$$\vec{Q} = (Q_x, Q_y, Q_z) = -k\left(\frac{dT}{dx}, \frac{dT}{dy}, \frac{dT}{dz}\right)$$

What are the units of *k* ?

$$[k] = \frac{[Q]}{[dT/dx]} = \frac{W m^{-2}}{K m^{-1}} = W m^{-1} K^{-1}$$

wind

(Fick's law) (Newton's law of viscosity)

**Similar mechanisms and laws for mass "diffusion" and viscous momentum transfer

B. Convection

Transport by bulk flow

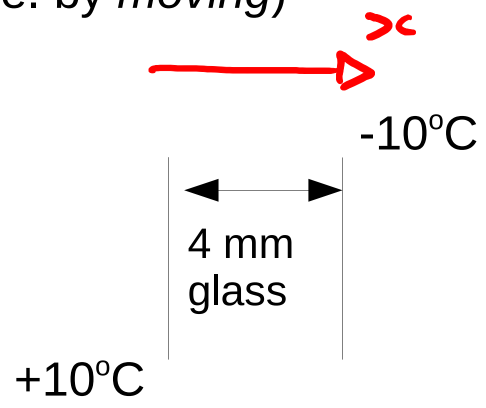
*Sec 4.4 is titled "Heat Transfer", but we'll cover the broader topic of "Natural mechanisms for heat, mass and momentum transfer"

C. Radiation (subject of Chapter 5) transports solar energy across the vacuum of space and powers the earth; we also deal with radiation from (and within) the earth-atmosphere system itself (terrestrial radiation). We later learn how greenhouse gases operate as selective absorbers of terrestrial (longwave) radiant energy

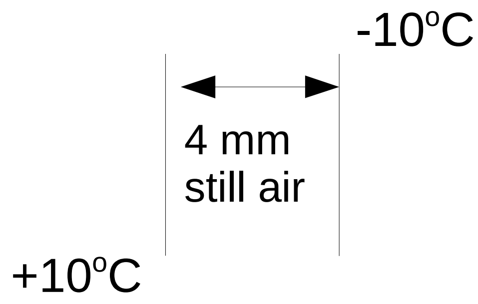
(Expanding on) Sec 4.4.1 Mechanisms for transport of energy – conduction

- by definition, poor conductors are good insulators – still air or fresh snow
- in the atmosphere, conduction is only important very close to surfaces (e.g. ground, or leaf surfaces) within the "laminar boundary layer". Air transports heat more effectively by convection (i.e. by *moving*)

Example:



$$|Q| \approx k\frac{\Delta T}{\Delta x} \simeq 1\,\frac{20}{.004} = 5000\ W m^{-2}$$



$$|Q| \approx .025 \times \frac{20}{.004} = 125\ W m^{-2}$$

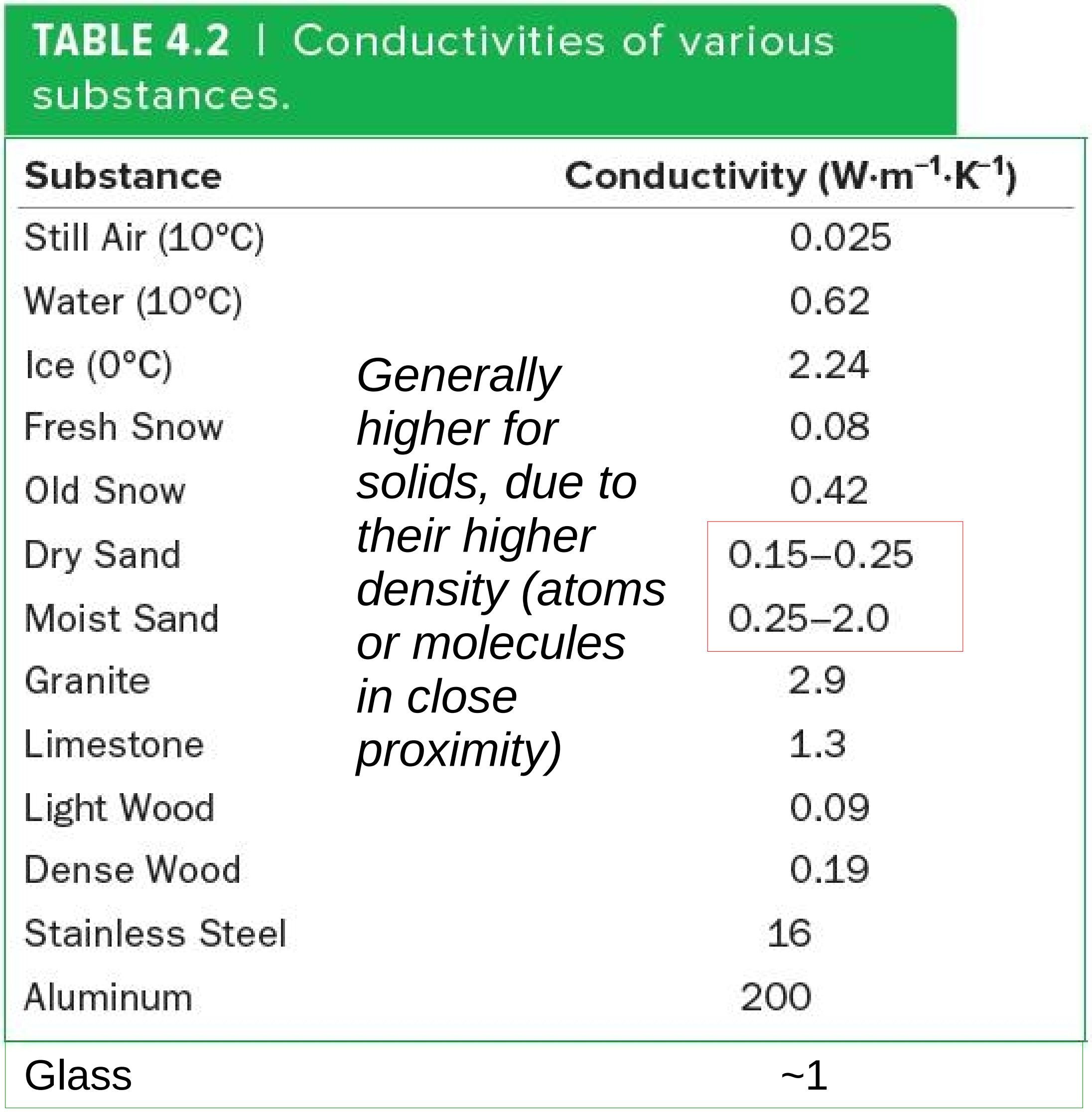


TABLE 4.2 | Conductivities of various substances.

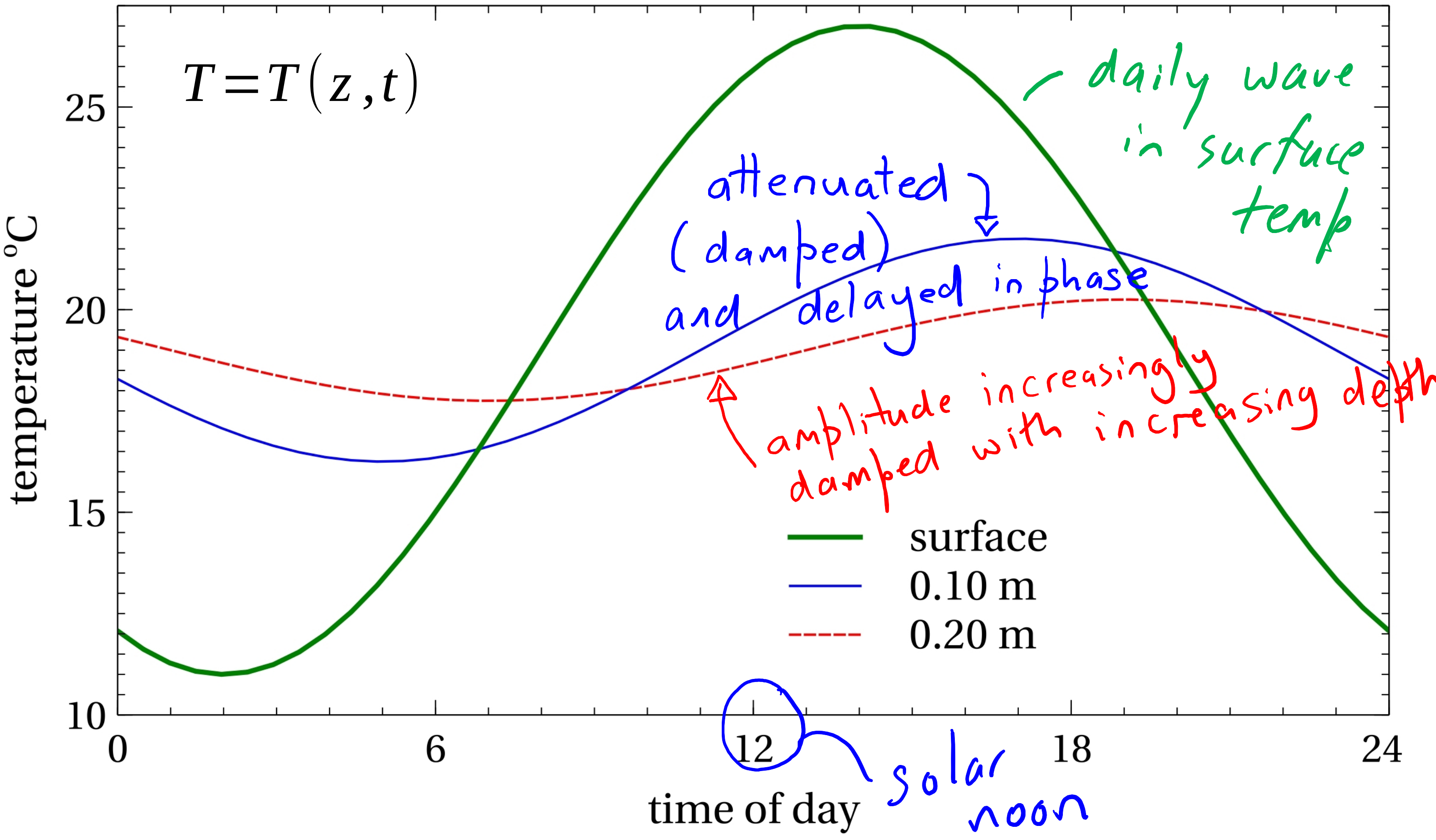
Substance	Conductivity (W·m^{-1}·K^{-1})
Still Air (10°C)	0.025
Water (10°C)	0.62
Ice (0°C)	2.24
Fresh Snow	0.08
Old Snow	0.42
Dry Sand	0.15–0.25
Moist Sand	0.25–2.0
Granite	2.9
Limestone	1.3
Light Wood	0.09
Dense Wood	0.19
Stainless Steel	16
Aluminum	200
Glass	~1

Generally higher for solids, due to their higher density (atoms or molecules in close proximity)

(Expanding on) Sec 4.4.1 Soil heat conduction

- conduction is the main mechanism for heat transfer within ground (percolation of rainwater – a convective process – can also transfer heat)
- daily (diurnal) and annual "temperature waves" are attenuated with increasing depth (z)
- ~ 10 cm participates diurnally; ~ few m annually
- soil conductivity a strong function of soil moisture content

Penetration of the daily temperature wave – increasing attenuation (i.e. amplitude reduction) and phase lag (i.e. time delay of the temperature peak) with increasing depth.



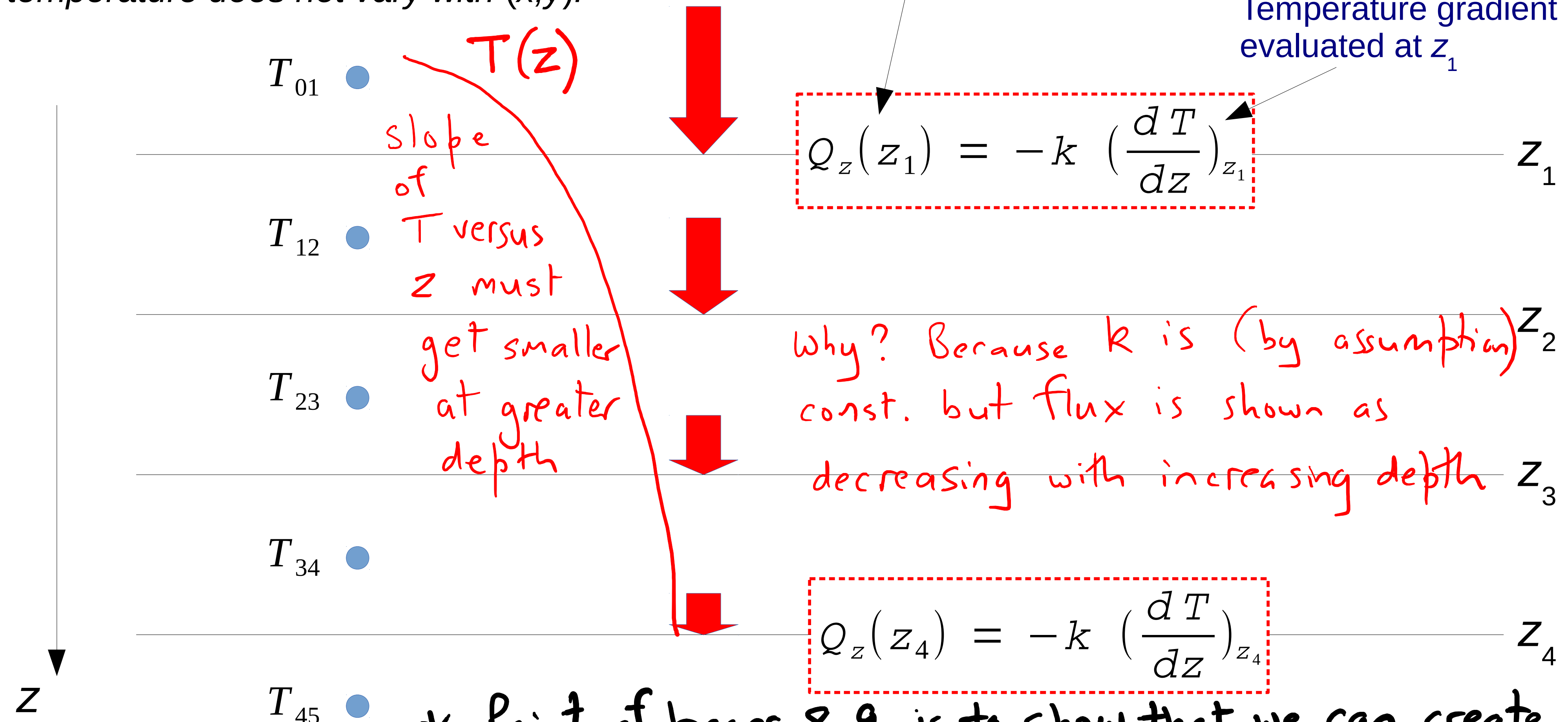
(Expanding on) Sec 4.4.1 Soil heat budget* – (novel form of the 1st law)

- consider <u>layers</u> of soil, assuming soil conductivity *k* is uniform (constant). The soil heat flux Q_z varies with depth *z* :

Implicitly we're assuming horizontal heat transport is negligible. This amounts to an assumption of symmetry, i.e. that soil temperature does not vary with (*x,y*).

Notation: Left Hand Side (LHS) is the heat flux <u>*evaluated at*</u> z_1 (not *times* z_1)

Temperature gradient evaluated at z_1



$$Q_z(z_1) = -k \left(\frac{dT}{dz}\right)_{z_1}$$

$$Q_z(z_4) = -k \left(\frac{dT}{dz}\right)_{z_4}$$

slope of T versus z must get smaller at greater depth

Why? Because k is (by assumption) const. but flux is shown as decreasing with increasing depth

* Point of pages 8, 9 is to show that we can create a quantitative model of the "meteorology" of the soil (i.e. the enviromental state).

(Expanding on) Sec 4.4.1 Soil heat budget equation in finite difference form

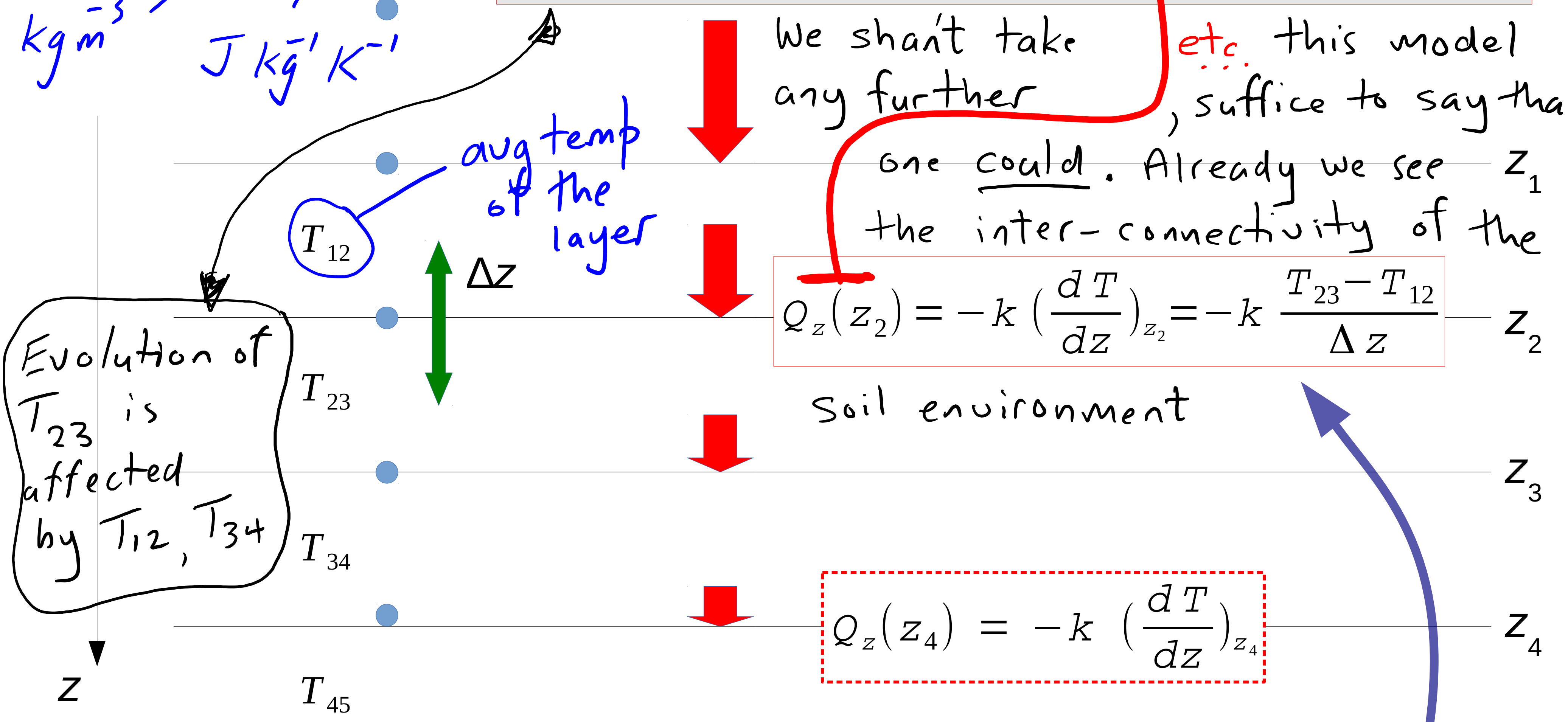
In time Δ*t* the volume *A* Δ*z* of the layer whose temperature is T_{23} gains an amount of heat $Q_z(z_2)\,A\,\Delta t$ **[J]** conducted across z_2, but loses heat $Q_z(z_3)\,A\,\Delta t$ **[J]** conducted across z_3

Soil density and heat capacity ρ_s and c_s

ρ_s: $kg\,m^{-3}$

c_s: $J\,kg^{-1}\,K^{-1}$

$$\rho_s\, c_s\, \overset{\text{volume}}{A\,\Delta z}\, \Delta T_{23} = A\, \overset{\text{time step}}{\Delta t}\,[\,Q_z(z_2) - Q_z(z_3)\,]$$



We shan't take this model any further, suffice to say that one could. Already we see the inter-connectivity of the soil environment

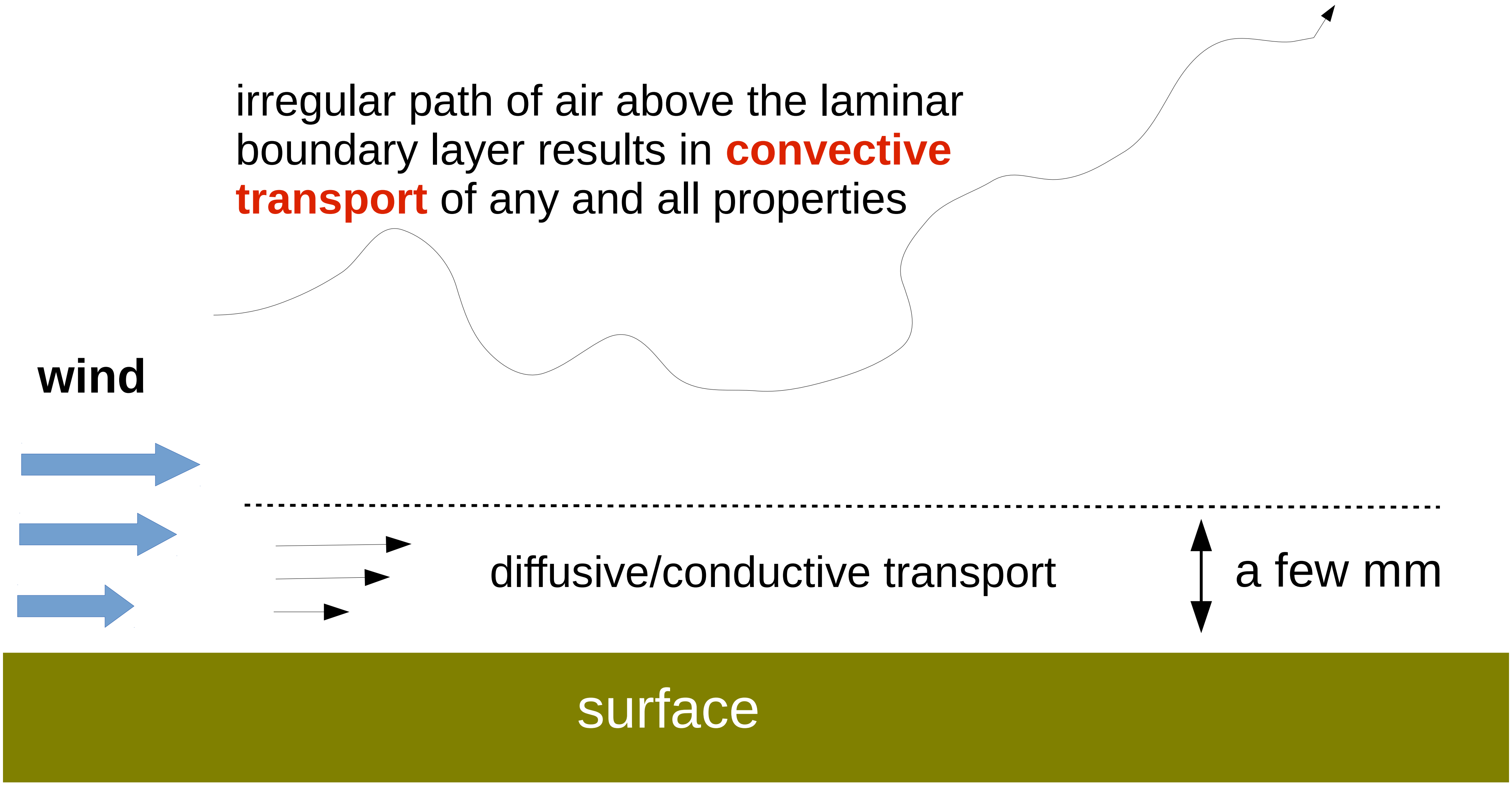
$$Q_z(z_2) = -k\left(\frac{dT}{dz}\right)_{z_2} = -k\,\frac{T_{23}-T_{12}}{\Delta z}$$

$$Q_z(z_4) = -k\left(\frac{dT}{dz}\right)_{z_4}$$

- at each interface, we can evaluate the needed heat flux using Fourier's law

Convection

When a fluid or gas flows over a solid surface, there is a thin layer adjacent to the surface, known as the laminar boundary layer, in which the flow is *parallel* to the surface, meaning there can be no convective transport to or from that surface – in that layer, only radiation & conduction can transport heat, and only diffusion can transport mass (e.g. water vapour)



irregular path of air above the laminar boundary layer results in **convective transport** of any and all properties

wind

diffusive/conductive transport

a few mm

surface

wind

- Transport of heat by "mass movement" of a fluid, i.e. organized (bulk) flow as opposed to the jitter of individual atoms, is named "convection"
- In the broadest context convection may transport *any* property in *any* direction
- Commonly the word is used to refer (more narrowly) to vertical transport of heat
- Must interpret the words "convection, convective" according to the context

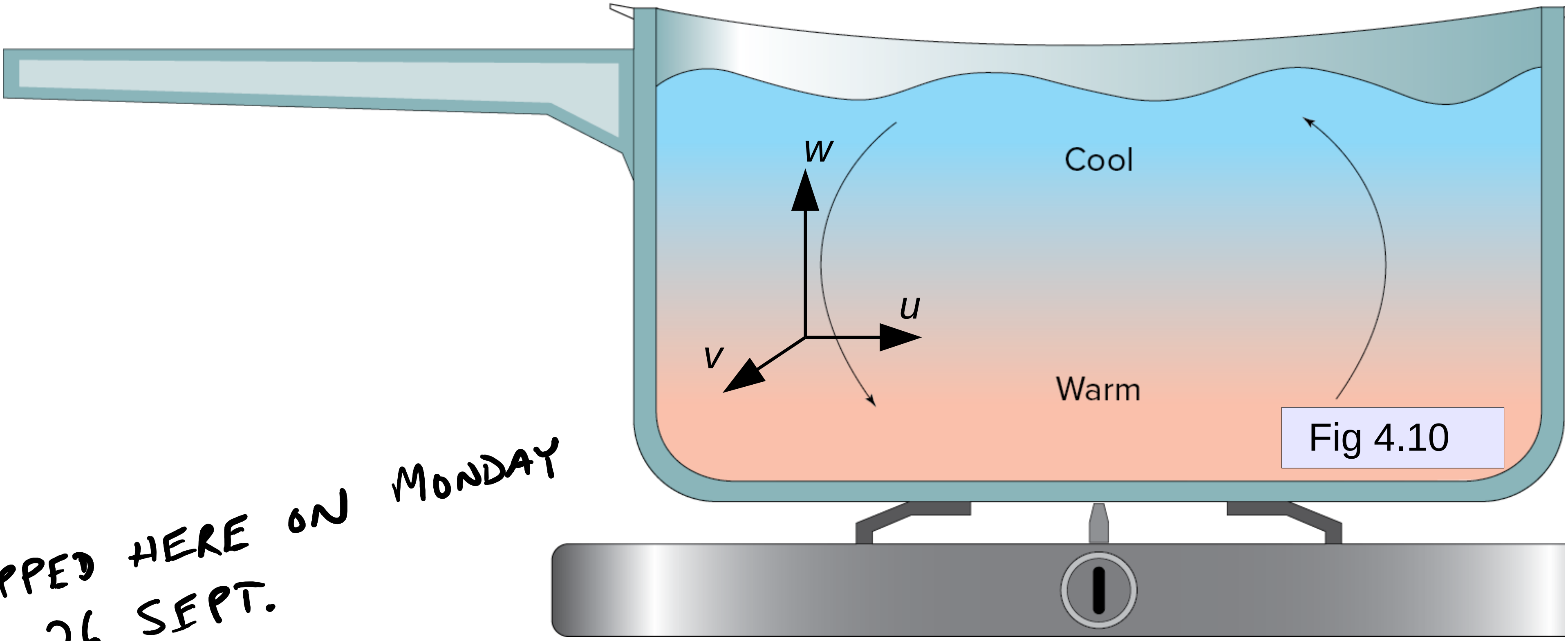


Fig 4.10

STOPPED HERE ON MONDAY
26 SEPT.

Sec 4.4.2 Rate of convective transport (vital definition NOT given in our textbook)

amount per cubic metre

- always wind velocity x volumetric concentration of the transported property

- e.g. let *w* be the vertical wind speed and *h* (as before) be the volumetric sensible heat content, then the rate of vertical convective transport of sensible heat is simply:

$$\underline{Q_z = w \quad h} = \rho \quad c_p \quad w \quad T \qquad [\mathrm{J\ s^{-1}\ m^{-2} = W\ m^{-2}}]$$

- this quantifies the vertical convection of energy in one form, i.e. sensible heat energy. To use all the adjectives, this is: the instantaneous local vertical convective flux density of sensible heat

- it's an important quantity that can be, and often is, measured (more in a later chapter). What is important here is the principle: product of wind velocity (in a given direction) times the amount of “stuff” per unit volume of air gives the rate at which that “stuff” is being transported (its “flux” in the given direction)

"absolute humidity"

Q: why is the sensible heat flux density evaluated using c_p rather than c_v?

because heat exchange is occurring at const. pressure

convective

Q: what formula would give the vertical flux density of water vapour [$\mathrm{kg\ m^{-2}\ s^{-1}}$]? $w\,\rho_v$

Natural mechanisms for heat, mass and momentum transfer

- conduction/diffusion mechanism for transport is of negligible importance in the interior of the atmosphere
- but may dominate within thin layers (mm to cm in depth) adjacent to surfaces

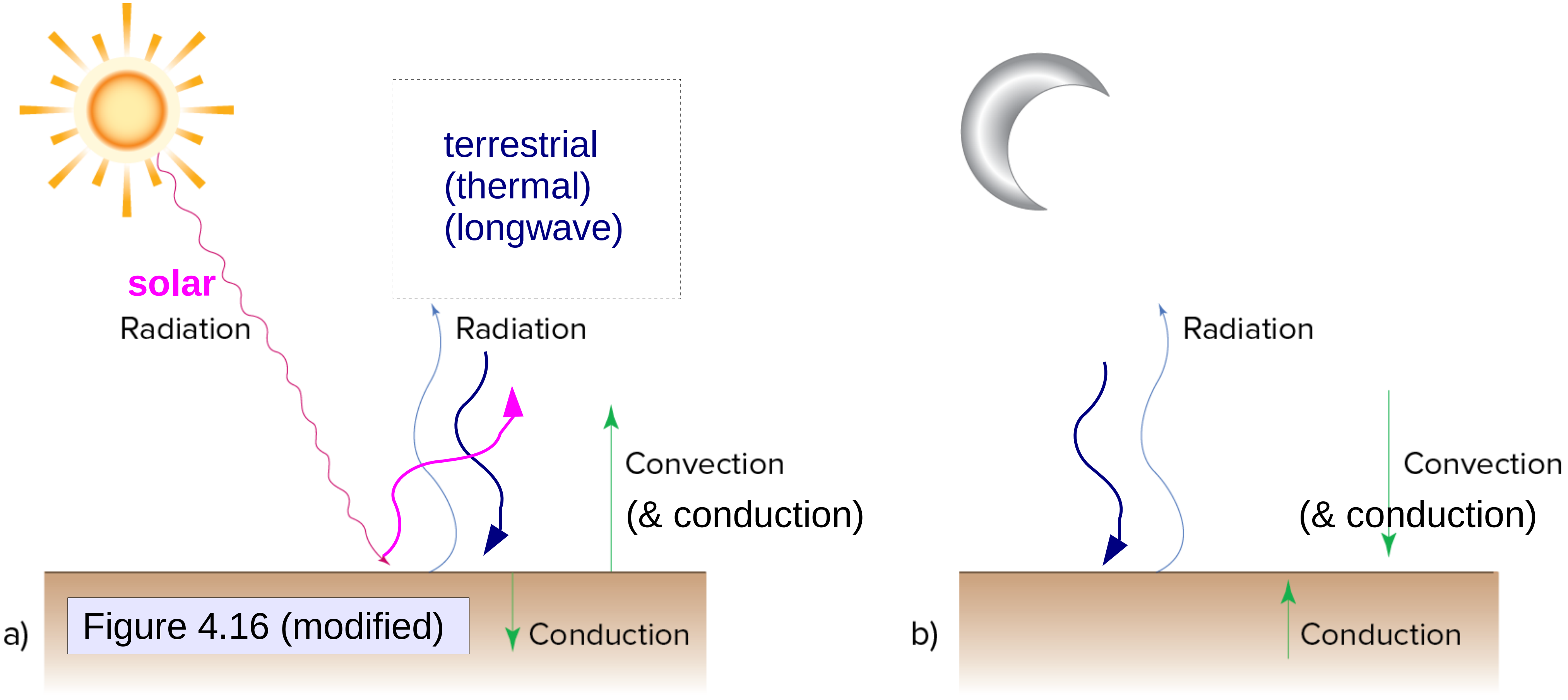


Figure 4.16 (modified)

Lecture of 26 Sept.

- temperature change versus pressure change (Poisson's equation)
- relative importance of conductive/convective/radiative fluxes in soil/atmosphere/space
- laws for conductive and convective heat fluxes (i.e. eqns. quantifying such fluxes)
- how an appropriate conservation law for heat energy is "built" in the case of a homogeneous soil (a more complex law would be needed if we had to handle exchange of heat along the horizontal axes)
- role of symmetry in science laws – here an *assumed* symmetry allowed us to remove two of the four space-time coordinates, i.e. we assumed T is "homogeneous" along x,y