| Absolute humidity | ρ_v | $\rho_v = \dfrac{\text{mass of water vapour}}{\text{volume of sample}}$ | $[\text{kg m}^{-3}]$ |

or "vapour density"

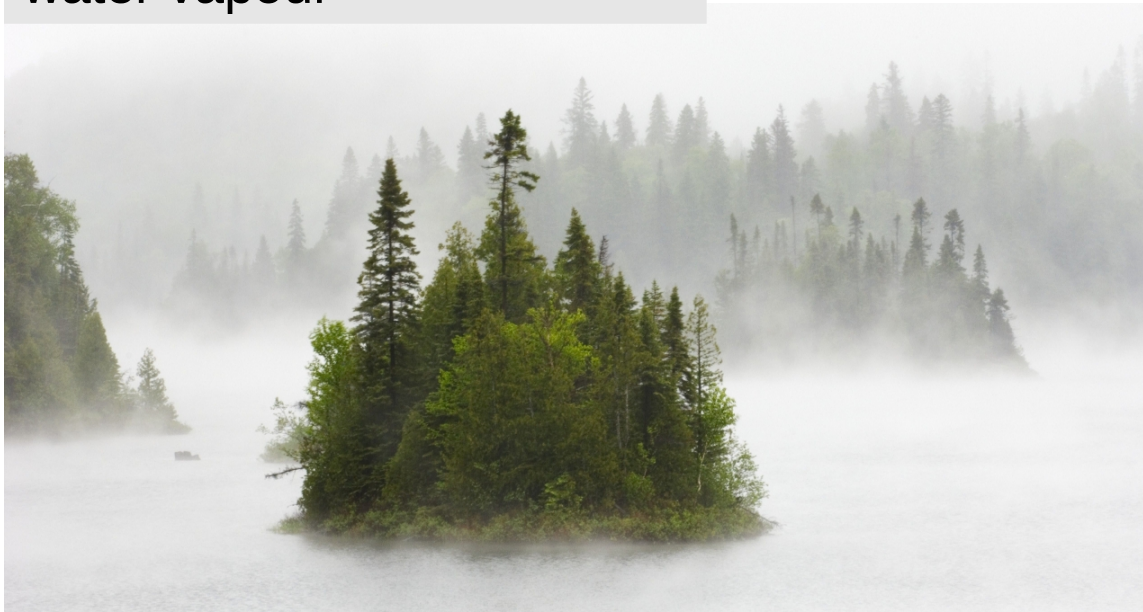| Specific humidity | q | $q = \dfrac{\text{mass of water vapour}}{\text{total mass of sample}} = \dfrac{\rho_v}{\rho}$ |

almost identical to

| Mixing ratio | r | $r = \dfrac{\text{mass of water vapour}}{\text{mass of dry air in sample}} = \dfrac{\rho_v}{\rho_d}$ |

$[\text{kg kg}^{-1}]$

or

$[\text{g kg}^{-1}]$

| Partial pressure of water vapour | e | | $[\text{Pa}]$ |

"At the same pressure and temperature, humid air weighs less (and has lower density) than dry air"

We shall prove this...

$$\rho = \rho_d + \rho_v$$
$$P = P_d + e$$

The ideal gas law inter-relates vapour pressure (e) & absolute humidity (ρ_v)

① $e = \rho_v R_v T$

$R^* = 8.314 \ \text{J mol}^{-1} \text{K}^{-1}$

② $P_d = \rho_d R_d T$

"vapour"

$$R_v = \frac{R^*}{MM} = 462 \ \text{J kg}^{-1}\text{K}^{-1}$$

③ $P = \rho R_d T_v$

$MM = 0.018 \ \text{kg mol}^{-1}$

"virtual"

TABLE 7.2 | Indicators of atmospheric water vapour.

Absolute Measures	Equation	Comments
Partial pressure of water vapour ("vapour pressure"), e	$e = P\left(\dfrac{r}{\varepsilon + r}\right)$ $\varepsilon = 0.622$ = ratio of R_d to R_v	not conserved in vertical motion
Absolute Humidity	$\rho_v = \dfrac{e}{R_v T}$	not conserved in vertical motion; can be measured directly
Mixing Ratio	$r = \varepsilon\left(\dfrac{e}{P - e}\right)$	conserved in vertical motion; used on thermodynamic diagrams
Specific Humidity	$q = \dfrac{r}{1 + r}$	conserved in vertical motion
Dew-Point Temperature	"temperature to which air must be cooled for saturation to occur (with no change in pressure or moisture content)"	not conserved in vertical motion; can be measured directly; reported as part of routine weather observations

Numerically, q and r are almost identical ($q \approx r$). Both "remain constant so long as water vapour is not added or removed from the parcel"

Specific humidity and mixing ratio behave as "tracers" so long as parcels remain unsaturated.

i.e. do not change

the parcel shown has expanded, but its specific humidity and mixing ratio are unchanged
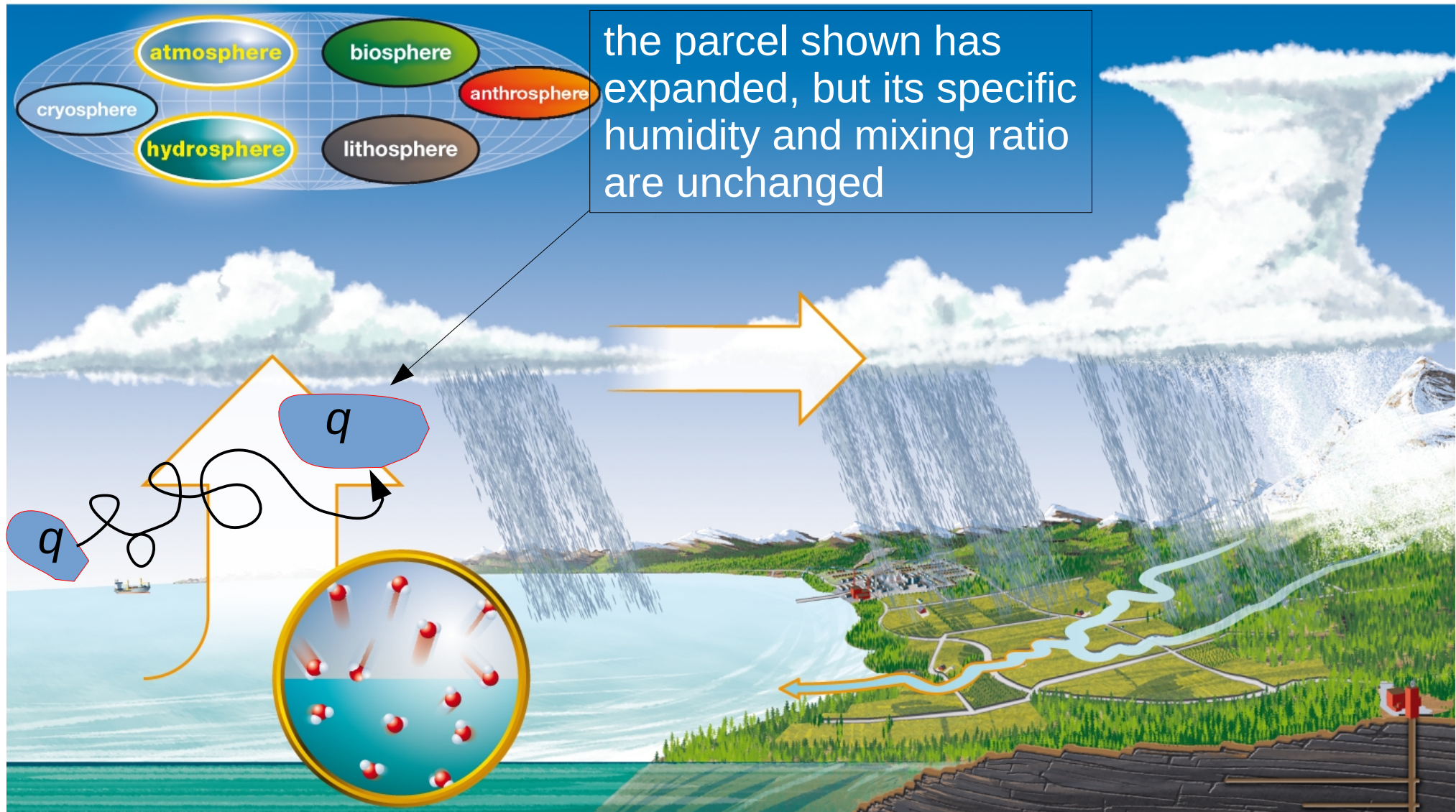
During calm, cloudless weather the late afternoon dewpoint is sometimes considered as a reasonable estimate for overnight minum temperature: e.g.

saturation ⟶ latent heat

⟶ result in droplets

that impede L↑ and enhance L↓

08:33 MDT Wed 26 Sept 2012

Past 24 Hour Conditions

Imperial Units

Date / Time (MDT)	Conditions	Temp (°C)	Humidity (%)	Dew Point (°C)
26 September 2012				
8:00	Sunny	6	91	4
7:00	Clear	6	89	4
6:00	Clear	6	87	4
5:00	Partly Cloudy	7	85	5
4:00	Mostly Cloudy	9	79	5
3:00	Mostly Cloudy	10	78	6
2:00	Mostly Cloudy	10	77	6
1:00	Cloudy	11	80	7
00:00	Cloudy	11	81	8
25 September 2012				
23:00	Cloudy	12	77	8
22:00	Mostly Cloudy	13	75	8
21:00	Clear	13	70	8
20:00	Clear	14	66	7
19:00	Sunny	15	59	7
18:00	Sunny	17	54	7
17:00	Sunny	17	56	8
16:00	Sunny	18	57	10
15:00	Sunny	18	58	9
14:00	Sunny	18	58	9
13:00	Sunny	18	56	9
12:00	Sunny	17	57	9
11:00	Sunny	15	68	9
10:00	Sunny	12	82	9
9:00	Sunny	10	88	8
8:00	Sunny	9	90	7

Proof:

R_* universal gas const.

$$P V = n R_* T$$

n no. of moles

$$\frac{P V}{R_* T} = n = \text{const.}$$

as we add moles of vapour we let moles of "dry air" escape.

$$n = n_v + n_d = \text{const.}$$

vapour   dry air

So if one adds Δn_v moles of vapour to dry air, keeping volume, pressure and temperature unchanged, then

$$\Delta n = 0$$

and so $\Delta n_d = - \Delta n_v$ < 0

$$\rho = \frac{MM_v \, n_v + MM_d \, n_d}{V}$$

Differentiating,

$$\Delta \rho = \frac{MM_v \, \Delta n_v + MM_d \, \Delta n_d}{V}$$

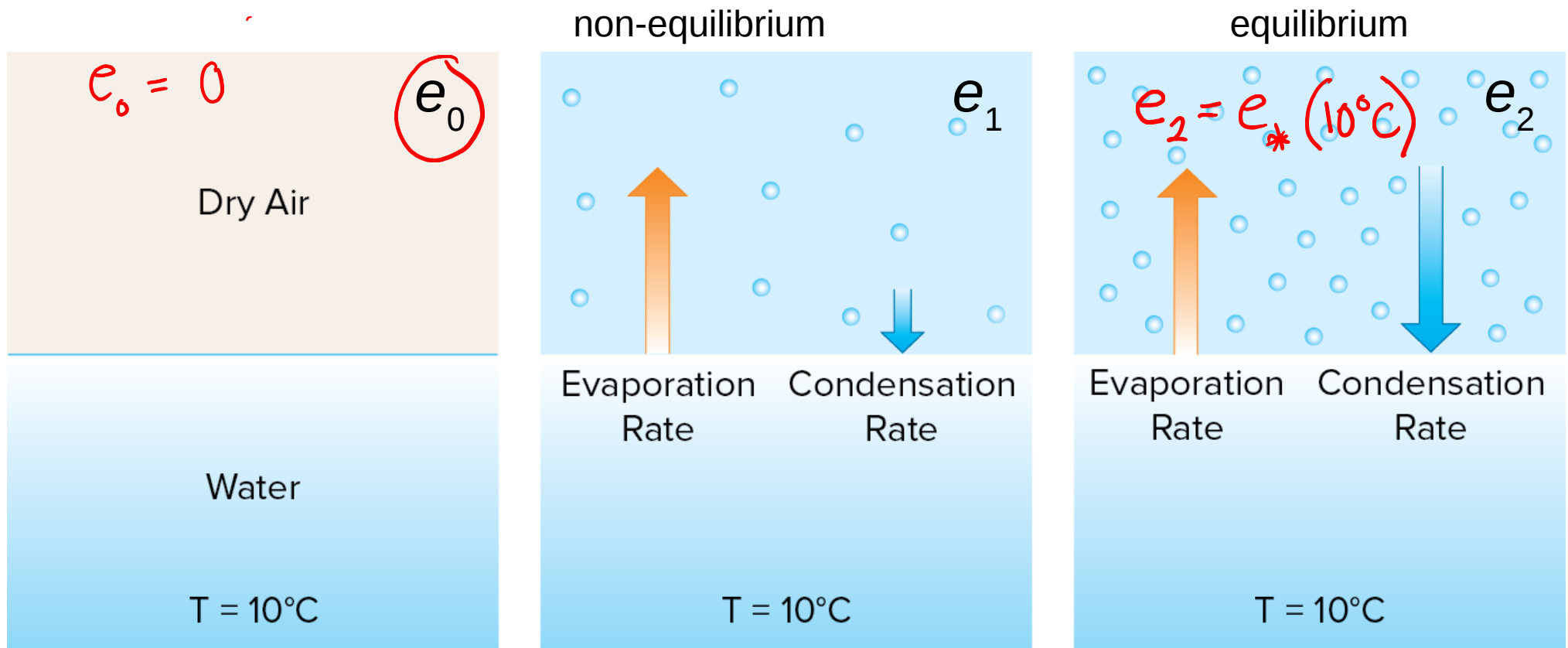$$\Delta \rho = \frac{MM_v \, \Delta n_v - MM_d \, \Delta n_v}{V}$$

$$\frac{\Delta \rho}{\Delta n_v} = \frac{1}{V} \left[MM_v - MM_d \right]$$

negative, because

$$MM_v = 0.018 \left[\text{kg mol}^{-1} \right]$$

$$MM_d = 0.029 \left[\text{kg mol}^{-1} \right]$$

(THOUGHT EXPT.) Only when the vapour pressure e is sufficiently large to assure that condensation rate = evaporation rate can this system be at equilibrium. The vapour pressure at which equilibrium occurs depends <u>only</u> on temperature, $e_* = e_*(T)$
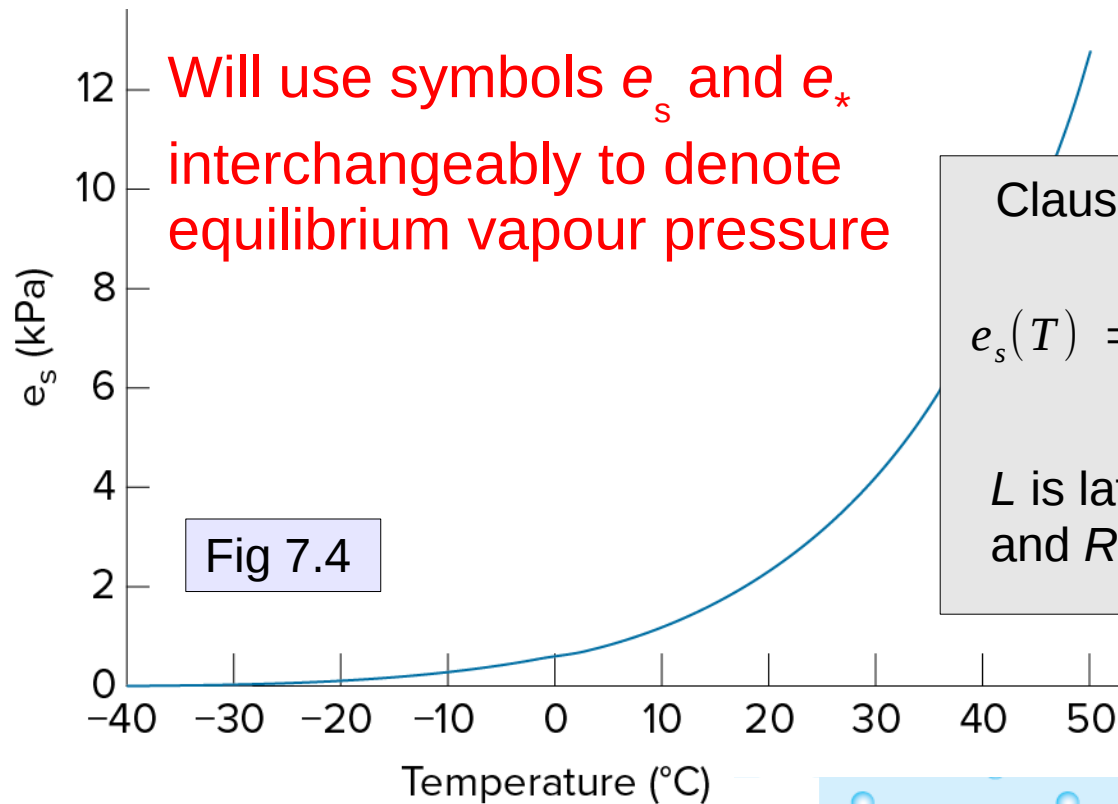
non-equilibrium

equilibrium

$e_o = 0$ e_0

Dry Air

Water

T = 10°C

a)

e_1

Evaporation   Condensation
Rate   Rate

T = 10°C

b)   Fig 7.2

$e_2 = e_*(10°C)$ e_2

Evaporation   Condensation
Rate   Rate

T = 10°C

c) STOPPED HERE 17 OCT.

Fig 7.2

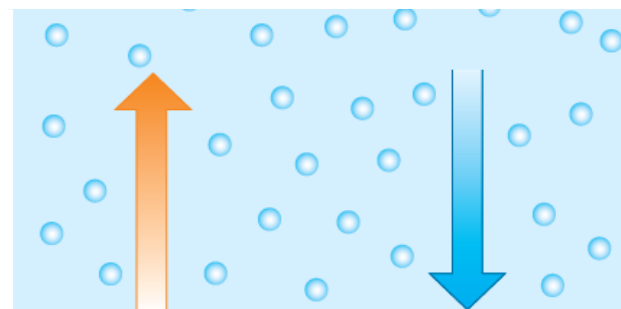Will use symbols e_s and e_* interchangeably to denote equilibrium vapour pressure

Fig 7.4

Clausius-Clapeyron eqn. gives $e_s(T)$ curve.

$$e_s(T) = 6.11 \ \exp\left[\frac{L}{R_v}\left(\frac{1}{273.15} - \frac{1}{T}\right)\right], \text{hPa} \quad \boxed{\text{Eq 7.1}}$$

L is latent heat (of vaporization or sublimation) and R_v=461.5 [J kg^{-1} K^{-1}] is gas const for w.v.
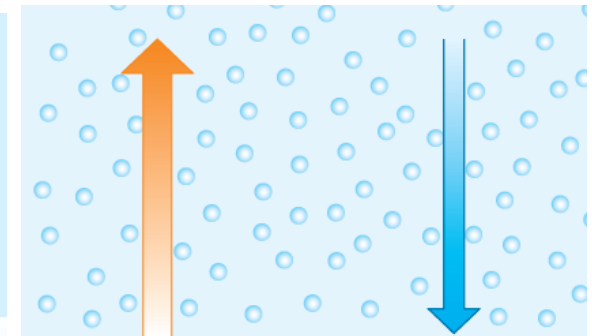
- alternative formulae for $e_s(T)$ exist

- will expect students to be able to use Table 7.1

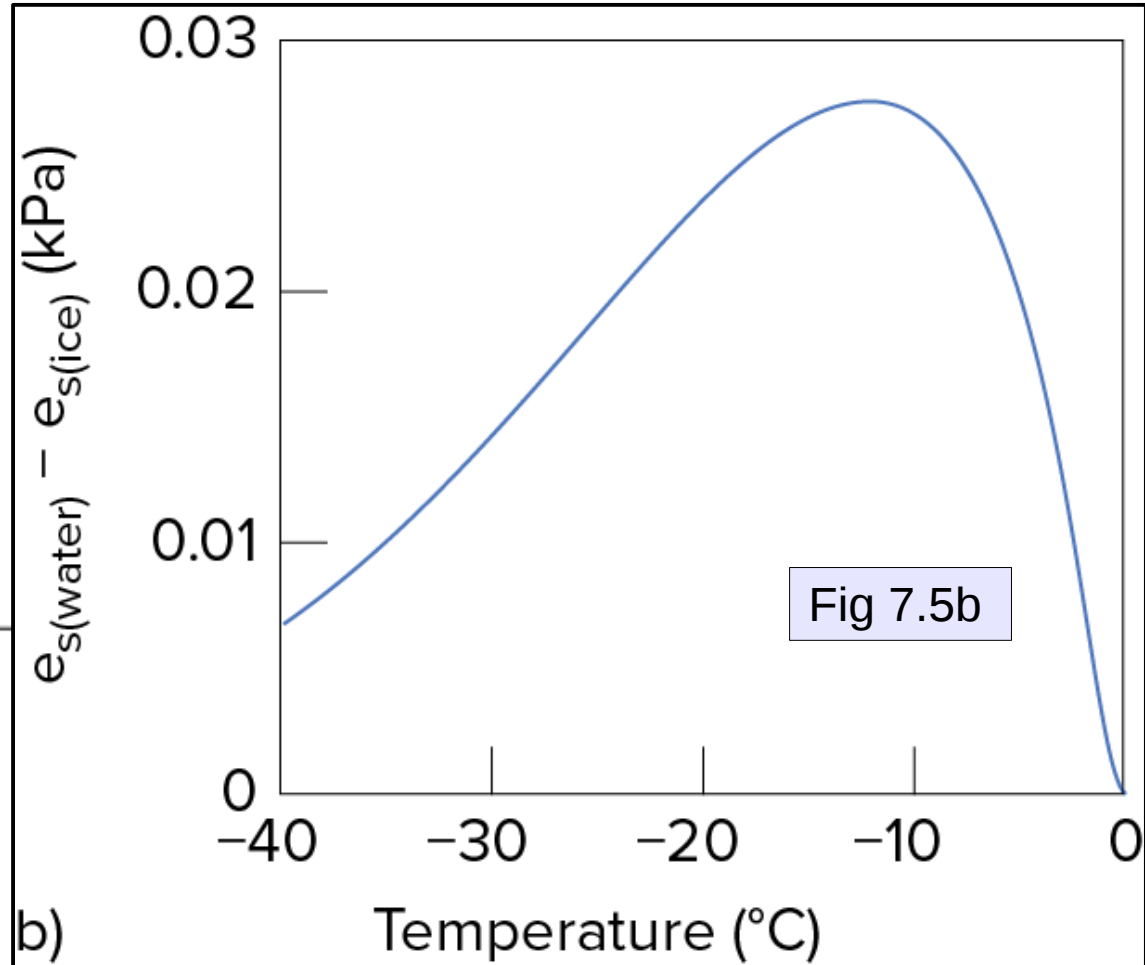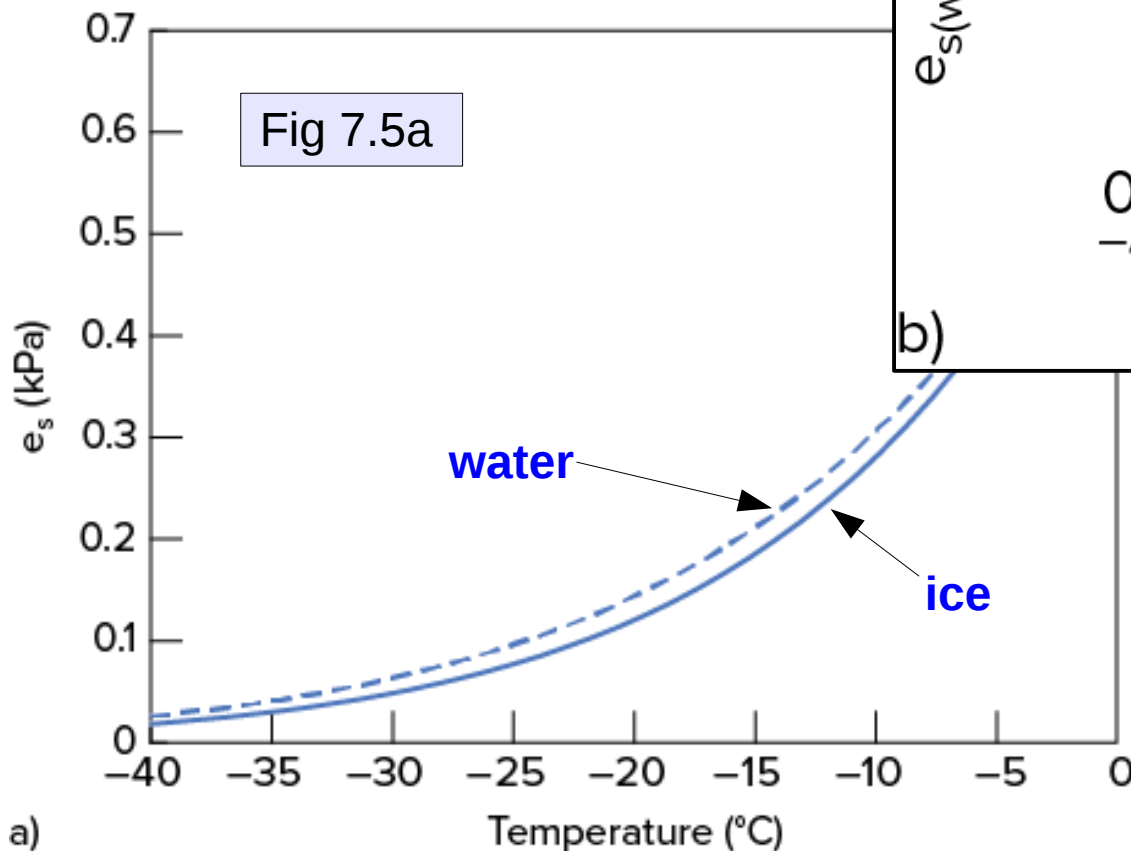Evaporation Rate   Condensation Rate

Fig 7.2c

T = 10°C

Evaporation Rate   Condensation Rate

Fig 7.3

T = 20°C

Tiny water droplets in the atmosphere can (and mostly do) remain unfrozen for temperatures far below 0°C, and are said to be "supercooled."

Perhaps $e_*(T)$ is a strange choice of "benchmark" for vapour pressure

Fig 7.5b

b)

Fig 7.5a

water

ice

a)

molecules held tighter in ice lattice

Vapour pressure *e* and dewpoint temperature T_d are in 1:1 relationship – they are not independent pieces of information. If you know one, you can deduce the other. Symbolically, we write
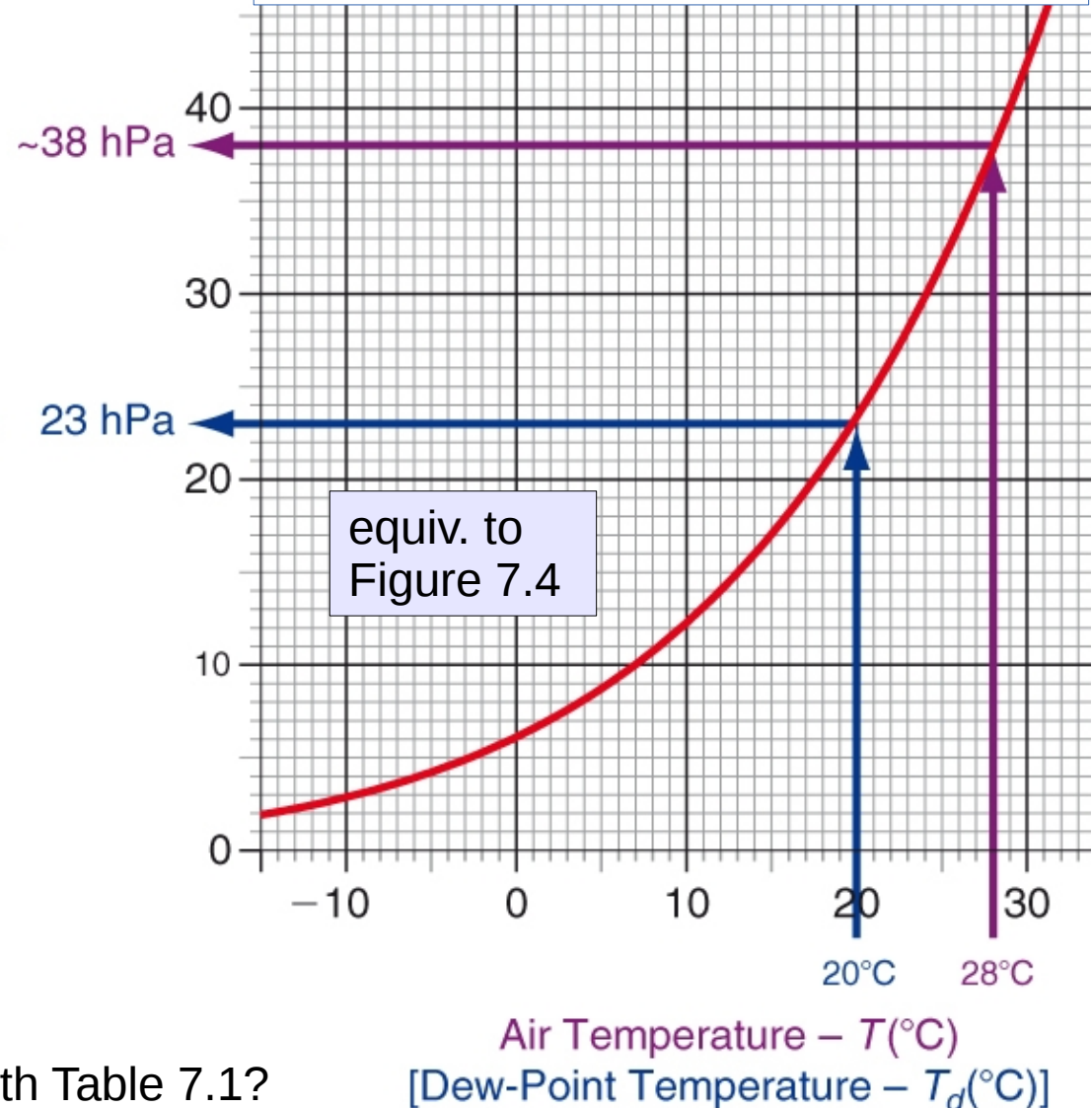
$$e \; = \; e_*(T_d)$$

meaning "*e* is a function of T_d" (the function being the e$_*$ curve). Similarly,

$$T_d \; = \; e_*^{-1}(e)$$

Check: is the adjacent figure consistent with Table 7.1?

Extracted from Table 7.1	Air Temperature/Dew-Point Temperature (°C)	Saturation Vapour Pressure/ Vapour Pressure (kPa)
	0	0.611
	1	0.657
	2	0.705
	3	0.758
	4	0.813

Saturation Vapour Pressure – e^* (hPa) [Actual Vapour Pressure – *e* (hPa)]

~38 hPa

23 hPa

equiv. to Figure 7.4

20°C   28°C

Air Temperature – *T* (°C)
[Dew-Point Temperature – T_d(°C)]

$$e_* = e_*(T)$$ 　insert　 T and get out equilibrium vapour pressure e_*
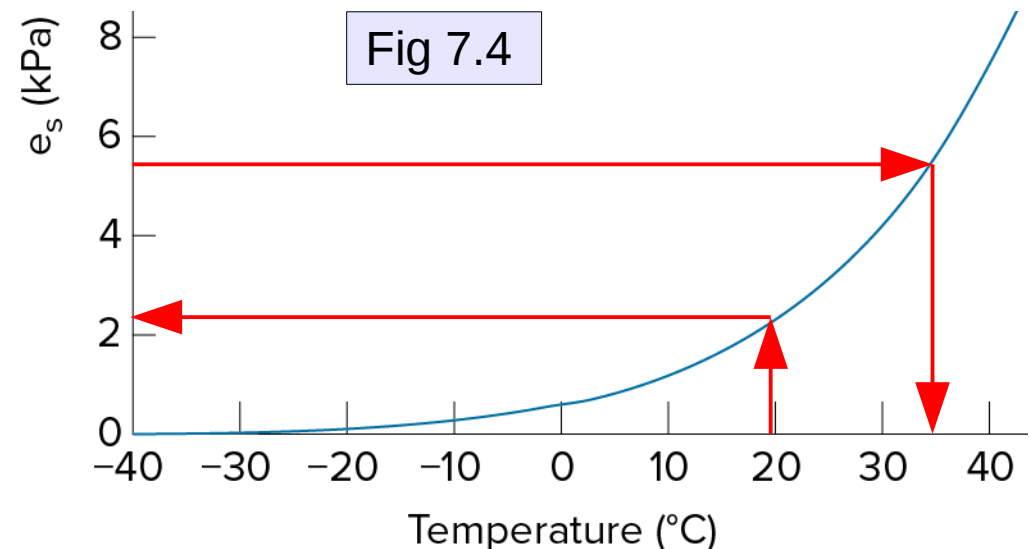
$$e = e_*(T_d)$$ 　insert　 T_d and get out actual vapour pressure e

$$T_d = e_*^{-1}(e)$$ 　insert　 e (actual vapour pressure) and get out T_d

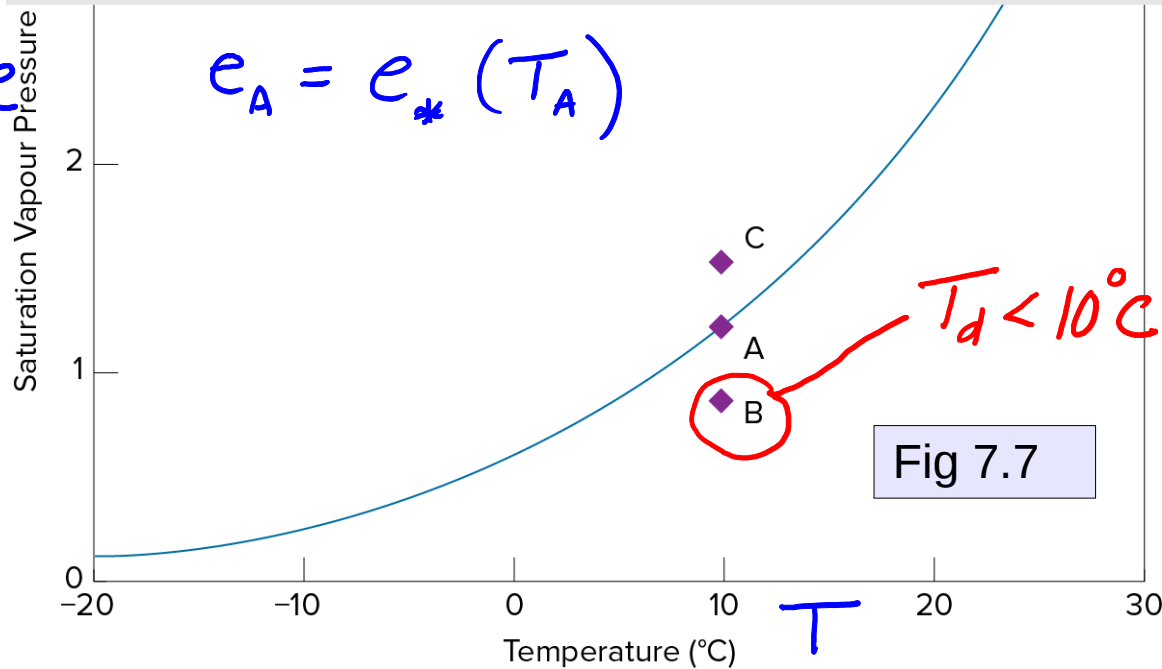$$T = e_*^{-1}(e_*)$$ 　insert　 e_* (equilibrium vapour pressure) and get out T

Fig 7.4

TABLE 7.1 | Saturation vapour pressure at different air temperatures, or vapour pressure at different dew-point temperatures,[a] over flat surfaces of pure liquid water or ice.[b]

Air Temperature/Dew-Point Temperature (°C)	Saturation Vapour Pressure/Vapour Pressure (kPa) over water (ice)	Air Temperature/Dew-Point Temperature (°C)	Saturation Vapour Pressure/Vapour Pressure (kPa)
		0	0.611
		1	0.657
−14	0.181 (0.208)	2	0.705
−13	0.198 (0.225)	3	0.758
−12	0.217 (0.244)	4	0.813
−11	0.238 (0.264)	5	0.872
−10	0.260 (0.286)	6	0.935
−9	0.284 (0.310)	7	1.001
−8	0.310 (0.335)	8	1.072
−7	0.338 (0.362)	9	1.147
−6	0.369 (0.391)	10	1.227
−5	0.402 (0.421)	11	1.312
−4	0.437 (0.455)	12	1.401
−3	0.476 (0.490)	13	1.497
−2	0.517 (0.528)	14	1.598
−1	0.562 (0.568)	15	1.704
0	0.611 (0.611)		

If $T=10$, what is the equilib.v.p.?

If $T_d=10$, what is the v.p.?

If $T=10$, what is the v.p.?

If $T_d=10$, what is the equilib.v.p.?

If $e=13.12$ hPa, what is T_d?

If $e^*=13.12$ hPa, what is T_d?

If $e^*=13.12$ hPa, what is T?

e

$$e_A = e_*(T_A)$$
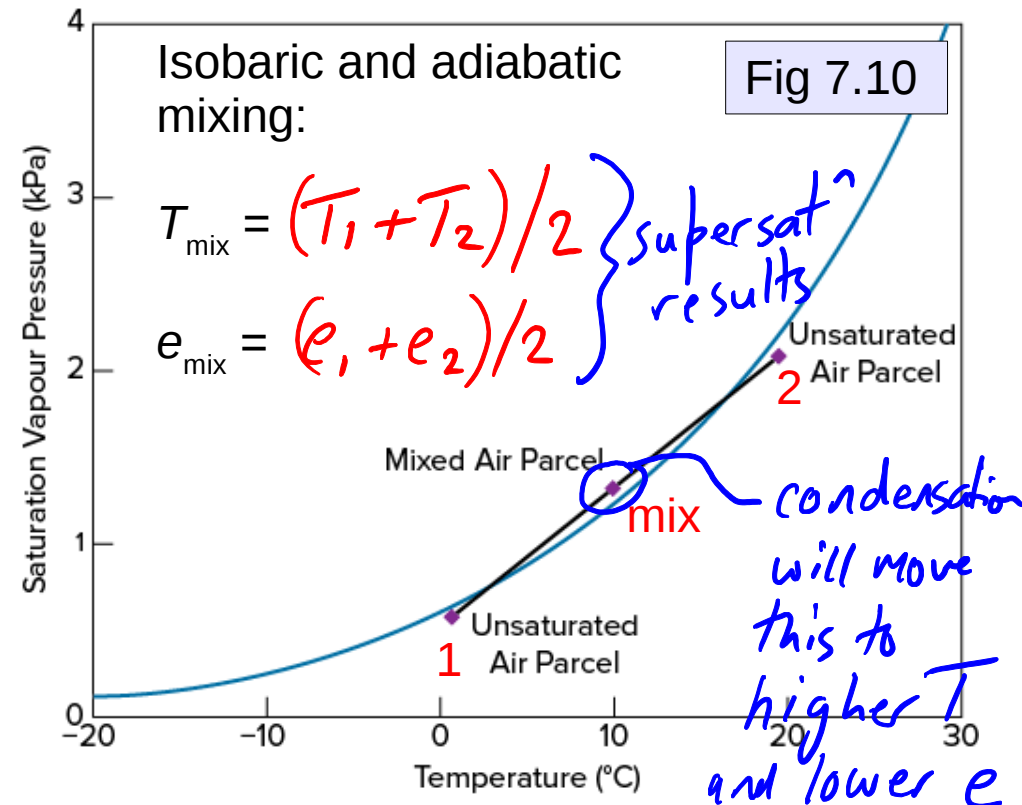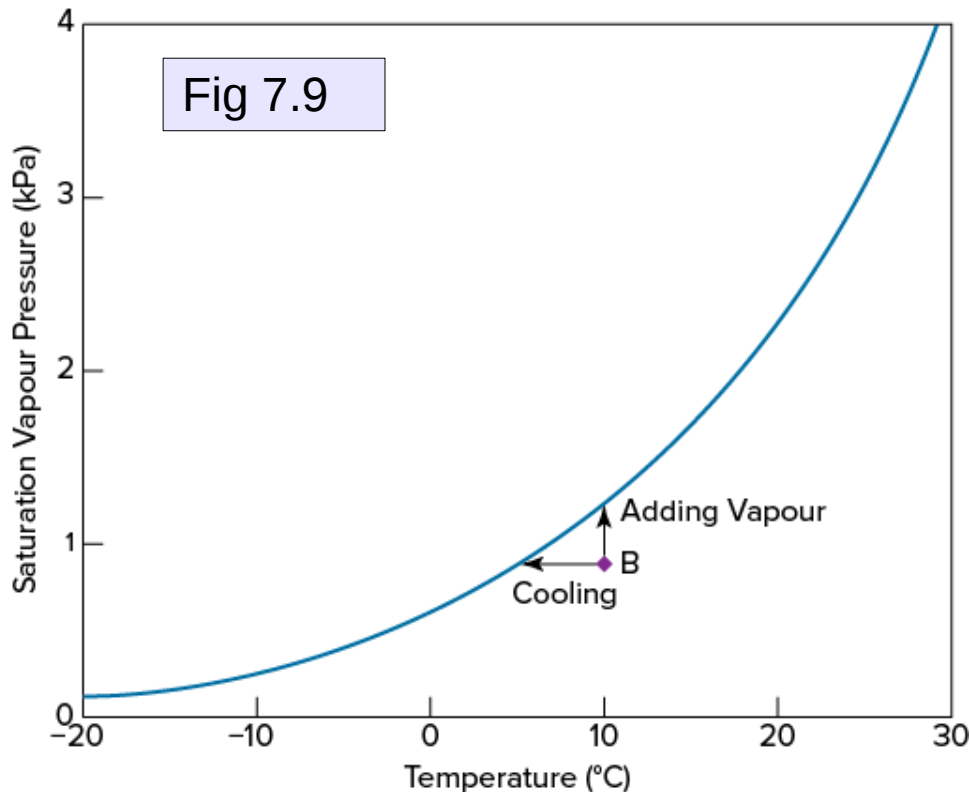
Saturation Vapour Pressure

$T_d < 10°C$

T

Temperature (°C)

Fig 7.7

Which statepoint on Fig 7.7 represents unsaturated air? B

Which represents supersaturated air? C

What three processes can bring air to saturation?

Fig 7.9

Saturation Vapour Pressure (kPa)

Adding Vapour

B

Cooling

Temperature (°C)

Isobaric and adiabatic mixing:

Fig 7.10

$$T_{mix} = (T_1 + T_2)/2 \quad \rbrace \text{supersat}^n \text{ results}$$

$$e_{mix} = (e_1 + e_2)/2 \quad \rbrace$$

Saturation Vapour Pressure (kPa)

Unsaturated Air Parcel

2

Mixed Air Parcel

mix — condensation will move this to higher T and lower e

Unsaturated

1 Air Parcel

Temperature (°C)

- humid parcels are positively buoyant (relative to dry parcels at same temperature and total pressure)

- common humidity variables, and their connection with one another

- notion of a "tracer" of parcel identity

- understanding definition/meaning of equilibrium vapour pressure

- 1:1 relationship between vapour pressure e and dewpoint T_d

- using the $e^*(T)$ table (or curve) to get e^* given T, or, e given T_d
  or T given e^*, or, T_d given e

"Because the relationship between dew-point temperature and vapour pressure is the same as that between air temperature and saturation vapour pressure, we can use Table 7.1 to determine dew-point temperature given vapour pressure" (p164)

TABLE 7.1 | Saturation vapour pressure at different air temperatures, or vapour pressure at different dew-point temperatures,[a] over flat surfaces of pure liquid water or ice.[b]

Air Temperature/Dew-Point Temperature (°C)	Saturation Vapour Pressure/ Vapour Pressure (kPa)	Air Temperature/Dew-Point Temperature (°C)	Saturation Vapour Pressure/ Vapour Pressure (kPa)
−40	0.013 (0.019)	0	0.611
−39	0.014 (0.021)	1	0.657