EAS327 "Environmental Instrumentation" Exam 16 Apr,2003

Professor: J.D. Wilson Time available: 120 mins Value: 30%

Closed book exam. Data and formulae given at back.

Multi-choice $(30 \text{ x } \frac{1}{2}\% \rightarrow 15\%)$

1. The vapour pressure in an unsaturated soil is determined by ____
   (a) soil gravimetric water content
   (b) soil volumetric water content
   (c) pH of the soil solution
   (d) salinity of the soil solution
   (e) soil water potential ✓✓

2. Soil water potential is usually expressed in the unit ____
   (a) kg kg^{-1}
   (b) m^3 per m^3
   (c) kg m^{-3}
   (d) m^3 kg^{-1}
   (e) MPa ✓✓

3. A tensiometer measures ____ and a TDR probe measures ____
   (a) volumetric water content; water potential
   (b) water potential; volumetric water content ✓✓
   (c) water potential; water potential
   (d) volumetric water content; volumetric water content
   (e) matric component of soil water potential; osmotic component of soil water potential

4. When we use a sensor at height h to directly measure the mean vertical flux of water vapour $E(h) = \overline{w' \rho_v '}$ (where w is the vertical velocity and ρ_v the absolute humidity) we are using the ____ method.
   (a) Bowen-ratio energy-balance
   (b) fast-hygrometry
   (c) eddy-covariance ✓✓
   (d) flux-gradient
   (e) surface water budget

5. With reference to the preceding question, the region of the surface upwind from the sensor that contributes to the flux $E(h)$ is called the ____

   (a) area source
   (b) plume
   (c) line source
   (d) footprint ✓✓
   (e) symmetric region

6. An important assumption made in this technique is that ____

   (a) a contribution $\overline{w}\,\overline{\rho}_v$ to the flux $E(0,h)$ can be neglected ✓✓
   (b) mean vertical velocity greatly exceeds the fluctuation ($\overline{w} >> w'$)
   (c) mean absolute humidity greatly exceeds the fluctuation ($\overline{\rho}_v >> \rho_v'$)
   (d) the flux is positive, ie. $E(0,h) \geq 0$
   (e) none of the above

7. Suppose that the variable $x(t)$ is defined as a sum of random steps, ie. $x(t+\Delta t) = x(t) + \Delta x$, where each step Δx is drawn independently from a Normal (ie. Gaussian) distribution having zero mean and unit standard deviation ($\sigma_{\Delta x} = 1$). According to the Central Limit Theorem, if initially $x = 0$ then after 16 steps, with 95% probability

   (a) $x \leq 2$ x 1/16
   (b) $x \leq 2$ x 1/4
   (c) $|x| \leq 2$ x 1/4 ✓✓
   (d) $x \geq 2$ x 1/4
   (e) $x = 0$

8. A soil heat flux plate and a thermometric net radiometer both produce a thermopile output signal responding to the temperature difference between upper and lower faces of a horizontal disc. The net radiometer has a transparent dome over its faces ____

   (a) to prevent deterioration of the specially-painted surface
   (b) to reject longwave photons
   (c) to trap longwave photons with 100% efficiency
   (d) to eliminate any signal dependency on windspeed ✓✓
   (e) to isolate it from electrical noise

9. Regarding the thermometric net radiometer, probability of absorption $a(\nu, \phi)$ of a photon of frequency ν and energy $h\nu$ (where h is Planck's constant) striking the plate at incidence angle ϕ (measured away from the normal to the plane of the radiometer), should ideally be ____

   (a) proportional to $h\nu$ and proportional to $\cos\phi$
   (b) independent of $h\nu$ and proportional to $\cos\phi$
   (c) independent of $h\nu$ and independent of ϕ ✓✓
   (d) proportional to $(h\nu)^2$
   (e) proportional to $h\nu$ and proportional to $\sin\phi$

10. In a wind which is so strong that it always exceeds the "threshold" of a propellor anemometer, which attribute(s) compromise its ability to measure the standard deviation of the wind velocity along its axis?

   (a) threshold error
   (b) inertia error
   (c) cosine error
   (d) threshold and inertia errors
   (e) inertia and cosine errors ✓✓

11. On the basis of the analogy between the propellor anemometer and the wind-driven flat plat fixed on a track so that it is free to run crosswind, the rotation rate (ω) of a propellor anemometer is probably in ____ relationship with the component ($U\cos\theta$) of the wind along its axis.

   (a) logarithmic
   (b) exponential
   (c) quadratic
   (d) non-linear
   (e) linear ✓✓

12. The "pitch angle" (θ) of each blade of a propellor anemometer varies with distance r from the axis of rotation, $\theta = \theta(r)$. This is arranged so that

   (a) at the equilibrium rotation rate, angle of incidence of the *relative* wind on the blade vanishes at all points along the blades ✓✓
   (b) calibration will be independent of air (or fluid) density
   (c) inertia of the anemometer is minimized
   (d) cosine error is minimized
   (e) none of the above

13. To measure temperature by exploiting the fact that in an ideal gas the speed of sound c is related to the (square root of the) temperature T, a sonic anemometer-thermometer

    (a) must measure backward **and** forward sound travel times along three paths
    (b) must measure backward **or** forward sound travel times along three paths
    (c) must measure backward **and** forward sound travel times along at least two paths
    (d) need only measure backward **and** forward sound travel times along a single path ✓✓
    (e) need only measure backward **or** forward sound travel time along a single path

14. For a 3-dimensional sonic anemometer providing all three components (u, v, w) of the velocity vector (where w is the vertical component), instantaneous azimuthal wind direction β (the direction a perfect wind vane would indicate) is

    (a) $\beta = \arctan(w/u)$
    (b) $\beta = \arctan\left(w/\sqrt{u^2 + v^2}\right)$
    (c) $\beta = \arctan(v/u)$
    (d) $\beta = \arctan(u/v)$
    (e) Either (c) or (d), depending on whether the wind direction $\beta = 0$ is defined as a wind parallel to or perpendicular to the u axis. ✓✓

15. An "analog output" from an environmental data-logger is driven by the logged number $V_i = V(i\Delta t)$ representing a signal V(t) at time $i\Delta t$ (where Δt is the sampling interval). If the signal has a varying component with standard deviation σ and autocorrelation timescale τ, such an analog output will exhibit a series of apparently random (uncorrelated) steps if

    (a) $\Delta t >> \tau$
    (b) $\Delta t = \tau$
    (c) $\Delta t << \tau$
    (d) $\Delta t >> \tau$, provided the full-scale-range of the analog output is set so as to discriminate changes of order σ ✓✓
    (e) none of the above

16. The time constant of a thermometer is affected by which factors?

    (a) Properties of the fluid in which it is placed
    (b) State of motion of the fluid in which it is placed
    (c) Its own bulk heat capacity
    (d) Its own geometry
    (e) All of the above ✓✓

17. In your view, which of the following should be the first line of investigation in response to the malfunction of a complex electronic measuring system:

   (a) improper electrical connections and/or improper programming ✓✓
   (b) deviation of resistors from the nominal values inscribed on them
   (c) deviation of capacitors from the nominal values inscribed on them
   (d) deviation of circuit relationships from Ohm's Law and other ideal characteristics of the components
   (e) noise picked up from other circuits

18. To examine a d.c. circuit to be sure of the absence of "noise", one should use a

   (a) multimeter
   (b) oscilloscope ✓✓
   (c) chart recorder
   (d) data-logger
   (e) grounded receiver

19. "Linearisation" refers to

   (a) choosing an appropriate full scale range on a voltage receiver
   (b) checking the input offset voltage of a voltage receiver
   (c) adjusting the calibration factors α, β of a linear sensor ($S = \alpha V + \beta$, where V is the voltage signal and S the property being measured)
   (d) selecting a.c. or d.c. coupling so as to best display a signal
   (e) representing a curve $y(x)$ by a straight line, over a narrow region of the x axis ✓✓

20. "Free convection" occurs when ____

   (a) the Grashof number is very large w.r.t. the Nusselt number
   (b) the Grashof number is very small w.r.t. the Nusselt number
   (c) the Grashof number is very large w.r.t. the thermal diffusivity
   (d) the Grashof number is very small w.r.t. the Reynolds number
   (e) none of the above ✓✓

21. To estimate the uncertainty in $\frac{a+b}{c}$ given the uncertainties in the factors, one should

   (a) sum the relative errors in a, b, c
   (b) sum the absolute errors in a, b, c
   (c) sum the absolute errors in a and b with the relative error in c
   (d) sum the absolute errors in a and b then subtract the relative error in c

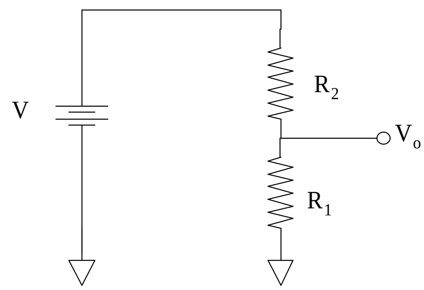
(e) none of the above ✓✓

22. A voltage receiver can be used to measure a current signal I by passing the current through a load resistor R_L connected across the input terminals of the receiver. If the input resistance of the receiver (before connection of R_L) is R_{in} then the input resistance R'_{in} with the load in place is

(a) $R'_{in} = R_{in}$
(b) $R'_{in} > R_{in}$
(c) $R'_{in} < R_{in}$ ✓✓
(d) $R_{in} + R_L$
(e) none of the above

23. With reference to the voltage divider shown, if $V = 6$ [volts], $R_1 = 2.5K\Omega$ and $R_2 = 7.5K\Omega$, then V_o [volts] is:

(a) 1.5 volts ✓✓
(b) 2 volts
(c) 4.5 volts
(d) 12 volts
(e) 18 volts



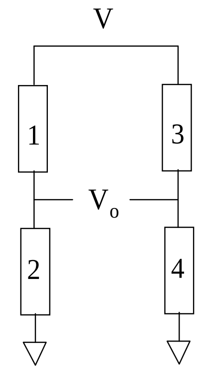
24. If the continuous random variable x is defined on range $a \leq x \leq b$ and has probability density function $f(x)$ then the variance of x is ____ (where μ_x is the mean).

(a) $\sigma_x = \int_a^b x\, f(x)\, dx$
(b) $\sigma_x^2 = \int_{-\infty}^{\infty} x^2\, f(x)\, dx$
(c) $\sigma_x = \int_a^b (x - \mu_x)\; f(x)\, dx$
(d) $\sigma_x^2 = \int_a^b (x - \mu_x)^2\; f(x)\, dx$ ✓✓
(e) $\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2\; f(x)\, dx$

25. A pitot tube is unsuitable as a velocity sensor for use in environmental science because

(a) its time constant is too long
(b) it is linear
(c) it is non-linear
(d) its directional response follows a cosine law
(e) its directional response does not follow a cosine law ✓✓

26. Given two identical thermistors R_{1T}, R_{2T} and two identical control resistors R_{1c}, R_{2c}, a differential temperature sensor could be constructed by placing ____ in the full bridge shown



(a) one thermistor in each of slots 1,2

(b) one thermistor in each of slots 3,4

(c) one thermistor in each of slots 1,3

(d) one thermistor in each of slots 2,4

(e) both (c) and (d) would work ✓✓

27. Regarding a differential voltage receiver, if one were to measure the input resistances R^+, R^- from the input terminals (labelled V^+, V^-) to the receiver ground (nb! without any signals connected),

(a) both would be zero

(b) one would be zero

(c) they would be equal and opposite

(d) they would be "large" and equal ✓✓

(e) none of the above

28. If the atmosphere is unstratified (ie. "neutral"), a measurement of the covariance $\overline{w'T'}$ would indicate that

(a) there is no energy supply to sustain evaporation

(b) the net radiation is zero

(c) the vertical velocity fluctuation $w' = 0$

(d) the vertical velocity statistic $\overline{w'} = 0$

(e) the eddy heat flux $Q_H = \rho c_p \, \overline{w'T'} = 0$ ✓✓

29. Ideally, so long as a battery retains some charge ($q > 0$), the voltage V across its terminals is constant. Thus the charge-voltage ($V - q$) characteristic of a battery is

(a) exactly that of a capacitor

(b) exactly that of a resistor

(c) unlike the $V - q$ characteristic of a capacitor ✓✓

(d) Ohm's Law

(e) none of the above

30. If the output $V_o(t)$ of the divider shown below is intended to integrate the input $V(t)$, slot "1" should take a ____ while slot "2" should take a ____

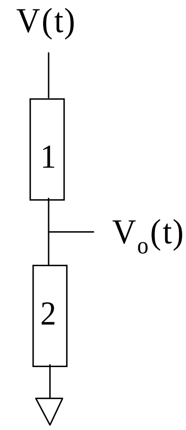
(a) resistor; resistor

(b) capacitor; capacitor

(c) resistor; capacitor ✓✓

(d) capacitor; resistor

(e) capacitor; short circuit



Calculations (2 x 4.5% → 9%)

Answer any **two** questions, showing your working.

1. You are given the following data pertaining to the surface energy balance on large flat field of a bare soil over a 30 minute interval on a summer afternoon: (i) A heat flux plate placed at a depth of 0.05 m in the soil and having calibration factor 0.75 [millivolt (W m^{-2})$^{-1}$] has given a mean signal over this period of 15 millivolts; (ii) Thermistors placed at depths of (0.01, 0.02, 0.04) m below the soil surface indicated temperature increases over the interval of $(2, 1, 0.5)^o$C; and (iii) Volumetric heat capacity of the soil is $C = 2$ x 10^6 J m^{-3} K^{-1}. From this information, estimate the change in the mean temperature of the soil layer above the heat flux sensor, and use that figure to estimate soil heat flux density $Q_G(0)$ at the soil-air interface.

   Answer: the soil heat flux at a depth $z = 0.05\ m$ is $Q_G(0.05) = 20$ [W m^{-2}], but you want the flux $Q_G(0)$. You plot the vertical profile of the temperature change over the 30 minutes, and formulate a strategy (I accepted any strategy!) to determine the area $A \sim 0.06$ [m K]... a summation of rectangles and triangles. The implied heat storage over the thirty minutes is $CA \sim 1.2$ x 10^5 [J m^{-2}] and dividing by $30 * 60$ [sec] this corresponds to a rate of storage ~ 65 [W m^{-2}]. So $Q_G(0)$ must exceed $Q_G(0.05)$ by this amount: hence, $Q_G(0) \sim 85$ [W m^{-2}].

2. Consider the temperature reading T of a cylindrical thermometer with radius $R = 1.5$ mm on a sunny afternoon when air temperature $T_a = 27^o$ C and the windspeed is only $U = 1$ m s^{-1}. Contrast the steady-state radiation error

$$\delta T = T - T_a = \frac{r_H Q^*}{\rho_a c_{pa}} \tag{1}$$

if the thermometer is unshielded (in which case assume the net radiation load on the thermometer is $Q^* = 700$ W m^{-2}) with the error if the thermometer is enclosed in a ventilated radiation shield. For the latter case assume the radiation shield has an interior surface temperature (as "seen" by the thermometer) $T_{int} = T_a + 1^oC$, and that the ventilation speed is $U = 4$ m s^{-1}; assume also that the shield is 100% effective at rejection of shortwave energy, ie. that $Q^* = L^*$, and that net longwave radiation can be calculated as $L^* = 4\sigma T_a^3 \left[T_{int} - T\right] = 4\sigma T_a^3[(T_{int} - T_a) - \delta T]$. To evaluate r_H, in both cases you may pre-suppose the situation is one of forced convection. Pick plausible values for all necessary variables or parameters that have not been provided.

Answer:

Unshielded: diameter $d = 0.003$ m giving $R_e = 200, N_u = 8.3, r_H = 17$ s/m. Assume $\rho c_p =$ (reasonable value, say, 1000)... $\delta T \sim 12^o$ C.

Ventilated, $R_e \rightarrow 800, N_u \rightarrow 17, r_h \rightarrow 8.4$ s/m. One has the equation

$$4\sigma T_a^3 \ (1 - \delta T) = \rho c_p \frac{\delta T}{r_H} \tag{2}$$

where the coefficient $4\sigma T_a^3 = 6.12...$ easily solved to get $\delta T \sim 0.05^oC$

3. A wet- and dry-bulb thermometer has indicated $T, T_w = 21, 17^o$C, and the local pressure is $p = 93$ kPa. Determine the vapour pressure e and dewpoint T_d, the absolute humidity ρ_v, the specific humidity q and the relative humidity RH.

Answer: Unfortunately, in the data where I gave you the equation for $e_S(T)$, I mistakenly cited the units for T in this equation as being Kelvin... they should have been Celcius. The consequence of using Kelvin is that one obtains vapor pressures that are far too large, as many of you realised. I accepted the answers either way.

Long Answer (6 %)

Answer **one** question from this section. Write up to two pages, including diagrams if they are helpful to your logic. (In the marking, I assigned two marks for the "structure"... is the response tidy, and does it have an orderly logic... a sense of an introduction, a main argument, and a conclusion?)

1. Making reference to hypothetical data containing random errors, explain under what circumstances it might be useful to determine the "least sum-of-squares fit" $T = \alpha V + \beta$ to a set of paired-values (V_i, T_i) obtained in a calibration of a thermometer (signal V [volts] representing temperature T [K]). Give your qualitative interpretation of the procedure involved in such a "best fit".

   Answer: (i) you suspect the relation between the V_i and the T_i is linear; (ii) you define a model $T_i^* = \alpha V_i + \beta$ in terms of which the errors are $e_i = T_i - T_i^*$ and you define a sum of squares of errors $SS = \sum_i e_i^2$ (squared distances of measured points T_i, V_i from the fitted line); (iii) this sum will depend on the as yet unspecified coefficients α, β of the model, and one finds that pair of values that make SS a minimum

   or

2. Consider a temperature signal $T(t) = \overline{T} + T'$ whose mean, standard deviation, and auto-correlation timescale over an interval (T_1, T_2) are $\overline{T}, \sigma_T, \tau_T$. Explain why it is **not** wrong to use a set of N samples ($T_i^{meas} = T^{meas}(t_i),\ i = 1...N$) from a "slow" temperature sensor (sensor time constant $\tau >> \tau_T$) to estimate the average temperature $\overline{T}$ over the interval, despite the fact that in general (due to the sensor's "lag") the signal $T^{meas}(t)$ from the sensor is not equal to the true temperature, ie. $T^{meas}(t_i) \neq T(t_i)$. (Hint: the Central Limit Theorem should star in your answer.)

   Answer: It is intuitive to most of us that although a "slow" sensor will not pick up (reveal) fast fluctuations, it will correctly indicate the underlying average signal - otherwise, for example, an ordinary mercury thermometer would be useless. *Technically*, the argument is as follows. The effect of the *inertia* of the sensor is that *each* *sample* T_i^{meas} is an *average* of the signal over a certain preceding interval of order τ in duration. When we average a bunch (N) of these to get $\overline{T} = (1/N) \sum_i T_i^{meas}$ we know from the Central Limit Theorem that the resulting estimate $\overline{T}$ is unbiased relative to the true underlying average, and has a sampling distribution about the true average that is proportional to $\sigma_T^{meas}/\sqrt{N}$, where the *measured* standard deviation σ_T^{meas} is far less than the true signal standard deviation σ_T because the sensor is slow.

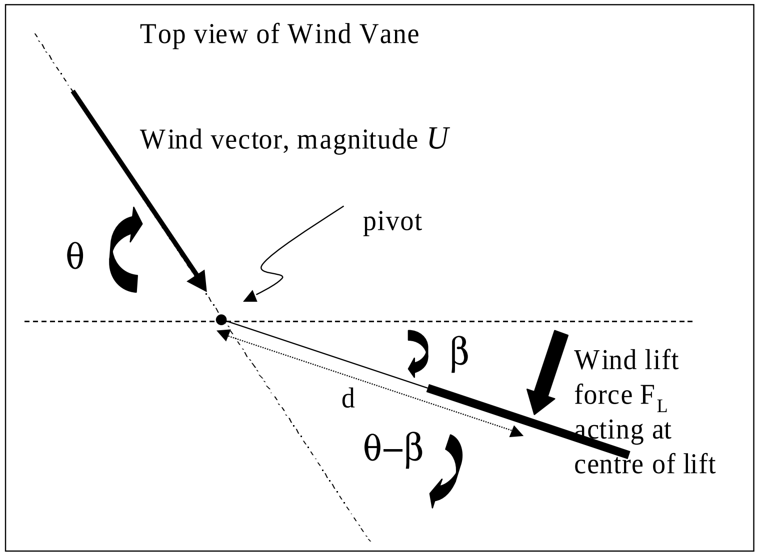
<u>or</u> (see next page)

3. The governing equation of a frictionless, undamped wind vane is

$$I \frac{d^2\beta}{dt^2} = d\ \rho\ A\ \alpha\ (\theta - \beta)\ U^2 \tag{3}$$

where I is the moment of inertia, and $\beta(t)$ is the orientation-angle of the vane with respect to north, which is intended to track the wind direction $\theta(t)$ (the equation is valid for small magnitudes of the "error angle" $\theta - \beta$). The vane is a thin flat plate of area A, mounted on a shaft that pivots on a bearing; ρ is the air density; U is the magnitude of the wind velocity vector, which we shall assume lies in the plane perpendicular to the axis of rotation of the shaft of the vane; α is a dimensionless constant; and d is a length.

From the description given above, create a large, tidy, well-labelled sketch of this wind vane, and making reference to your sketch, comment on the origin and meaning of the terms in the governing equation. (SEE OVER FOR ANSWER IN BRIEF)



Answer: In addition to the 2 marks reserved for structure of the response, I assigned 2 marks for the diagram (see above, or see the course web page), and the other 2 marks for covering the origin of the terms, roughly as follows...

The l.h.s. is the rate of change of the angular momentum $I\dot{\beta}$, and the r.h.s. is the wind torque driving the angular acceleration, ie. the wind "lift" force $F_L = \rho A \alpha (\theta - \beta) U^2$ times the distance d to the point of application of the lift force (roughly, the distance from the pivot to the centre of the plate of the vane).

Wind "pressure" on an object scales with ρU^2 [Pa], and we obtain a quantity with units of force upon multiplying by the area A of the wing or plate. But to get the lifting force F_L we must multiply by a dimensionless "lift coefficient" c_L and this coefficient depends on the angle of attack of the wind relative to the plate $(\theta - \beta)$; for small values of $(\theta - \beta)$ one may assume the lift force is linearly related to the error angle, ie. write $c_l = \alpha(\theta - \beta)$ where α is dimensionless.

We have accounted for all symbols appearing on the r.h.s. Concerning the l.h.s., the moment of inertia I depends on the mass distribution of the vane as a function of radial distance from the pivot, a reasonable guess being $I = md^2$ where m is the mass of the plate (this neglects the mass of the shaft of the vane; and assumes the centre of lift and the centre of mass of the vane plate coincide).

Data:

- Saturation vapour pressure $e_S(T)$ in [millibars] versus temperature T [^{o}C]

$$e_S(T) = 6.112 \exp\left(\frac{17.67\,T}{243.5 + T}\right) \tag{4}$$

- Dewpoint temperature [^{o}C] for a given value of the actual vapour pressure e [mb]

$$T_d(e) = \frac{243.5\ \ln(e/6.112)}{17.67 -\ \ln(e/6.112)} \tag{5}$$

- Kinematic viscosity of air: $\nu \approx 1.5 \text{ x } 10^{-5}\ [m^2 s^{-1}]$

- Thermal diffusivity[1] of air: $\kappa \equiv D_H \approx 2.1 \text{ x } 10^{-5}\ [m^2 s^{-1}]$

- Specific heat capacity of air at constant pressure: $c_p \approx 1010\ [J\ kg^{-1} K^{-1}]$

- Latent heat of vaporization: $L_v \approx 2.5 \text{ x } 10^6\ [J\ kg^{-1}]$

- The equation to determine the vapour pressure e [Pa] from wet- and dry-bulb temperatures T_w, T is

$$e = e_S(T_w) - \gamma\ (T - T_w) \tag{6}$$

  where the psychrometric constant $\gamma = \frac{p\ c_p}{0.622\ L_v}$

- $P = \rho\, R\, T$

  The ideal gas law. P [Pascals], pressure; ρ, $[kg\ m^{-3}]$ the density; T [Kelvin], the temperature; and $R = 287\quad [J\ kg^{-1}\ K^{-1}]$, the specific gas constant for air.

- $e = \rho_v\ R_v\ T$

  The ideal gas law for water vapor. e [Pascals], pressure; ρ_v, $[kg\ m^{-3}]$ the absolute humidity (ie. vapor density); T [Kelvin], the temperature; and $R_v = 462\quad [J\ kg^{-1}\ K^{-1}]$, the specific gas constant for water vapor.

- $Q_H = \rho\ c_p\ \frac{T_1 - T_2}{r_H}$

  Ohm's law model for sensible heat exchange.

[1] Symbols κ, D_H are both used for this quantity.

- $Q^* = Q_H + Q_E + Q_G$

  Surface energy balance on the ground-air interface plane. All fluxes are in $[W\ m^{-2}]$. Q^* the net radiation, positive if directed towards the ground surface; Q_H, Q_E the sensible heat flux and the latent heat flux, positive if directed away from the ground surface; Q_G the soil heat flux at the ground-air interface, positive if directed away from the ground surface; Q_S, the storage term.

- $N_u = 0.32 + 0.51\ R_e^{0.52}\ (0.1 \leq R_e \leq 1000),\ N_u = 0.24\ R_e^{0.6}\ (10^3 \leq R_e \leq 5 \text{ x } 10^4)$

  Nusselt number versus Reynolds number ($R_e = Ud/\nu$) for a cylinder (diameter d) in a current of air of speed U (forced convection).

- $r_H = \frac{d}{D_H N_u}\ [s\ m^{-1}]$

  Resistance r_H for heat transfer.

- $a = \overline{a} + a'$

  "Reynolds decomposition" of a variable $a(t)$ into the sum of its mean $\overline{a}$ and the instantaneous fluctuation $a'(t)$

- $E[g(x)] = \int_a^b\ f(x)\ g(x)\ dx$

  Formula for the expected value $E[\ ]$ of any function $g(x)$ of the random variable x which is defined on the range $a \leq x \leq b$ and which has probability density function $f(x)$.

- $m = (\overline{xy} - \overline{x}\ \overline{y}) \,/\, \left(\overline{x^2} - (\overline{x})^2\right),\ b = \overline{y} - m\ \overline{x}$

  Formulae for the slope (m) and intercept (b) of the "best least squares" straight line fit to a set of given data (y_i, x_i).