EAS327, "Environmental Instrumentation" <u>Mid-term</u> 7 Apr, 2005

<u>Professor</u>: J.D. Wilson <u>Time available</u>: 80 mins <u>Value</u>: 20%

Instructions: Closed book exam. Please record your answers in the exam booklet. Pertinent data are at the back.

Multi-choice (20 x $\frac{1}{2}$% → 10%)

1. If $x(t)$ is a stationary random signal with zero mean and unit standard deviation, the autocorrelation function for x is $R(\xi) =$ ____

   (a) $\overline{x(t) - x(t+\xi)}$
   (b) $\overline{x^2(\xi)}$
   (c) $\overline{x(t)\, x^{-1}(t+\xi)}$
   (d) $\overline{x(t)\, x(t+\xi)}$ ✓✓
   (e) $\sqrt{\overline{x(t)\, x(t+\xi)}}$

2. A random variable x is decomposed into the sum of its mean m_x and a deviation x', ie. $x = m_x + x'$. The mean value E$[x']$ of x' satisfies:

   (a) $\mathrm{E}[x'] = m_x$
   (b) $\mathrm{E}[x'] = s_x$, the standard deviation of x
   (c) $\mathrm{E}[x'] = 0$ ✓✓
   (d) $\mathrm{E}[x'] = s_x^2$, the variance of x
   (e) none of the above

3. The uncertainty due to random error, in a single measurement of temperature by a particular noisy (but unbiased) thermometer, is $\delta T = 1^o$ C. If the true temperature is constant, and it is estimated by an average $\overline{T}$ over a sample $T_1, T_2, T_3 ... T_N$ taken with this instrument, then if $N = 25$, the uncertainty in $\overline{T}$ is ____ ^{o}C

   (a) 0.04
   (b) 0.2 ✓✓
   (c) 0.5
   (d) 2.5
   (e) 5

4. Suppose temperature is fluctuating about its mean value with standard deviation $\sigma_T = 3^o$ C, and that $\overline{T}$ is an average over $N = 25$ independent samples, taken with an unbiased, noise-free sensor able to follow all fluctuations ($T_{meas} = T_{true}$). Then with 95% probability $\overline{T}$ is no further than $\epsilon =$ ____ ^{o} C from the true mean

   (a) 1/5
   (b) 2/5
   (c) 3/5
   (d) 6/5 ✓✓
   (e) 6/25

5. A cylindrical chamber containing volume V and having basal area A is placed over the soil at time $t = 0$. The air inside the chamber is continuously mixed by a fan, and the concentration (C, kg m^{-3}) of a particular gas escaping from the soil and accumulating inside the chamber is observed to increase over the next T [s] by amount ΔC. The average rate of release of the gas, per unit ground area, over the period T was ____

   (a) $\frac{V}{A}\,\frac{\Delta C}{T}$ ✓✓
   (b) $V\,\frac{\Delta C}{T}$
   (c) $\frac{\Delta C}{T}$
   (d) $\frac{A}{V}\,\Delta C$
   (e) $TV\,\frac{\Delta C}{A}$

6. Suppose it is known that (on theoretical grounds) the current output I from a certain electromagnetic absorption hygrometer varies with absolute humidity ρ_v according to $\log_e I \propto -\,\rho_v$. A two-point calibration based on the observations of the table below indicates the calibration law is ____

I [mA]	ρ_v [g m^{-3}]
1.00	0.0
0.05	10.0

   (a) $\rho_v = -\,0.30\ \log_e I$
   (b) $\rho_v = -\,0.095\ \log_e I$
   (c) $\rho_v = -\,10.5\ \log_e I$
   (d) $\rho_v = -\,3.33\ \log_e I$ ✓✓
   (e) $\rho_v =\ 0.30\,(\log_e I)^{-1}$

7. For many simple instruments the input-output relationship can be represented

$$y(t) = \int_{\xi=0}^{\infty} x\,(t-\xi)\ \exp(-\xi/\tau)\ d\xi$$

(this is the I/O characteristic of a linear, 1st-order system). The input signal is ____ and the "system weighting function" is ____

(a) y; x

(b) x; y

(c) x; τ

(d) y; $\exp(-\xi/\tau)$

(e) x; $\exp(-\xi/\tau)$ ✓✓

8. An example of an environmental sensor having the above temporal response characteristic is ____

(a) a wind vane

(b) a sonic anemometer

(c) an ordinary thermometer (eg. mercury-in-glass) ✓✓

(d) a cup anemometer

(e) none of the above

9. Let $\dot{\theta}$ be the rotation rate of a cup anemometer, and let I be the moment of inertia of the cup-assembly. The statement that $I\ d\dot{\theta}/dt = 0$ implies ____

(a) there is no wind

(b) the cups are not rotating

(c) the cups are in a state of steady rotation

(d) the angular acceleration of the cup assembly is zero

(e) both (c) and (d) apply ✓✓

10. The provision of air bearings for a propellor anemometer overcomes which error?

(a) inertial error

(b) cosine error

(c) non-linearity at high speeds

(d) threshold error ✓✓

(e) both (a) and (b)

11. The "pitch angle" (θ) of each blade of a propellor anemometer varies with distance r from the axis of rotation, $\theta = \theta(r)$. This is arranged so that

   (a) at the equilibrium rotation rate, angle of incidence of the *relative* wind on the blade vanishes at all points along the blades ✓✓

   (b) calibration will be independent of air (or fluid) density

   (c) inertia of the anemometer is minimized

   (d) cosine error is minimized

   (e) none of the above

12. A propellor velocity-sensor has a sensitivity of 10 [mV (m s^{-1})$^{-1}$]. It is placed obliquely in a uniform stream, at an angle of 30^o with respect to the flow. If its output is 20 mV, then the velocity of the stream, assuming an ideal cosine response, is ____ [m s^{-1}]

   (a) $20/\cos(30)$

   (b) $20\ \cos(30)$

   (c) $20/(10\cos(30))$ ✓✓

   (d) $20\ \tan(30)$

   (e) $\sqrt{20^2 + \cos^2(30)}$

13. Suppose the random variable x has p.d.f. $f(x) = \begin{cases} 0 & x < -\frac{1}{2} \\ \alpha\ (1-x) & -\frac{1}{2} \le x \le \frac{1}{2} \\ 0 & x > \frac{1}{2} \end{cases}$

   In order that $f(x)$ should be a p.d.f., $\alpha =$ ____ (Hint: you can get the answer either by geometric reasoning, or by doing the Calculus)

   (a) $1/x$

   (b) x

   (c) x^2

   (d) 1 ✓✓

   (e) 0

14. The variable x has been measured to have the value $X \pm \epsilon_x$, where ϵ_x is the absolute uncertainty. The absolute uncertainty in the derived variable $y = (2/x) + 1$ is ____

   (a) $2\epsilon_x$

   (b) $2/\epsilon_x$

   (c) $2/(1+\epsilon_x)$

   (d) $2\epsilon_x/X^2$ ✓✓

   (e) X/ϵ_x

15. The units of spectral radiative intensity I_ν are ____

   (a) W m^{-2} μm^{-1}
   (b) J s^{-1} m^{-2} steradian^{-1}
   (c) W s^{-1} m^{-2} μm^{-1}
   (d) J s^{-1} m^{-2} steradian^{-1}
   (e) J s^{-1} m^{-2} steradian^{-1} μm^{-1} ✓✓

16. In the longwave band, absorbtivity a is equal to emissivity ϵ. Suppose a perfect radiometer is placed in an isothermal room (temperature T) and, facing down at a plane surface whose emissivity is $\epsilon < 1$, measures the upwelling hemispheric longwave radiant energy flux density $L\uparrow$. The radiometer will detect a flux $L\uparrow=$ ____

   (a) $\epsilon\sigma T^4$
   (b) $(1-\epsilon)\sigma T^4$
   (c) $(1-a)\sigma T^4 + \epsilon\sigma T^4$
   (d) σT^4
   (e) (c) and (d) are equivalent, and correct ✓✓

17. A sphere of radius R is held suspended inside a closed, evacuated chamber. The entire system is allowed to come to an isothermal equilibrium at 0^oC. The net radiative energy flux density Q^* [W m^{-2}] at the surface of the sphere is ____

   (a) zero ✓✓
   (b) $\sigma\ 273^4$
   (c) $\sigma\ 273^4\ \pi R^2$
   (d) $\sigma\ 273^4\ 4\pi R^2$
   (e) $\sigma\ 273^4\ \frac{4}{3}\pi R^3$

18. The allwave absorbtivity of a surface exposed to solar and terrestrial radiation can be written $\alpha = (aK + \epsilon L)\,/\,(K + L)$ where K, L are the incident shortwave and longwave radiant energy fluxes, and a is the ____ of the surface

   (a) shortwave reflectivity
   (b) shortwave absorbtivity ✓✓
   (c) shortwave transmissivity
   (d) emissivity
   (e) net radiation

19. A simple scientific model of the net radiometer (with isolating domes) suggests the calibration relationship $\alpha Q^* = (4\epsilon\sigma T^3 + 2k/d)(T_t - T_b)$ where T_t, T_b are the temperatures of the upper and lower surfaces (sensed by a thermopile), α, ϵ are respectively the allwave and longwave absorbtivities, d is the separation of the planes of thermocouple junctions, and k is the conductivity of the slab between upper and lower surfaces. Suppose that rather than being separated from each other by a (somewhat) thermally conductive matrix, the upper and lower planes of thermocouple junctions were thermally isolated ($k = 0$). Where for finite k (which ensures $4\epsilon\sigma T^3 << 2k/d$) one had had a 1:1 (unique) relationship between $T_t - T_b$ (thermopile measurement) and the inferred environmental variable (Q^*), introducing thermal isolation between the junctions would ____ the relationship between $T_t - T_b$ and Q^*.

   (a) enhance
   (b) render more accurate
   (c) render more precise
   (d) render ambiguous ✓✓
   (e) reverse

20. The energy balance of a wet-bulb thermometer, operating ideally at steady state, reduces to ____

   (a) $C\,\frac{dT}{dt} = 0 = A\,(Q^* + Q_H + Q_E) + \ 0$
   (b) $C\,\frac{dT}{dt} = A\,(0 + Q_H + Q_E) + \ P$
   (c) $C\,\frac{dT}{dt} = 0 = P$
   (d) $|Q_H| = |Q_E|$ ✓✓
   (e) $C\,\frac{dT}{dt} = A\,(Q_H + Q_E)$

Short Answer (10 %)

Answer **two** questions from this section. Give diagrams where appropriate. Justify any assumptions or simplifications you make. (**13 April 2005: Answers given at back**)

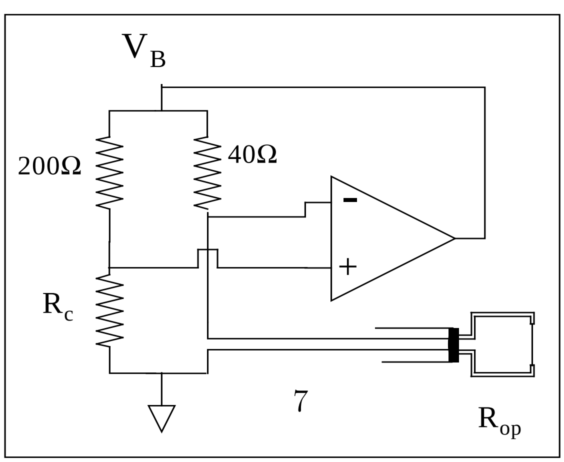
1. You have just bought a cup of coffee from Java-Jive, and you have left your cup, which we shall treat as a styrofoam (conductivity $k = 0.025$ W m^{-1} K^{-1}) cylinder of mean diameter $d = 6$ cm, height $h = 10$ cm and wall thickness $d^* = 2$ mm, sitting freely exposed on a bench outside to a wind of speed $U = 1$ m s^{-1} and temperature $T_a = 20^o$C (assume air density $\rho_a = 1.1$ kg m^{-3}). The interior temperature T_i can be assumed to be well-mixed (uniform), and at the instant you sat your cup down $T_i = 85^o$C.

   Derive a formula for the surface temperature $T_w(t)$ of the outside of the cup, under the approximation that heat is lost down a transfer pathway $T_i \rightarrow T_w \rightarrow T_a$ under the control of two resistances $R_{iw} \equiv d^*/k$ (across the styrofoam) and $R_{wa} = r_H/(\rho_a c_{pa})$ (from outer surface to bulk airstream), where r_H [s m^{-1}] is the conventional convective heat transfer resistance appearing in the Ohm's Law analogy (see data at back; assume fully forced convection). The two resistances are configured in series, and a constant "current" (sensible heat flux Q_H) flows through them.

   From your formula, compute the surface temperature T_w of the cup at $t = 0$ and at $t = 15$ min.

2. A cylindrical hot-film anemometer has a "cold" resistance of $R_0 = 6\Omega$ at temperature $T_0 = 6^o$ C, and its resistance vs. temperature calibration is $R(T) = R_0[1 + \alpha(T - T_0)]$, where $\alpha = 0.0018$ [K^{-1}].

   - calculate the required operating resistance R_{op} if the operating temperature T_{op} is to be 300^oC
   - calculate the required value of the control resistor R_c to ensure balance of the bridge circuit shown below
   - The hot-film has diameter $d = 0.2$ mm. Assuming perfect balance of the bridge in an airstream of velocity $U = 5$ m s^{-1} having temperature $T_a = 20^o$C and density $\rho = 1.1$ kg m^{-3}, estimate the Nusselt number for convective heat loss and thence the required rate (P) of power dissipation in the sensor to balance convective cooling. Deduce the heating current (i) through the sensor, and the bridge voltage V_B.



3. Derive a formula for the rate of heat-loss [W] from the bare, exposed fist (sphere of diameter $d = 10$ cm, approximated as being at temperature T_{sk}) of a gloveless hang-glider pilot, who is at an altitude of about 7000' ASL where air pressure and temperature are about $(p, T_a) = (80$ kPa, -10^o C$)$, and who is flying at speed $U = 10$ m s^{-1} (about 20 mph). Evaluate your solution for the case that $T_{sk} = 10^o$C. Explain in physical terms, perhaps making reference to the "half bridge" (Fig. 1), what controls the skin temperature T_{sk}.

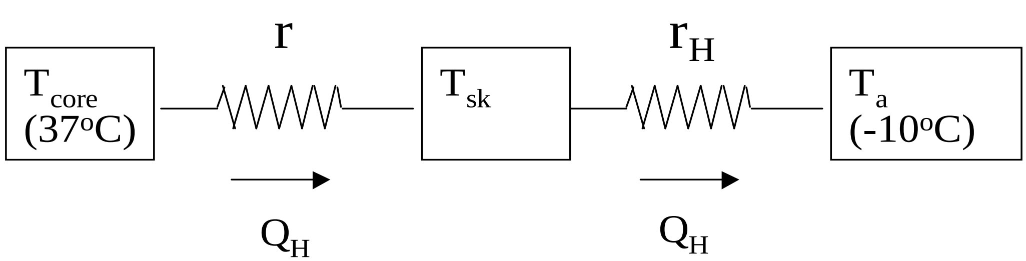


Figure 1: Resistor network model for factors determining skin temperature.

Data:

- Kinematic viscosity of air: $\nu \approx 1.5 \text{ x } 10^{-5}$ [m^2 s^{-1}]
- Thermal diffusivity[1] of air: $\kappa \equiv D_H \approx 2.1 \text{ x } 10^{-5}$ [m^2 s^{-1}]
- Specific heat capacity of air at constant pressure: $c_{pa} \approx 1000$ [J kg^{-1}K^{-1}]
- Ideal gas law for air: $P = \rho_a\ R_d\ T$ ($R_d = 287$ J kg^{-1} K^{-1})
- Stefan-Boltzmann constant $\sigma = 5.67 \text{ x } 10^{-8}$ W m^{-2} K^{-4}
- $Q_H = \ k\ \frac{\Delta T}{\Delta x}$

  Fourier's Law of conduction for the heat flux resulting from a temperature gradient of strength $\Delta T/\Delta x$, where k [W m^{-1} K^{-1}] is the thermal conductivity
- $Q_H = \rho\ c_p\ \frac{T_1 - T_2}{r_H}$

  Ohm's law model for sensible heat exchange by bulk convection/conduction to/from a body to a fluid.
- $r_H = \frac{d}{D_H N_u}$ [s m^{-1}]

  Relationship between heat transfer resistance r_H and the Nusselt number.
- $N_u = \begin{cases} 0.32 + 0.51\ R_e^{0.52} & \text{if} \quad 10^{-1} \leq R_e \leq 10^3 \\ 0.24\ R_e^{0.6} & \text{if} \quad 10^3 \leq R_e \leq 5 \text{ x } 10^4 \end{cases}$

  Nusselt number versus Reynolds number ($R_e = Ud/\nu$) for a cylinder (diameter d) in a current of air of speed U (forced convection).
- $N_u = \begin{cases} 2 + 0.54\ R_e^{0.5} & \text{if} \quad 0 \leq R_e \leq 3 \text{ x } 10^2 \\ 0.34\ R_e^{0.6} & \text{if} \quad 5 \text{ x } 10^1 \leq R_e \leq 1.5 \text{ x } 10^5 \end{cases}$

  Nusselt number versus Reynolds number ($R_e = Ud/\nu$) for a sphere (diameter d) in a current of air of speed U (forced convection).
- $C\ \frac{dT}{dt} = A\ (\ Q^* + Q_H + Q_E\ ) +\ P$

  Energy balance for a thermometer (A the surface area, C the bulk heat capacity, P internal heat production or consumption, other terms conventional).

[1] Symbols κ, D_H are both used for this quantity.

Answers to short-answer questions

No penalty for cumulative errors (ie. an error early in the calculation counts only once). Also, no penalty if (eg.) you took the $N_u(R_e)$ formula for a cylinder instead of a sphere, because the information was laid out on the exam in a way that made this an easy mistake to make.

1. The coffee cooling question. This was harder than I realized when I posed it; it was marked by assigning 1 per significant step towards solution. Note that in the "live" exam, the formula for R_{iw} was given incorrectly as $R_{iw} \equiv k/d^*$, which has units [W m^{-2} K^{-1}]. However clearly, in order to have the correct units in a heat transfer model of form Q_H [W m^{-2}] $= \Delta T/R$, it is $1/R$ that must have units of [W m^{-2} K^{-1}].

   - Give a diagram of the two resistors in series, with "voltages" $T_i,\ T_w,\ T_a$ controlling the current (Q_H) of heat (the heat transfer diagram is the same in principle as that given in Question 3 for heat loss from an exposed hand; only the labeling differs). Note that the voltage at the bottom of the pathway, T_a, is analogous to 'ground' in an electrical circuit
   - Then apply the voltage divider formula, $T_w = T_a + \alpha\ (T_i - T_a)$ where $\alpha = R_{wa}/(R_{wa} + R_{iw})$. Determine the two resistances, to get α:
   - The resistance across the wall of the cup is easy, $R_{iw} = d^*/k = 0.08$ K W^{-1} m^2. To get the aerodynamic resistance R_{wa}: $R_e = 4000 \rightarrow N_u = 0.24 R_e^{0.6} = 34.8 \rightarrow r_H = 82$ s m^{-1} $\rightarrow R_{wa} = 0.075$ K W^{-1} m^2. Hence, $\alpha = 0.48$
   - Thus at $t = 0$, $T_w = 20 + 0.48 * (85 - 20) = 51^o$ C
   - **Because the remainder of the problem requires information not given, I marked such that you could get 5/5 if you got this far without error.** For your interest, the balance of the calculation goes as follows:
   - The coffee will cool according to

     $$C\,\frac{dT_i}{dt} = A\ \left(T_i(t) - T_a\right) / \left(R_{iw} + R_{wa}\right) \tag{2}$$

     where $C =$ J kg^{-1} K^{-1} is the bulk heat capacity of the coffee, and $A = \pi dh + 2\ \pi (d/2)^2 = 0.0245$ m^2. Then the time constant for cooling is

     $$\tau = \frac{C\ (R_{iw} + R_{wa})}{A} \tag{3}$$

   - To evaluate τ I should have given you the information that

     $$C = 4186\ \rho_w\ \pi (d/2)^2 h = 1184 \text{ J kg}^{-1} \text{ K}^{-1} \tag{4}$$

     (where 4186 J kg^{-1} K^{-1} is the specific heat capacity of water). The result is that $\tau = 6900$ s, which does seem a little too long.
   - and since I didn't give you the formula you could hardly have been expected to guess that

     $$T_i(t) = 20\ \left(1 -\ e^{-\,t/\tau}\right) +\ 85\ e^{-\,t/\tau} \tag{5}$$

     and hence deduce that

     $$T_i(900\text{s}) = 77 \text{ C} \tag{6}$$

2. Hot film anemometer calculations. Marked by subtracting 1/2 per significant mistake.

   - $R_{op} = 9.175\Omega$
   - $40R_c = 200R_{op} \rightarrow R_c = 45.88\Omega$
   - $R_e = 66.7 \rightarrow N_u = 4.85 \rightarrow r_H = 1.96$ s m^{-1}
   - $Q_H = \rho c_p \ (T_{op} - T_a)/r_H = 1.51$ x 10^5 W m^{-2} $\rightarrow P = (\pi d\ell) \ Q_H$, where (my slip-up) ℓ was not stated
   - $P = i^2 R_{op} \rightarrow i = \sqrt{\pi d\ell Q_H/R_{op}}$
   - $V_B = i(R_{op} + 40)$

3. Compute the rate of heat loss from a hand modeled as a sphere. Marked by subtracting 1/2 per significant mistake.

   - The rate of heat loss is $P = AQ_H$
   - surface area $A = 4\pi(d/2)^2 = 0.0314$ m^2
   - sensible heat flux density $Q_H = \rho c_p(T_{sk} - T_a)/r_H$
   - $\rho = p/RT = 1.06$ kg m^{-3}
   - $R_e = 6.67$ x $10^4 \rightarrow N_u = 267 \rightarrow r_H = 17.8$ s m^{-1}
   - substituting, $P = 37$ W
   - skin temperature is controlled by the relative values of r, r_H for according to this model

     $$T_{sk} = T_a + \ (T_{core} - T_a) \ \frac{r_H}{r_H + r} \tag{7}$$

     Note that the resistance r is controlled by internal (bodily) properties