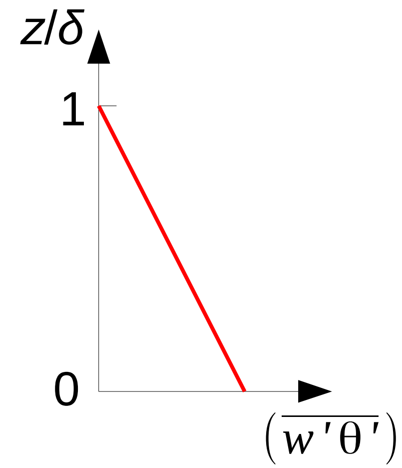
Exercise – heat budget of the ideal ABL

By averaging the equation $\frac{D\theta}{Dt} = 0$



we found

$$\frac{\partial\overline{\theta}}{\partial t} + \overline{u}\,\frac{\partial\overline{\theta}}{\partial x} + \overline{v}\,\frac{\partial\overline{\theta}}{\partial y} + \overline{w}\,\frac{\partial\overline{\theta}}{\partial z} = -\frac{\partial\overline{u'\theta'}}{\partial x} - \frac{\partial\overline{v'\theta'}}{\partial y} - \frac{\partial\overline{w'\theta'}}{\partial z}$$

Assuming horizontal uniformity (i.e. horizontal gradients vanish) and that $\overline{w} = 0$,

this simplifies to $$\frac{\partial\overline{\theta}}{\partial t} = -\frac{\partial\overline{w'\theta'}}{\partial z}$$

Suppose the eddy heat flux decays linearly with height across the boundary layer

$$\overline{w'\theta'} = \left(\overline{w'\theta'}\right)_0 \left[1 - z/\delta\right]$$

Then (1) if the rate of warming is 2 K hr^{-1} while the ABL depth is δ = 500 m, what is the value of the kinematic eddy heat flux density at the surface, $\left(\overline{w'\theta'}\right)_0$? And (2) if $Q_{H0} \equiv \rho\, c_p \left(\overline{w'\theta'}\right)_0 = 300\ \mathrm{W\,m^{-2}}$ and δ = 1750 m, what is the rate of warming?

Exercise – plot a wind profile (Project Prairie Grass run 57)

z [m]	U [m s^{-1}]
16	9.89
8	8.79
4	8.24
2	7.20
1	6.42
0.5	5.56
0.25	4.69

Project Prairie Grass was a tracer dispersion experiment performed over ideal uniform terrain; gas was released continuously from a point 0.46 m above ground, and the resulting 10-min average concentration field was measured on arcs at radii R = (50, 100, 200, 400, 800) m downwind. The table gives the wind profile measured during run 57, for which the Obukhov length was effectively infinite (i.e. the surface layer was neutrally stratified**). Plot this wind profile on log-linear graph paper, and determine the friction velocity graphically from the slope (rise-over-run) of a fit to the data.

** in a neutral surface layer, the wind profile is $$\frac{U}{u_*} = \frac{1}{k_v} \ln \frac{z}{z_0}$$ (where k_v=0.4) , and this implies that $$\frac{\Delta U}{\Delta \ln z} = \frac{u_*}{k_v}$$

Exercise – calculations relating to the neutral wind profile

Suppose a neutrally stratified ABL is blowing over an open plain whose surface aerodynamic roughness length is z_0=0.05 m. The surface pressure and temperature are 980 hPa and 17°C. If measurements within the surface layer** give the values in the table then:

z [m]	U [m s^{-1}]
15	4.28
3	3.07

(1) what was the friction velocity u_* ?

(2) what was the drag τ on ground?

(3) what was the wind speed at standard reporting height (10 m)?

(4) what would be a plausible value for the standard deviation σ_w of vertical velocity?

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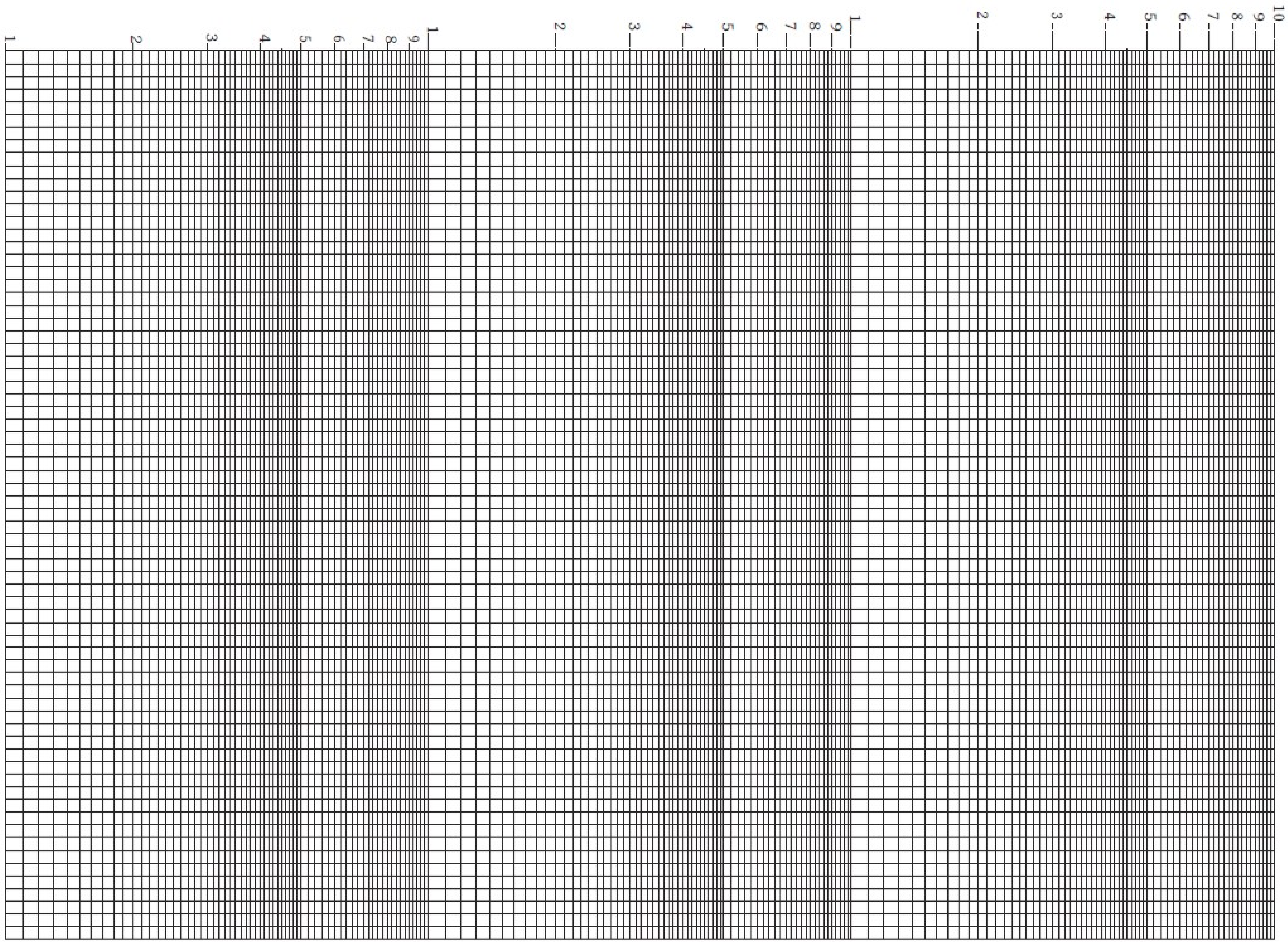
(Again, to determine u_* you *might* use log-linear graph paper; but it can also be done without plotting the data)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