General comments regarding NWP Models

Integration times (ie. forecast range) short (~48 hours to 2 weeks or less), so processes with longer time scales excluded (whereas active in climate simulations, e.g. CO₂ cycle)

Key aspects:

- Domain global, hemispheric, regional? Resolution horiz. (Δ) and vertical
- Lateral boundary conditions (if needed)
- Dynamics hydrostatic (considered inapprop. for Δ <10 km) or non-hydrostatic?
- Horiz. discretization finite difference, finite element, spectral?
- Vertical coordinate usually related to p/p_{sfc} and discretization
- Representation of terrain
- Coupling to lower boundary static ocean?, cryosphere?, vegetation?..
- Initialization and data-assimilation (4D-Var now usual)
- Numerics e.g. order of approx. of operators, control of numeric noise?
- Parameterizations for unresolved processes ("model physics")
 - solar and longwave radiation
 - vertical transport by unresolved motion (esp. in friction layer)
 - unsaturated convection, convective cloud, stratiform cloud
 - coupling to surface (air- ground or ocean exchange fluxes)
 - gravity wave drag

Aside on dynamics

Hydrostatic approximation not realistic if aim is to resolve atmosphere down to scales on which convection occurs. Let total pressure $p = p_0 + \tilde{p}$ where $p_0(z)$ denotes the pressure of a hydrostatic reference atmosphere

• under Boussinesq** approx., vertical acceleration of a parcel depends on deviations \tilde{T} , \tilde{p} of the parcel's state from the reference state p_0 , T_0 at that level...

$$\frac{d w}{d t} = -\frac{1}{\rho_0} \frac{\partial \tilde{p}}{\partial z} + g \frac{\tilde{T}}{T_0}$$

vert. accel'n PGF buoyancy

• versus hydrostatic approximation

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial z} + g$$

Molinari (1993; in *Representation of Cumulus Convection in Numerical Models*, Am. Meteor. Soc.) defines mesoscale models as hydrostatic models with horiz. gridlength $10 \le \Delta \le 50\,$ km

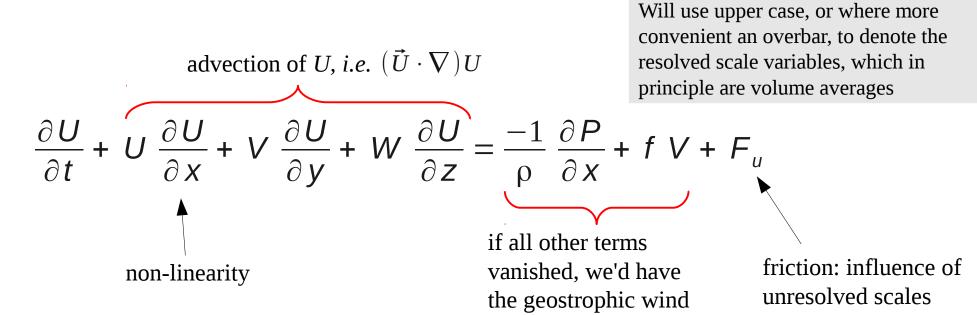
By this criterion both the Global (25 km) and Regional (10 km) runs of CMC's GEM (Global Environmental Multiscale) NWP model are mesoscale models...

"At a grid spacing of 10 km, the grid scale approaches the preferred scale for instability of convection in nature."

(Molinari)

**Boussinesq approx. suitable for shallow layer (ABL) only. NWP models (e.g. WRF) fully compressible

Reynolds-averaged zonal momentum equation (in Cartesian coords.)



or using the Lagrangian derivative

$$\frac{dU}{dt} = \frac{-1}{\rho} \frac{\partial P}{\partial x} + f V + F_u$$

The friction term is (formally) the divergence of the unresolved momentum flux

$$F_{u} = -\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{v'u'}}{\partial y} - \frac{\partial \overline{w'u'}}{\partial z}$$
these terms neglected

vertical gradient of the mean vertical flux of *u*-momentum carrried by the unresolved scales of motion

CMC's GEM - Global Environmental Multiscale - model: common elements

• primitive equations model, formulated in "horiz." velocity components (U, V), the vertical "velocity" $\dot{\eta} \equiv d \, \eta / d \, t$, the virtual temperature** T_v and specific humidity Q

• hydrostatic *or* non-hydrostatic (hydrostatic for the GDPS and RDPS, non-hydrostatic for the HRDPS (GEM-LAM 2.5 km) forecasts

• vertical coord*
$$\eta = \frac{P - P_T}{P_S - P_T}$$

 $P_{\rm S}$, surface pressure, evolves $P_{\rm T}$, pressure at top of domain, fixed

 $0 \le \eta \le 1$

• top level P_T = 10 hPa

**Temperature of dry air having same P and ρ as sample:

$$T_{v} = T (1+0.61Q)$$

^{*}changed July 2014 to "terrain following vertical coordinate of the log-hydrostatic-pressure type vertically discretized on a Charney-Phillips grid" (source: an internal CMC report)

- ullet in the operational hydrostatic GEM the coordinate η is based on total pressure
- in non-hydrostatic version it is based on the dry, hydrostatic component of the pressure (see NAM/WRF model later) as introduced by Laprise (1992, MWR Vol. 120). Note that $\dot{\eta} = d\eta/dt = 0 \qquad \text{at the surface} \quad (\eta = 0 \quad) \text{ and top of model domain} \quad (\eta = 1 \quad)$

Dynamics/physics terminology

$$\frac{\partial U}{\partial t} = -U \frac{\partial U}{\partial x} - V \frac{\partial U}{\partial y} - W \frac{\partial U}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial x} + f V + F_u$$

$$\frac{\partial U}{\partial t} = \left(\frac{\partial U}{\partial t}\right)_{\text{dyn}} + \left(\frac{\partial U}{\partial t}\right)_{\text{phy}}$$

CMC's GDPS-4.0.0 (Global Deterministic Prediction System) as of 18 Nov. 2014

- run four times a day in analysis mode (centered at 00, 06, 12, 18 UTC)
- run twice a day with initial times at 00 and 12 UTC, in addition to providing analysis and first-guess fields ("background state") to its own forecast component, the GDPS analysis component also provides the initial conditions to the regional deterministic prediction system (RDPS) assimilation cycle
- forecast range to 10 days (Saturday, range to 15 days)
- global domain
- horizontal resolution $\Delta = 25$ km at mid latitudes
- 79 levels
- timestep 12 min

Spatial discretization (grid) for GDPS

Squares: scalar fields (*P* etc.)

Circles: *U* nodes

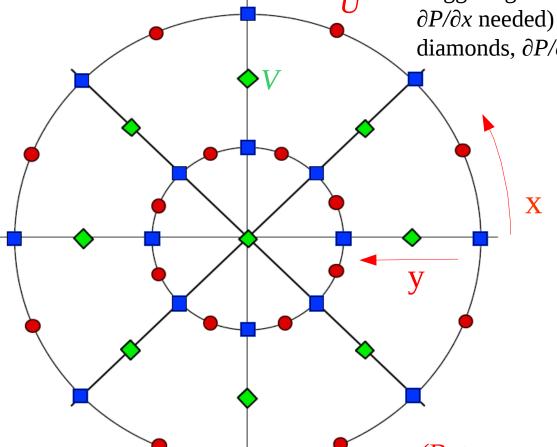
Diamonds: V nodes

• domain is separated by imaginary lines into finite elements

• values of *U*, *V*, *W*, *P*... at the nodes are the basic unknowns (resolved variables)

• an interpolating function is used to provide the values of U, V (etc.) wherever needed within each finite element in terms of nodal values (e.g. at U gridpoints we need V to compute $V \partial U / \partial y$

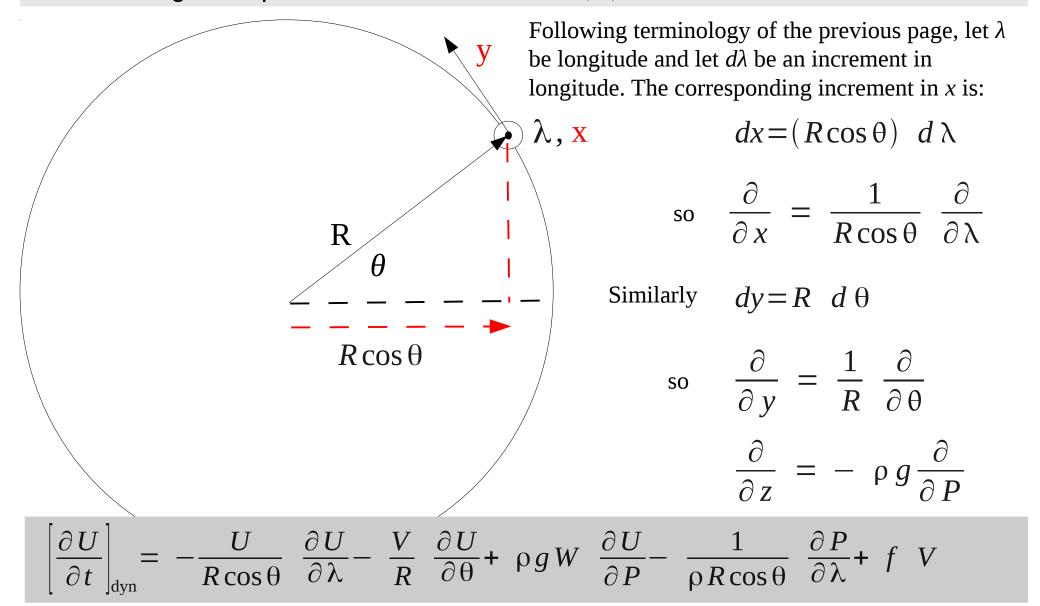
Each cell is a "finite element." With pressure P placed at the blue squares we have nice staggering for U (red circles, $\partial P/\partial x$ needed) and V (green diamonds, $\partial P/\partial y$ needed)



(But we need to transform this into θ , λ , P coords)

$$\left[\frac{\partial U}{\partial t}\right]_{\text{dyn}} = -U \frac{\partial U}{\partial x} - V \frac{\partial U}{\partial y} - W \frac{\partial U}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial x} + f V$$

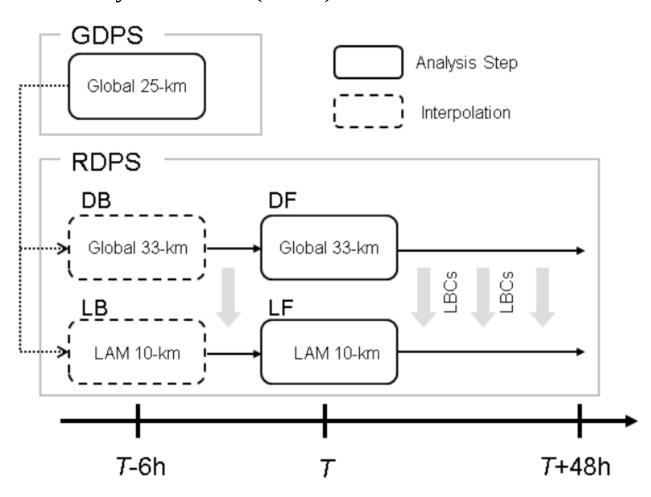
Transforming the eqn for resolved *U*-mtm into θ , λ , *P* coordinates:



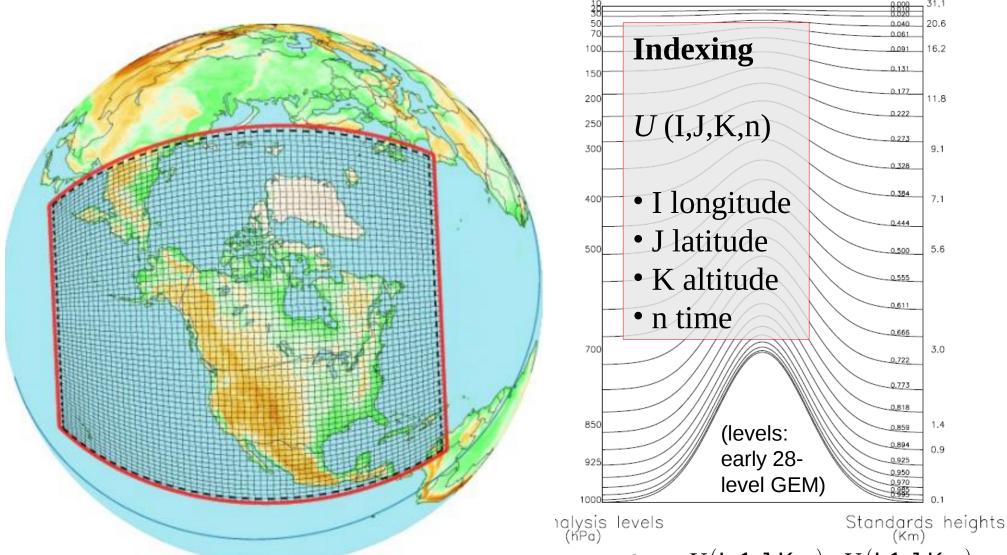
And the effect of unresolved motion? $\left[\frac{\partial U}{\partial t}\right]_{\text{phy}} \approx -\frac{\partial \overline{w'u'}}{\partial z} = -\frac{\partial}{\partial z}\left(-K\frac{\partial U}{\partial z}\right) = \rho g^2 \frac{\partial}{\partial P}\left(\rho K\frac{\partial U}{\partial P}\right)$ Treated as:

CMC's RDPS 4.0.0 (Regional Deterministic Prediction System) as of 18 Nov. 2014

- launched at [00,06,12,18] Z+2:00, with forecast range to 48 hours (sometimes 54 hours)
- 25 km analysis from GDPS initializes a 6 h limited area mesh (LAM) forecast starting *T*-6h (LB). This forecast serves as the background state for the analysis at time T (LF). Same procedure is applied to a **global GEM driving model** (33 km). The synchronous global driving analysis (DF) and forecast allow obs. outside the LAM domain to influence the LAM forecasts through the lateral boundary conditions (LBCs).
- core is a LAM calculation
- LAM grid uniform Δ =10 km
- b/conds for LAM provided by global "driving" model
- both have same 80 levels (7 or more below 850 hPa)
- timestep of the LAM 5 min



Domain of the limited area model in RDPS (Δ =10 km)



Derivatives are approximated by finite differences, e.g. $\frac{\partial U}{\partial x} = \frac{U(I+1,J,K,n) - U(I-1,J,K,n)}{x(I+1,J) - x(I-1,J)}$

with the result that the governing differential equations are transformed into a coupled set of non-linear algebraic equations

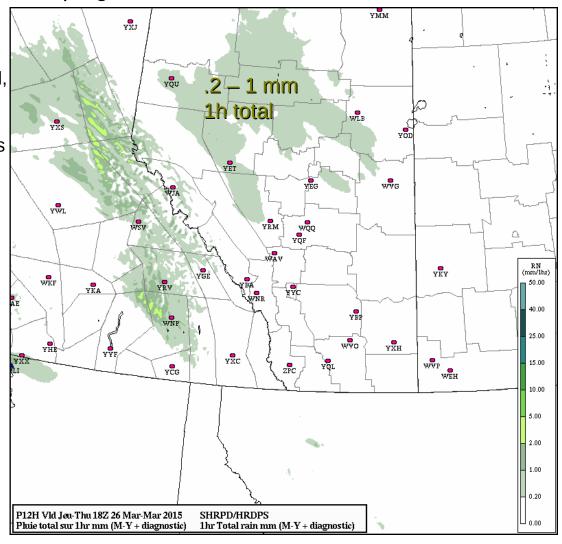
High Resolution Deterministic Prediction System (HRDPS)

- one way nested LAM (Limited Area Model) non-hydrostatic implementation of GEM's eqns
- horizontal resolution 2.5 km
- 4 x daily on four domains:

arctic, east, west & maritimes

two-way nesting, if permitted, allows hi-resolution fields computed on sub-domain to modify lower resolution fields Outer domain over that region of the outer domain Nested subdomain

12h prog. LAM 2.5 km west vld 18 Z Thurs 26 Mar.



From: Belair et al., 2005, Monthly Weather Review

These specs. pertain to the twice-daily Global runs (to 10 days*) – note the coarser resolution and timestep relative to the Regional run

*A 15-day run is made on Saturdays

TABLE 1. Summary of the GEM forecast system.

Dynamics/numerics

- Hydrostatic primitive equations;
- Global uniform resolution of 0.45° longitude and 0.30° latitude (800×600): 1024x800
- Variable vertical resolution with 80 levels; model top at 10 hPa;
- Time step 12 min);
- Cell-integrated finite-element discretization on Arakawa C grid;
- Terrain-following hydrostatic pressure vertical coordinate;
- Two-time-level semi-implicit time scheme;
- 3D semi-Lagrangian advection; (see over)
- ∇⁶ horizontal diffusion on momentum variables; increased horizontal diffusion (sponge) for the four uppermost levels;
- Periodic horizontal boundary conditions;
- No motion across the lower and upper boundaries.

Physics

- Planetary boundary layer based on TKE with statistical representation of subgrid-scale cloudiness (MoisTKE);
- Fully implicit vertical diffusion;
- Stratified surface layer, distinct roughness lengths for momentum and heat/humidity;
- Four types of surface represented: land, water, sea ice, and glaciers;
- Solar/infrared radiation schemes with cloud-radiation interactions based on predicted cloud radiative properties;
- Kuo Transient scheme for shallow convection;
- Kain-Fritsch scheme for deep convection;
- Sundqvist scheme for nonconvective condensation.

Semi-Lagrangian treatment of advection?

Strategy to overcome limitation imposed by the Courant condition, which demands

$$\frac{|U| \Delta t}{\Delta x} \le 1$$

$$\frac{|V| \Delta t}{\Delta y} \le 1$$

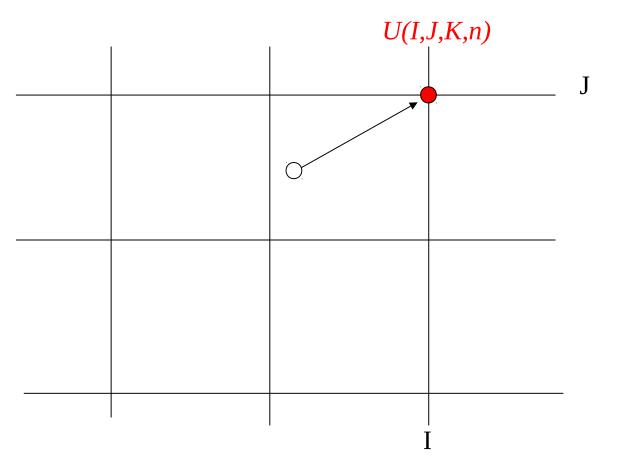
Evaluate this with

 $\Delta x = 25 \text{ km}$

 $\Delta t = 720 \text{ s}$

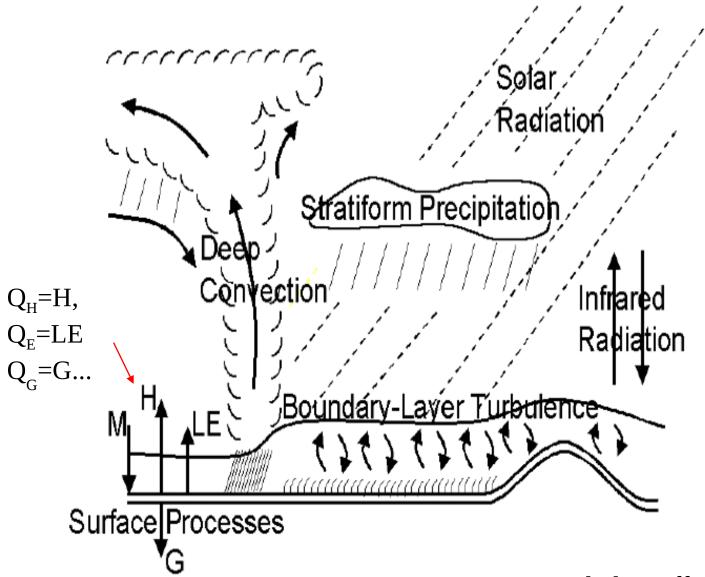
 $U=50 \text{ m s}^{-1}$

Computed path of a fluid element backwards in time from t to t-\Delta t such that U(I,J,K,n) is evaluated by taking the value at the upwind point (open circle) for time level n-1. The latter is evaluated by cubic interpolation from the gridded values



• of course other factors, notably pressure gradient and Coriolis force, demand an adjustment to this advected value

Overview of Physical Processes parameterized in GEM



Thanks to Stephane Belair (CMC) for permission to use this and other sketches

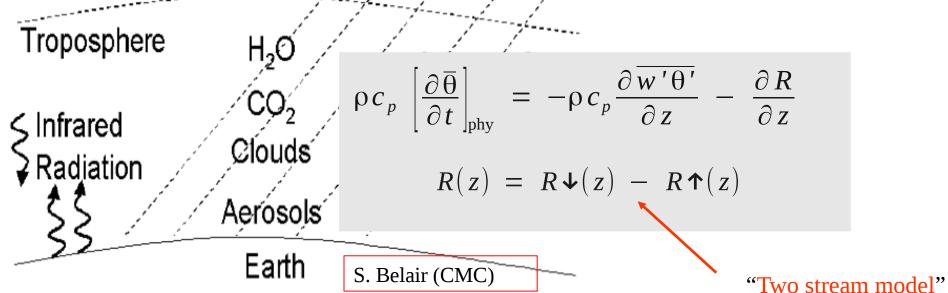
* including effects of unresolved (subgrid scale) motion

Atmospheric Radiation

GEM radiation calculations once each 2 hours (to reduce computation load) **SOLAR**

Sun

- single waveband
- sun-earth geometry
- multiple scattering
- absorption by "<u>model</u> clouds" in rel'n to diagnosed fractional sky coverage & effective cloud liquid water content
- climatol. profiles of ozone, CO2; no scheme for aerosols



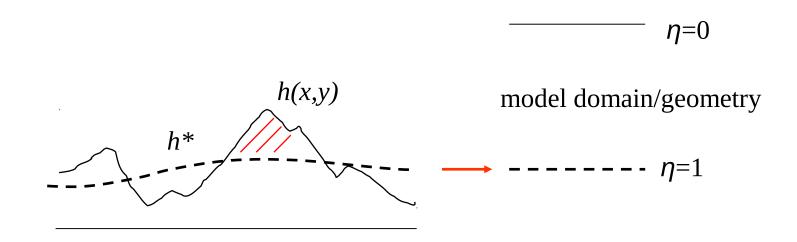
Şoʻlar-

Radiation

(R is the net radiation)

LONGWAVE

- four wavebands; interaction with water vapour, $\mathbf{O_3}$, $\mathbf{CO_2}$, clouds
- climatological O₃; [CO₂] treated as uniform



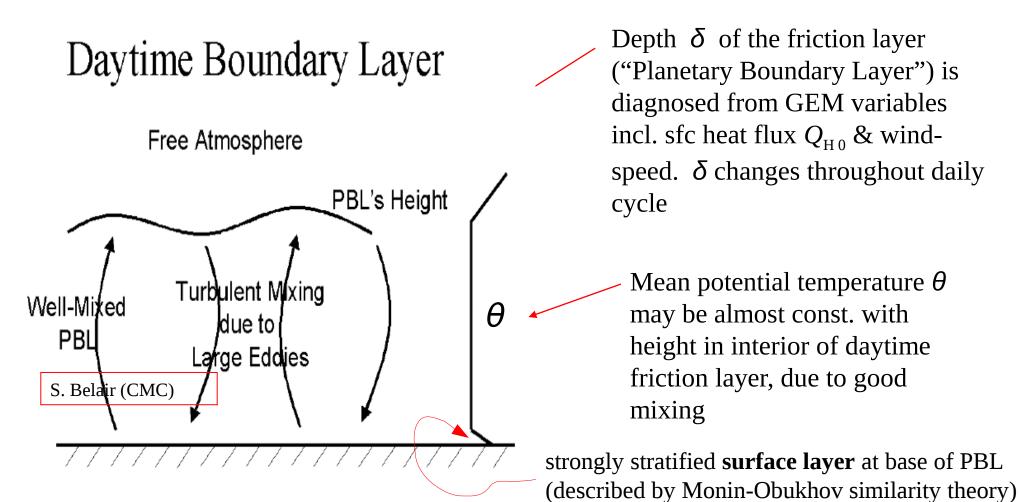
Resolved terrain $h^*(x,y)$ "disappears" in the (terrain following) eta (η) coordinate system. How are mountains "felt"? New terms appear in the momentum equations when they are transformed into the η coord. system ("metric terms")*

Additional - parameterized - effects of terrain

- gravity wave drag slows stratospheric winds (GEM dynamical eqns suppress gravity waves, thus need for parameterization; Dr. Sutherland EAS/Physics supervising studies of this)
- "blocking" parameterization recognizes influence of unresolved terrain reduces the low level winds in mountainous regions*

Subgrid (unresolved) transport in the boundary-layer (ie. friction layer, PBL)

- sub-grid scale motion transports heat, vapour, momentum... (eg. redistributes heat and vapour added at ground). Consider vertical exchange only, i.e. the "grid-point computations" involve local column only, no lateral coupling.
- in analogy with molecular mixing, subgrid transport is represented as "diffusion." Eddy diffusivity K is function of kinetic energy of turbulence, and stratification



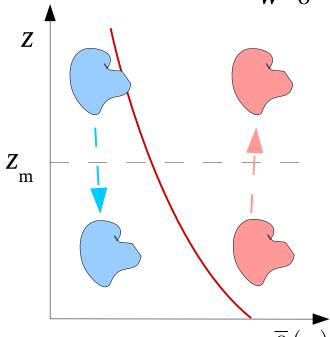
Parameterizing unresolved vertical fluxes by eddy viscosity/diffusion paradigm

- mean vertical convective heat flux due to the unresolved vertical motion is average of the w'T product (or one can equally write $w'\theta$)... thus, need $w'\theta$
- unresolved fluctuations w' carry heat, vapour, CO₂, etc. to and from the surface

• eddy-diffusion model postulates that the direction of the mean flow of heat will be from warm to cold, and introduces as proportionality constant an "eddy diffusivity" (for heat) with the same units as, but vastly greater magnitude than, the molecular diffusivity. That is, one adopts the model

 $\overline{w'\theta'} = - K_h \frac{\partial \theta}{\partial z}$

(Dimensionally, K_h is [velocity x length]; numerically, it vastly exceeds the molecular thermal diffusivity; furthermore, it is a property of the flow, not of the fluid itself)

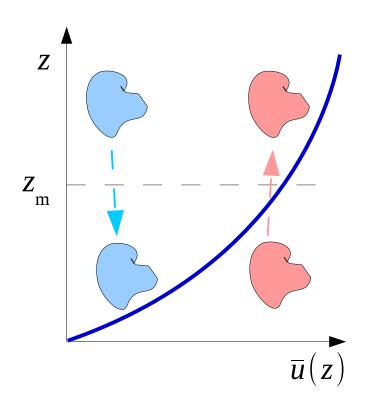


In the case shown (unstable stratification), the covariance (eddy heat flux) is +ve (upward). Why*?

- \bullet No net volume flux across measurement level $\boldsymbol{z}_{_{\boldsymbol{m}}}$
- For each cold parcel (θ' <0) crossing z_m downward (w'<0) a warm parcel (θ' > 0) of equal volume crosses z_m moving upward (w'>0)

*This is is not a proof, merely a *plausibility* argument

Parameterizing unresolved vertical fluxes by eddy viscosity/diffusion paradigm



- On average, for each fast-moving parcel (u '>0) crossing z_m downward (w'<0) a slow-moving parcel (u '<0) of equal volume crosses z_m moving upward (w'>0)
- Thus the eddy momentum flux (covariance) is negative. The simplest model, "first order closure,"

is
$$\overline{w'u'} = -K_m \left[\frac{\partial \overline{u}}{\partial z} + \frac{\partial \overline{w}}{\partial x} \right]$$

Now we have the problem of how to rationally model the eddy viscosity and eddy diffusivity! Their magnitude must depend on some measure of the "amount" of vertical motion (loosely, of "mixing"), and this is often expressed by the "turbulent kinetic energy" $k = \frac{\sigma_u^2 + \sigma_v^2 + \sigma_w^2}{2}$ (one half the sum of the variances of the unresolved velocity components)

The eddy diffusivity and eddy viscosity are usually assumed proportional or even equal, and typically written $K_{m,h} \propto \lambda(z) \sqrt{k}$, where λ is the "length scale"

TKE budget equation – assuming horizontal homogeneity

Buoyant production,
$$\mathbf{P}_{\mathrm{B}}$$

$$\frac{\partial k}{\partial t} = -\overline{u'w'} \frac{\partial U}{\partial z} - \overline{v'w'} \frac{\partial V}{\partial z} + \frac{g}{\theta_0} \overline{w'\theta'} - \varepsilon - \frac{\partial}{\partial z} \overline{w'(\frac{p'}{\rho} + \frac{u'u' + v'v' + w'w'}{2})}$$
Shear production, \mathbf{P}_{S}
Dissipation (conversion of TKE to heat)

Assume

$$\overline{u'w'} = -K \frac{\partial U}{\partial z}, \ \overline{v'w'} = -K \frac{\partial V}{\partial z}, \ \overline{w'\theta'} = -K \frac{\partial \overline{\theta}}{\partial z}, \ \epsilon = \frac{(\alpha k)^{3/2}}{\lambda}$$

(note the assumption that eddy viscosity and eddy diffusivity are equal)

Resulting TKE equation:

$$\frac{\partial k}{\partial t} = K \left[\left| \frac{\partial U}{\partial z} \right|^2 + \left| \frac{\partial V}{\partial z} \right|^2 \right] - \frac{g}{\theta_0} K \frac{\partial \overline{\theta}}{\partial z} - \frac{(\alpha k)^{3/2}}{\lambda} + \frac{\partial}{\partial z} K \frac{\partial k}{\partial z}$$
Shear production. Buoyant

Shear production, P.

production, P.

Flux Richardson number:
$$R_i^f = -\frac{P_B}{P_S} = \frac{g}{\theta_0} \, \frac{\partial \overline{\theta}/\partial z}{(\partial U/\partial z)^2 + (\partial V/\partial z)^2} \quad \text{Unstable, } R_i^f < 0$$
 Stable, $R_i^f > 0$

Exercise: calibrating the TKE budget equation (a manipulation using calculus & algebra)

$$\frac{\partial k}{\partial t} = K \left[\left| \frac{\partial U}{\partial z} \right|^2 + \left| \frac{\partial V}{\partial z} \right|^2 \right] - \frac{g}{\theta_0} K \frac{\partial \overline{\theta}}{\partial z} - \frac{(\alpha k)^{3/2}}{\lambda} + \frac{\partial}{\partial z} K \frac{\partial k}{\partial z}$$

In the ideal neutral surface layer

- k = const.
- $K = k_{\nu} u_* z$
- $U = \frac{u_*}{k_v} \ln \frac{z}{z_0}$ (coords. chosen such that V=0)
- $\overline{\theta} = \text{const.}$
- $\lambda = k_{\nu} z$

Assuming stationarity, express the coefficient α in the TKE eqn. in terms of u_*^2/k

Solution: "calibrating the TKE budget equation"

$$\frac{\partial k}{\partial t} = K \left[\left| \frac{\partial U}{\partial z} \right|^2 + \left| \frac{\partial V}{\partial z} \right|^2 \right] - \frac{g}{\theta_0} K \frac{\partial \overline{\theta}}{\partial z} - \frac{(\alpha k)^{3/2}}{\lambda} + \frac{\partial}{\partial z} K \frac{\partial k}{\partial z}$$

Differentiating the wind profile
$$U = \frac{u_*}{k_v} \ln \frac{z}{z_0}$$
 we get $\frac{\partial U}{\partial z} = \frac{u_*}{k_v z}$

Substituting this into the TKE eqn. and implementing the assumptions that define this ideal neutral surface layer,

$$0 = k_{v} u_{*} z \left[\frac{u_{*}}{k_{v} z} \right]^{2} - 0 - \frac{(\alpha k)^{3/2}}{k_{v} z} + 0$$

Multiplying through by $k_{_{V}}z$, this simplifies to $u_{*}{}^{3}=(\alpha k)^{3/2}$

and so
$$\alpha = u_*^2/k$$

GEM's formulation** of the eddy viscosity/diffusivity K and length scale λ

$$K = c \lambda \sqrt{k} = \frac{\lambda^{\text{neut}}(z)}{1 - R_i^f} \sqrt{c^2 k}$$

 $1\!-\!R_i^f$, "stability (correction) function"

c , dimensionless constant

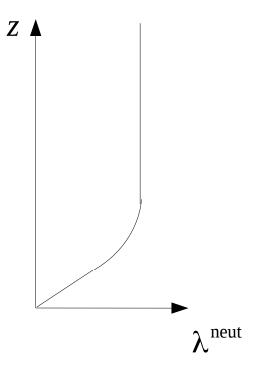
Require that if $R_i^f \rightarrow 0$ with $z \rightarrow 0$

then
$$K \rightarrow K^{\text{neut}} = k_{v} u_{*} z$$

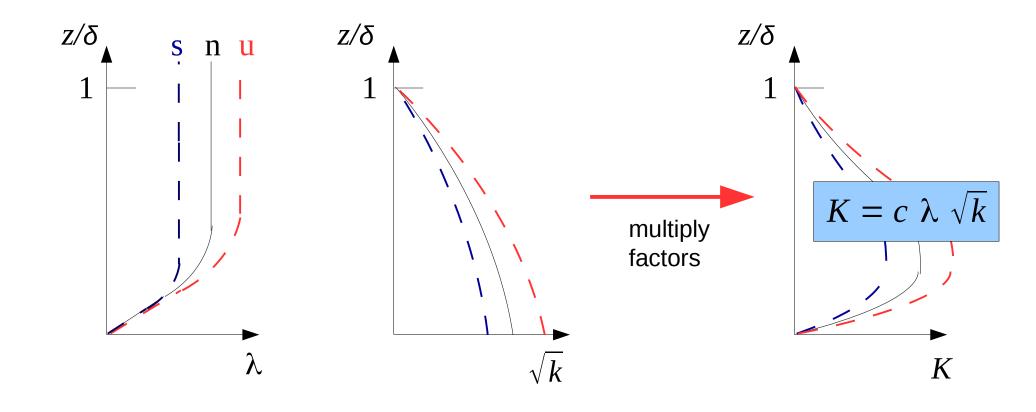
Thus require $c^2 = [u_*^2/k]^{\text{neut}}$

$$\lambda^{\text{neut}} = \min[k_v z, 200 \,\text{m}]$$

(arbitrarily imposed maximum value for the length scale in a neutral ABL)



^{**}http://collaboration.cmc.ec.gc.ca/science/rpn/physics/physic98.pdf



GEM's coupling to the surface

- enforce surface energy balance $Q^* = K^* + L^* = Q_{H0} + Q_{E0} + Q_{S0}$
- detailed map of (time-evolving) surface type/condition
- prognostic variables for surface and soil temperatures, and soil moisture
- "ISBA" scheme (Interaction Soil Biosphere Atmosphere): three soil layers, vegetation canopy, interaction of radiation and vegetation canopy (surface albedo), vertical diffusion of heat and moisture between the soil layers, treatment of snow on canopy, inclusion of precip infiltration, runoff, and drainage
- static analyzed ocean/lake ice field and ocean/lake temperature (SST)
- lake/ocean surface roughness length (" z_0 ") responds to surface windspeed

GEM's treatment of clouds and precip – see table on a previous page



Turbulent Hysteresis in a TKE-based Boundary Layer Scheme

SCMO 2012 Montréal

Ron McTaggart-Cowan 1, Ayrton Zadra 1, Jocelyn Mailhot 1, André Plante 2 (Presented by André Plante) 1 Recherche en Prévision Numérique, Service

météorologique du Canada

2 Centre météorologique canadien, Service météorologique du Canada

Contact: andre.plante@ec.gc.ca

http://web2.sca.uqam.ca/~wgne/CMOS/PRESENTATIONS/5807 1c3.3 Andre Plante.pdf

Vertical Diffusion Revisited

TKE tendency equation

$$\frac{\partial E}{\partial t} = BE^{1/2} - CE^{3/2} + \frac{\partial}{\partial z} \left(K_M \frac{\partial E}{\partial z} \right)$$

$$B = (1 - Ri_f) \cdot \left| \frac{\partial V}{\partial z} \right|^2 \cdot c\lambda_{\text{mix}} \quad \leftarrow \text{Positive}$$

$$Ri_f = \frac{\text{(buoyant suppression)}}{\text{(shear generation)}}$$

 $Ri_c < 1 \rightarrow B > 0$ TKE production $Ri_{\star} > 1 \rightarrow B < 0$ TKE destruction

Page 7 - 20 juin 2012

Canada

Vertical Diffusion Revisited

 With its turbulent kinetic energy (TKE) closure, the PBL scheme estimates the diffusion coefficients using

$$K = \frac{c\lambda\sqrt{E}}{\Phi}$$

- Stability function
- Mixing length (vertical scale of turbulent eddies)
- Constant of proportionality (currently 0.516)
- Turbulent kinetic energy (TKE)

Page 5 - 20 juin 2012



Ongoing / Planned Work

- Implementation in RDPS this fall, in GDPS early in 2013
- Modernization of the PBL code:
- Implementation of distributed drag to change lower boundary condition for the PBL scheme
- The presentation did not prescribe the length scale profile λ
- clearly C has a factor $1/\lambda$
- $BE^{1/2} = K |d\vec{V}/dz|^2 = E^{1/2} (c \lambda/\Phi) |d\vec{V}/dz|^2$
- $B = (c \lambda/\Phi) |d \vec{V}/dz|^2$
- $1 Ri_f = 1/\Phi$