$$\frac{\sigma}{2}\left[\nabla^2 + \frac{f_0^2}{\sigma}\,\frac{\partial^2}{\partial p^2}\right]\omega = -\nabla\cdot\vec{Q}$$

If RHS of this eqn is positive (convergent **Q** vectors) expect negative ω (ascent)

$$(\omega = -\rho\, g\, w)$$

or

$$\frac{\sigma\,\rho\, g}{2}\left[\nabla^2 + \frac{f_0^2}{\sigma}\,\frac{\partial^2}{\partial p^2}\right] w = \nabla\cdot\vec{Q} = \left[\frac{\partial|\vec{Q}|}{\partial s} + |\vec{Q}|\,\frac{\partial\beta}{\partial n}\right]$$

divergence of **Q** expressed in the natural coord. system **aligned with isotherms**

or, symbolically

$$\frac{\sigma\,\rho\, g}{2}\left[\frac{\overline{w}_{\mathrm{nbrs}} - w_{\mathrm{P}}}{\Delta^2}\right] \approx \nabla\cdot\vec{Q}$$

Note: here giving Martin's Eqs (6.40, 6.44). Holton's Eq 6.53 includes an extra term that vanishes in the *f* = const. approx.

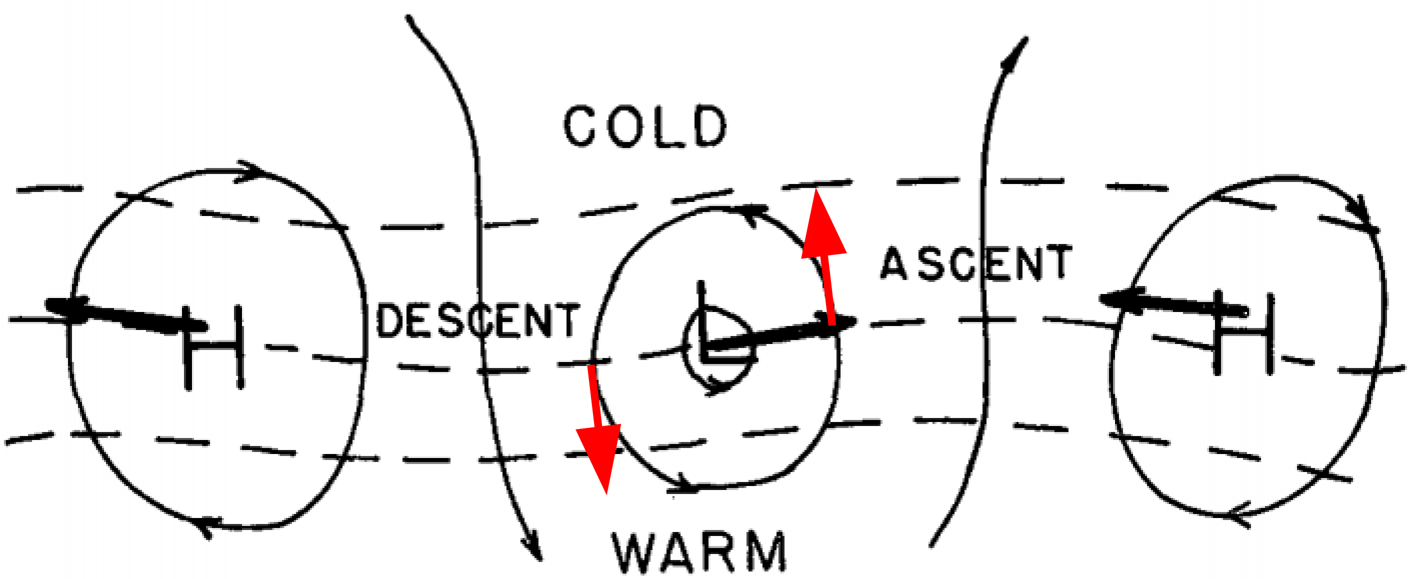
where

$$\vec{Q} = \frac{-R}{p}\left|\frac{\partial T}{\partial n}\right|\left[\hat{k}\times\frac{\partial \vec{V}_g}{\partial s}\right] = \frac{-R}{p}\left|\frac{\partial T}{\partial n}\right|\left[-\hat{s}\,\frac{\partial v_g}{\partial s} + \hat{n}\,\frac{\partial u_g}{\partial s}\right]$$

"The **Q** vector can be obtained by evaluating the vectorial change of $\mathbf{V}_g$ along the isotherm (with cold air on the left), rotating this change vector by 90° clockwise, and multiplying the resulting vector by $|\partial T/\partial n|$." (Holton)

Wherever the pattern of the **Q** vector is convergent $(\nabla\cdot\vec{Q} < 0)$ the Laplacian of *w* at P is negative, suggesting *w* is more positive at P than in the neighbourhood... suggests ascent

$$\vec{Q} = \frac{-R}{p}\left|\frac{\partial T}{\partial n}\right|\left[\hat{k}\times\frac{\partial \vec{V}_g}{\partial s}\right] = \frac{-R}{p}\left|\frac{\partial T}{\partial n}\right|\left[-\hat{s}\,\frac{\partial v_g}{\partial s} + \hat{n}\,\frac{\partial u_g}{\partial s}\right]$$



Modified from Sanders & Hoskins (1990; Weather & Forecasting Vol. 5, Fig. 3)

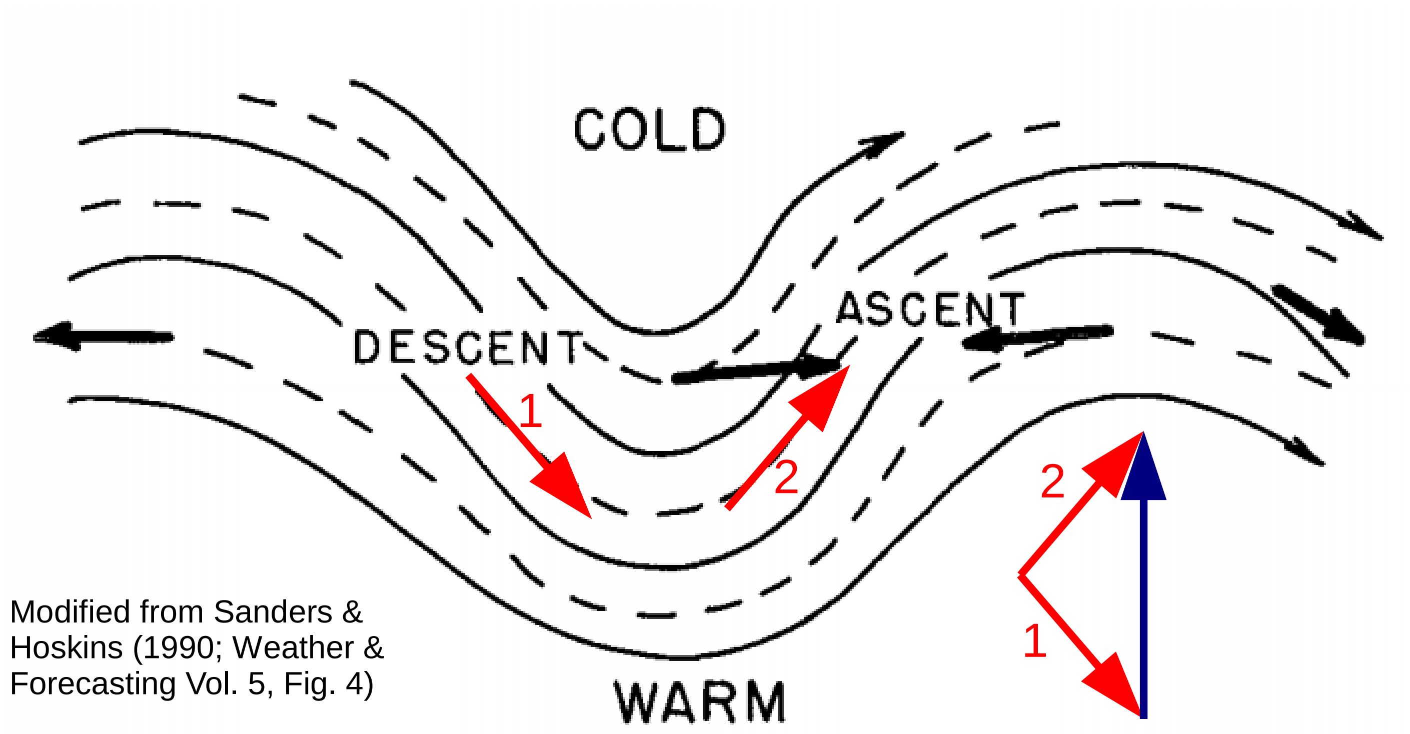
- Closed isobars of MSLP (solid)
- isotherms (dashed)
- geostrophic wind vectors
- **Q vectors**

West of the low, v_g is northerly (negative), east of the low it is southerly (positive) so that $\partial v_g/\partial s > 0$; but u_g is zero on west and east sides). Thus only the term along **s** contributes, and the two negatives cancel

"The **Q** vector can be obtained by evaluating the vectorial change of $\mathbf{V}_g$ along the isotherm (with cold air on the left), rotating this change vector by 90° clockwise, and multiplying the resulting vector by $|\partial T/\partial n|$." (Holton)

$$\frac{\sigma\,\rho\,g}{2}\left[\frac{\overline{w}_{\text{nbrs}} - w_{\text{P}}}{\Delta^2}\right] \approx \nabla\cdot\vec{Q}$$

$$\vec{Q} = \frac{-R}{p}\left|\frac{\partial T}{\partial n}\right|\left[\hat{k}\times\frac{\partial \vec{V}_g}{\partial s}\right] = \frac{-R}{p}\left|\frac{\partial T}{\partial n}\right|\left[-\hat{s}\,\frac{\partial v_g}{\partial s} + \hat{n}\,\frac{\partial u_g}{\partial s}\right]$$

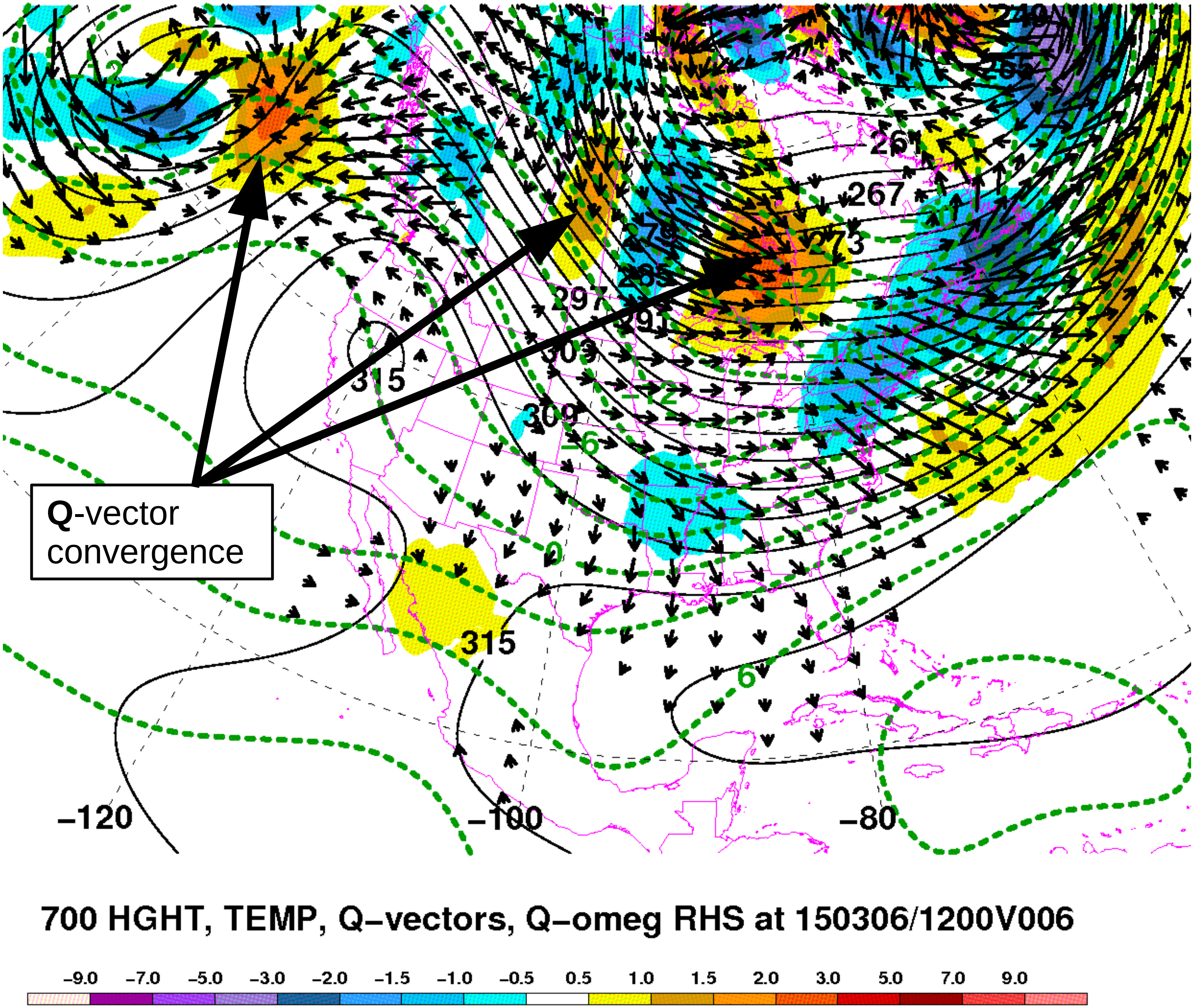


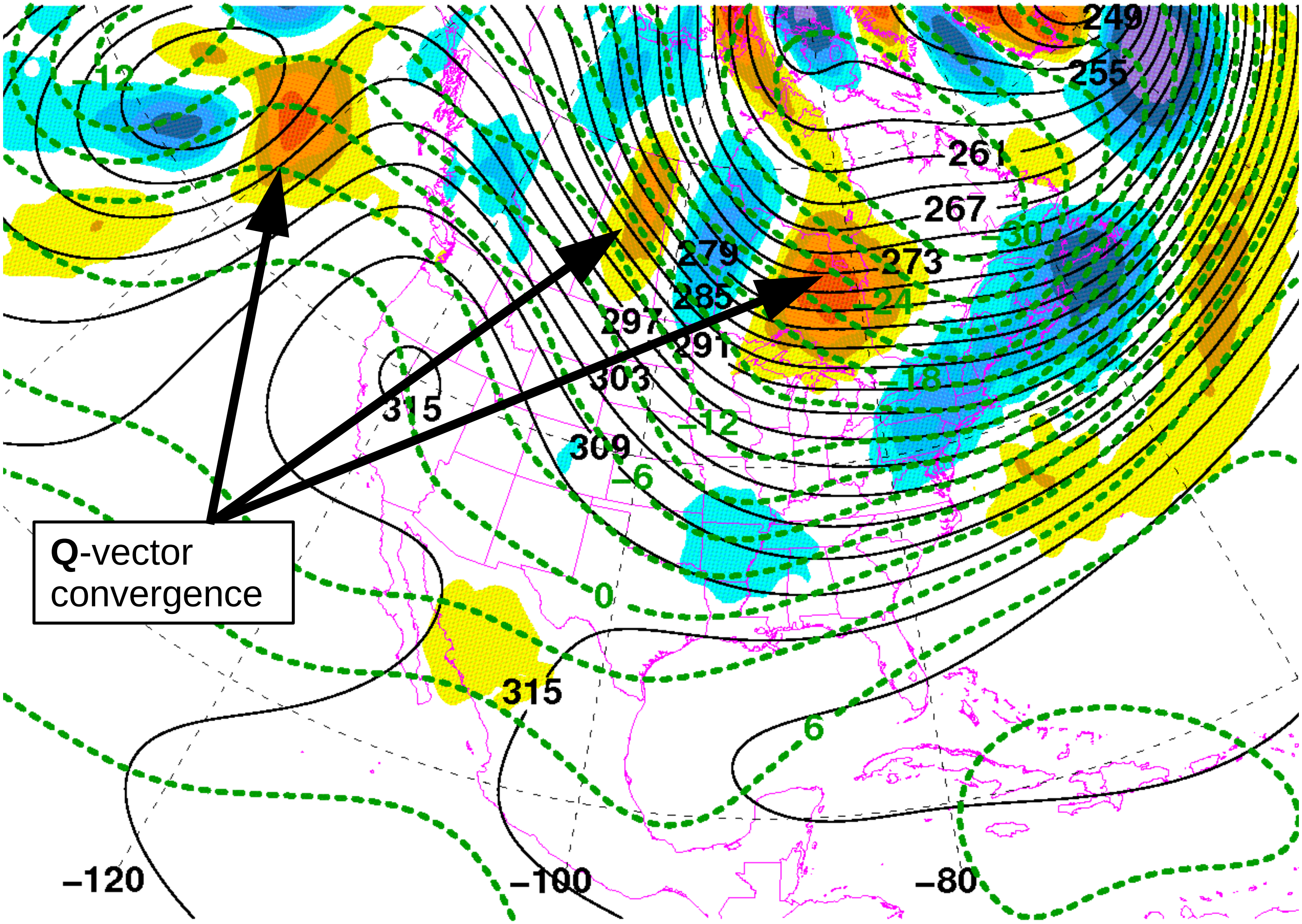
Modified from Sanders & Hoskins (1990; Weather & Forecasting Vol. 5, Fig. 4)

- Height contours (solid)
- isotherms (dashed)
- geostrophic wind vectors
- **Q vectors**

Another idealized example (upper trough & ridge). Note the absence of thermal advection (isotherms parallel to height contours). The v_g component of the geostrophic wind, i.e. the component normal to the isotherms, is zero everywhere. The u_g component, i.e. the component projected onto the (curvy) **s** axis is constant in magnitude, but its orientation changes; that change (across the trough) is given by the **blue vector**, which we rotate 90° clockwise to get **Q**.

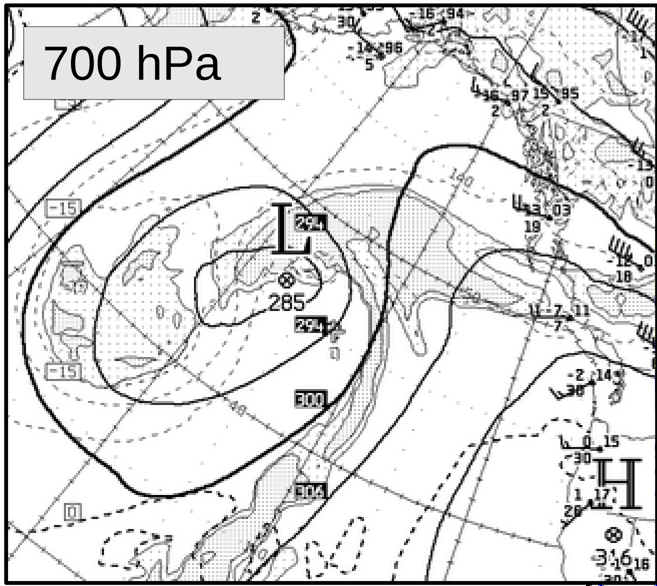
"The **Q** vector can be obtained by evaluating the vectorial change of $\mathbf{V}_g$ along the isotherm (with cold air on the left), rotating this change vector by 90° clockwise, and multiplying the resulting vector by $|\partial T/\partial n|$." (Holton)

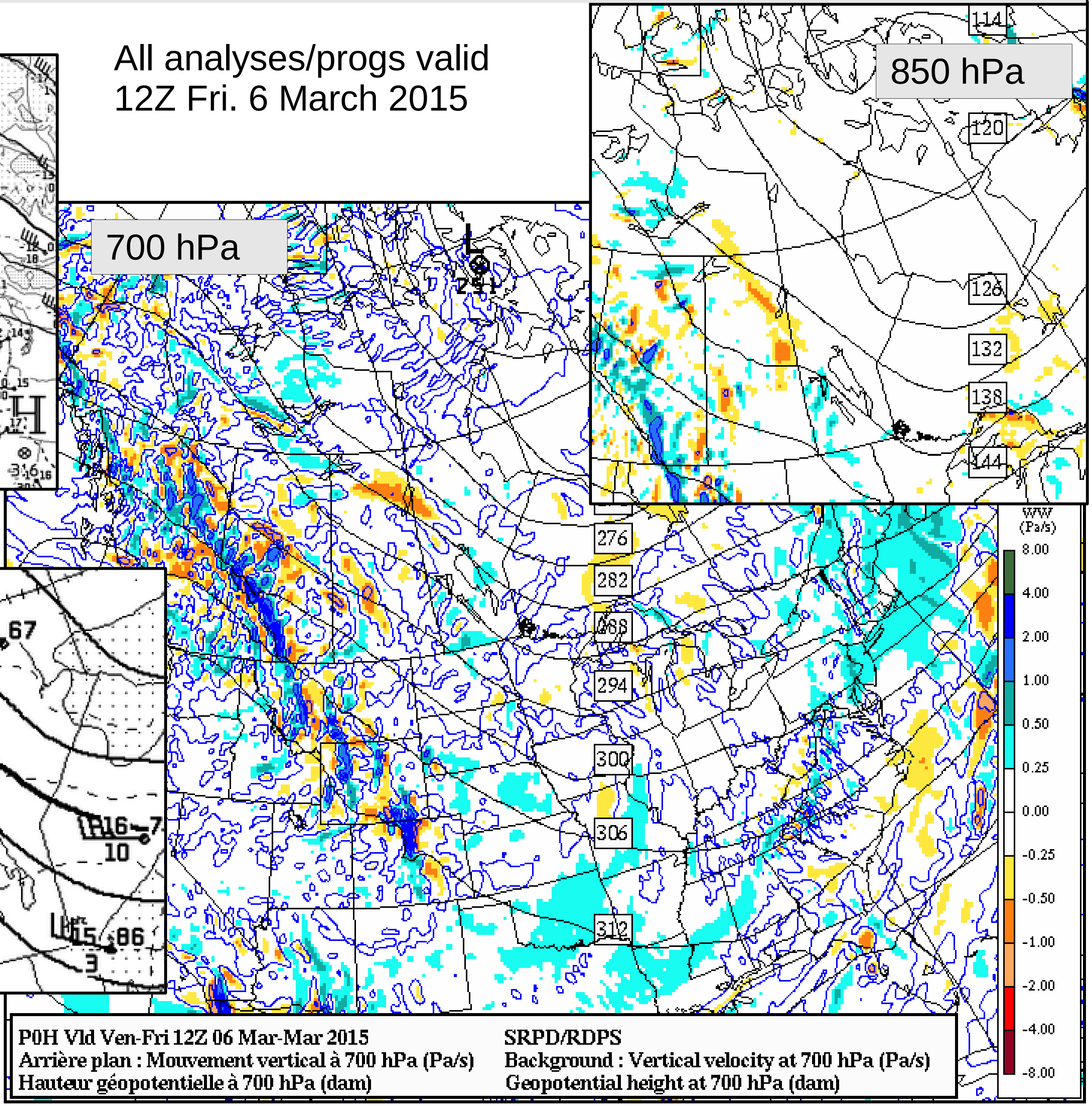

Q-vector
convergence
261
267
273
279
285
291
297
303
309
315
315
315
–24
–18
–12
–6
0
6
–120
–100
–80
700 HGHT, TEMP, Q–vectors, Q–omeg RHS at 150306/1200V006
–9.0 –7.0 –5.0 –3.0 –2.0 –1.5 –1.0 –0.5 0.5 1.0 1.5 2.0 3.0 5.0 7.0 9.0

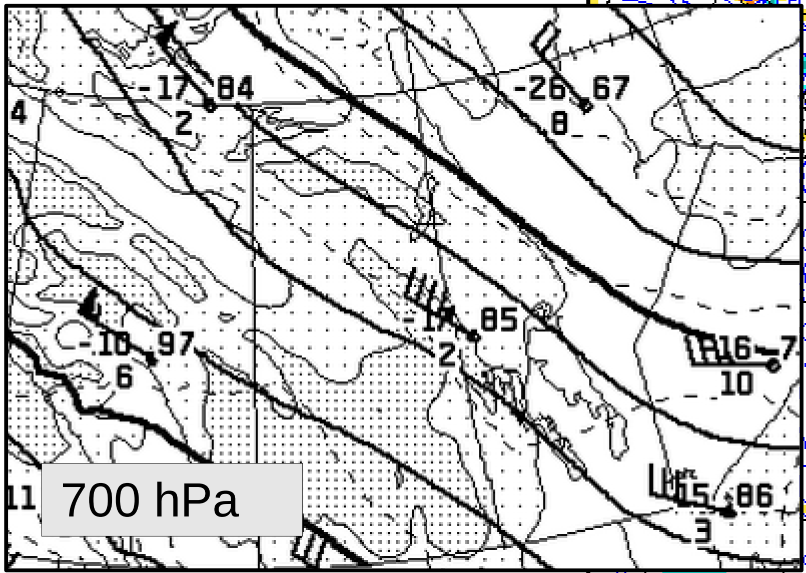


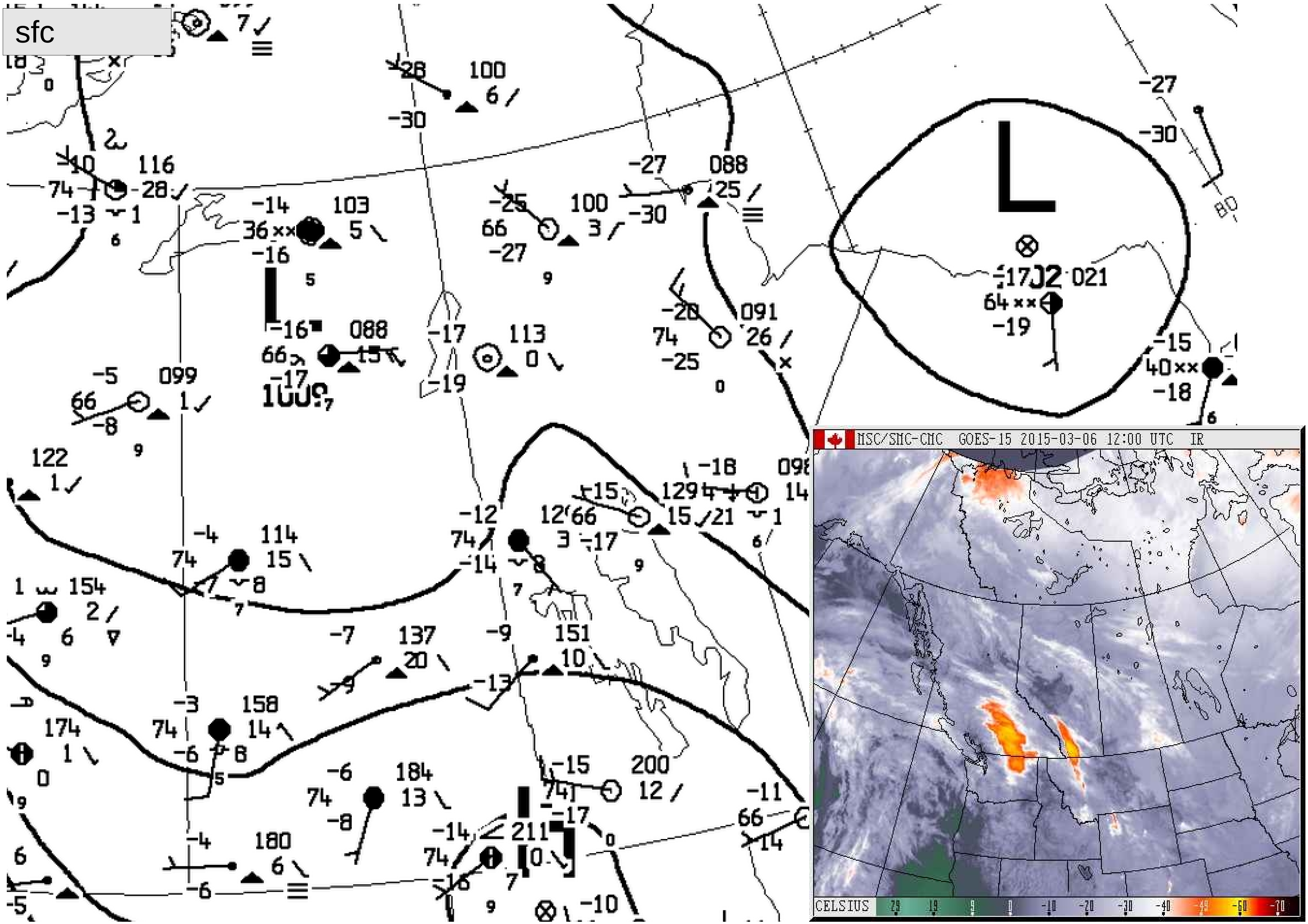
700 HGHT, TEMP, Q–omeg RHS at 150306/1200V006

All analyses/progs valid
12Z Fri. 6 March 2015





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