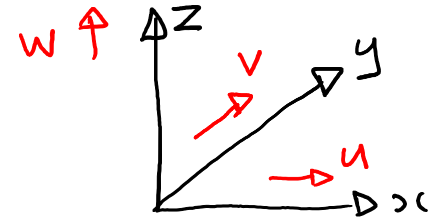
EAS 372, 12 JAN 2017



A. LAGRANGIAN DERIVATIVE

Arbitrary variable $q = q(\underbrace{x, y, z, t}_{\text{independent variables}})$ (alt. θ, ϕ, p, t)

$$dq = \frac{\partial q}{\partial x} dx + \frac{\partial q}{\partial y} dy + \left(\frac{\partial q}{\partial z}\right)_{x,y,z} dz + \frac{\partial q}{\partial t} dt$$

Constrain dx, dy, dz to be $d\vec{x} = \vec{u}\, dt$ where $\vec{u} = (u, v, w)$

$$Dq = \frac{\partial q}{\partial x} u\, dt + \frac{\partial q}{\partial y} v\, dt + \frac{\partial q}{\partial z} w\, dt + \frac{\partial q}{\partial t} dt$$

$$\frac{Dq}{Dt} = \frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x} + v \frac{\partial q}{\partial y} + w \frac{\partial q}{\partial z}$$

Lagrangian (or "material") derivative

$$\frac{Dq}{Dt} = \frac{\partial q}{\partial t} + \vec{u} \cdot \nabla q$$

local tendency in time

∇ is the gradient operator "grad"

∇ has different representation in different coord systems. ②

In Cartesians, $\nabla = \left(\partial/\partial x, \partial/\partial y, \partial/\partial z \right)$

Operates on a scalar to produce a vector, e.g.

$$\nabla q = \left(\partial q/\partial x, \partial q/\partial y, \partial q/\partial z \right)$$

Operates on a vector to produce a scalar, e.g.

$$\nabla \cdot \vec{u} = \left(\partial/\partial x, \partial/\partial y, \partial/\partial z \right) \cdot (u, v, w) = \underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}}_{\text{3D velocity divergence}}$$

($\nabla \cdot$ divergence operator)

$$\nabla \cdot \vec{u} = \underbrace{\nabla_H \cdot \vec{u}_H}_{\text{horiz. divergence}} + \frac{\partial w}{\partial z} \qquad \vec{u}_H = (u, v)$$

B. CONSERVED VARIABLES

③

If "q" is a conserved variable, then $\dfrac{Dq}{Dt} = 0$ ‡

e.g. Potential temperature if motion is adiabatic & unsaturated

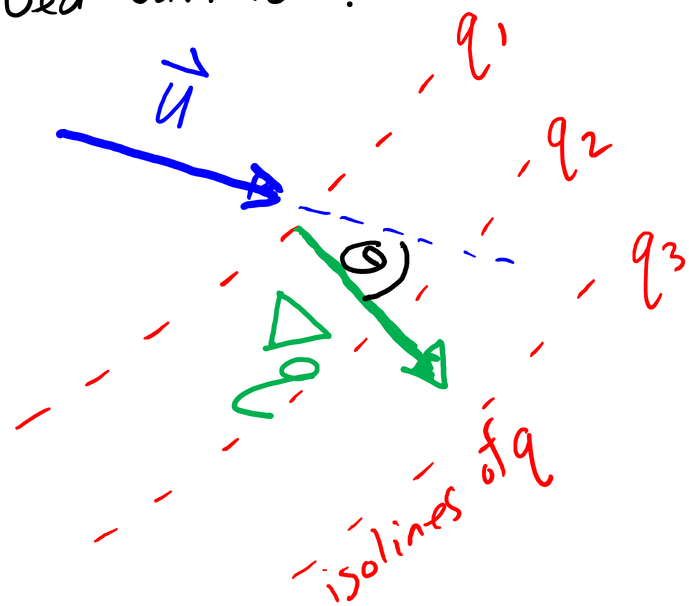
Mixing ratio, specific humidity " " unsaturated

T? NO, in general, but yes in unsat., adiab., horiz motion

‡ $$\frac{\partial q}{\partial t} = -\vec{u}\cdot\nabla q$$

Local tendency — advection (transport by the fluid)

The local tendency is due entirely to advection, for a conserved variable.



$$\vec{u}\cdot\nabla q = |\vec{u}|\,|\nabla q|\cos\theta$$