A. NATURAL COORD. SYSTEM

Recall that if q is a conserved variable then $\frac{Dq}{Dt} = 0$ and so

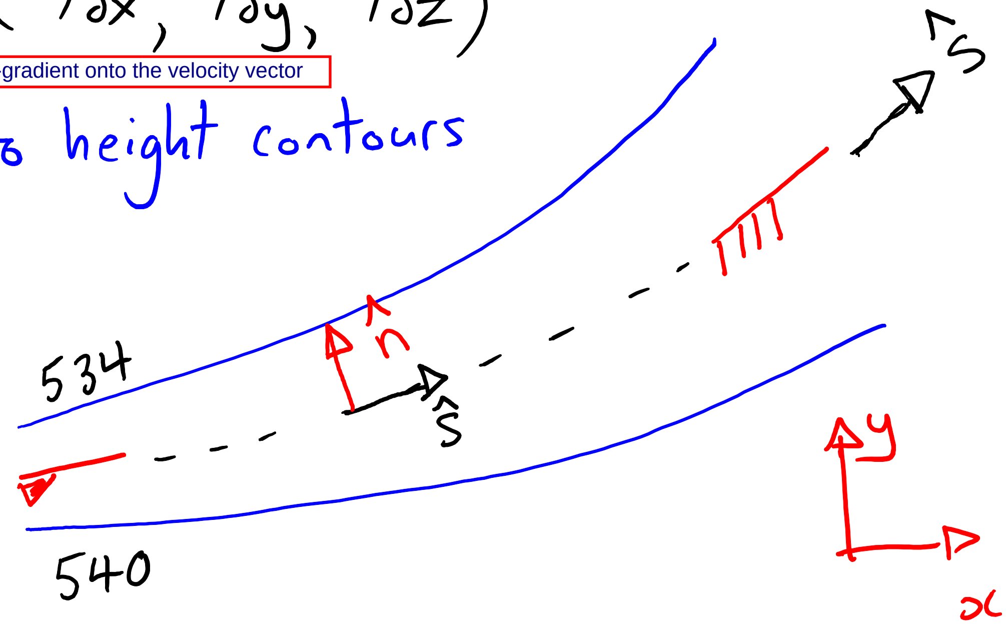
$$\frac{\partial q}{\partial t} = -\vec{u}\cdot\nabla q \quad \text{where} \quad \nabla = \left(\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z\right)$$

this quantity is the projection of the q-gradient onto the velocity vector

Natural coord system is referred to height contours

$\hat{s}$ points along the stream, and is a unit vector

$\hat{n}$ is $\perp^r$ to $\hat{s}$



By definition in $\hat{n}-\hat{s}$ coord system $\vec{u} = (u, v) \longrightarrow (|\vec{u}|, 0)$

along stream , across stream

$\therefore\ \vec{u}\cdot\nabla \longrightarrow \underbrace{|\vec{u}|}_{v} \frac{\partial}{\partial s}$

In natural coord system then,

$$\boxed{\frac{\partial q}{\partial t} = -v\frac{\partial q}{\partial s}}$$

To some approximation*, then, $\dfrac{\partial T}{\partial t} = \underbrace{-v\dfrac{\partial T}{\partial s}}_{\text{advective rate of change of temp.}}$

* in horizontal, unsaturated adiabatic motion its a good approx.

B. Derivation of Hypsometric Eqn

$$\frac{\partial p}{\partial z} = -\rho g \quad \text{and} \quad p = \rho \underbrace{R_d}_{\substack{\text{dry air gas const.}\\ 287\ \mathrm{J\,kg^{-1}\,K^{-1}}}} T_v \quad \text{with } T_v = T(1+0.61r)$$

r mixing ratio kg/kg

$$dz = \frac{-dp}{\rho g}$$

$$= -dp\,\frac{R_d T_v}{g\,p} = -\frac{R_d T_v}{g}\,\frac{dp}{p}$$

$\dfrac{dp}{p}$ — $d\ln p$

$y = \ln x$

$\dfrac{dy}{dx} = \dfrac{1}{x}$

$dy = \dfrac{dx}{x}$

$d\ln x = \dfrac{dx}{x}$

$$\int_{z_1}^{z_2} dz = z_2 - z_1$$

$$= -\frac{R_d}{g}\int_{p_1}^{p_2} T_v\, d\ln p, \quad T_v = T_v(p)$$

$$= -\frac{R_d}{g}\,\overline{T_v}\left[\ln p_2 - \ln p_1\right]$$ *

by 1st Mean Value theorem

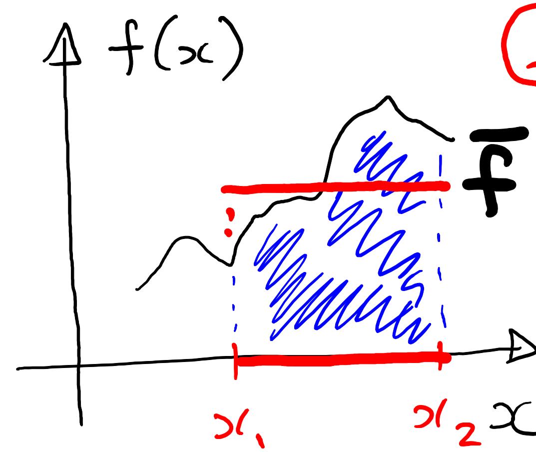
z_2 — p_2

z_1 — p_1

$\uparrow z$

(3)

$$\int_{x_1}^{x_2} f(x)\,dx = \overline{f}\,(x_2 - x_1)$$ 1ST Mean Value Thm



$$z_2 - z_1 = \frac{R_d}{g}\,\overline{T_v}\,(\ln p_1 - \ln p_2)$$

$$= \frac{R_d}{g}\,\overline{T_v}\,\ln p_1/p_2$$

$$* \quad \overline{T_v} = \frac{\int_{p_1}^{p_2} \frac{T_v}{p}\,dp}{\int_{p_1}^{p_2} dp/p}$$

Suppose $p_1 = 1000$ hPa, $p_2 = 500$ hPa

$$\underset{m}{\Delta z} = \left(\frac{287}{9.81}\ln 2\right)\overline{T_v} = 20.3\,\underset{K}{\overline{T_v}}$$

If we measure thickness in dam, then

$$\Delta z\,[\text{dam}] = 2.0\,\overline{T_v}\,[K]$$

But what is the nature of this average?

For each 1 K change in $\overline{T_v}$ we expect a 2 dam change in thickness

$$\overline{T_v} = \frac{\int_{p_1}^{p_2} \frac{1}{p} \, T_v(p) \, dp}{\int_{p_1}^{p_2} \frac{1}{p} \, dp}$$

simply $\ln p_2/p_1$

This mean virtual temperature is a "weighted average" and the "weighting function" is $1/p \times 1/\ln p_2/p_1$

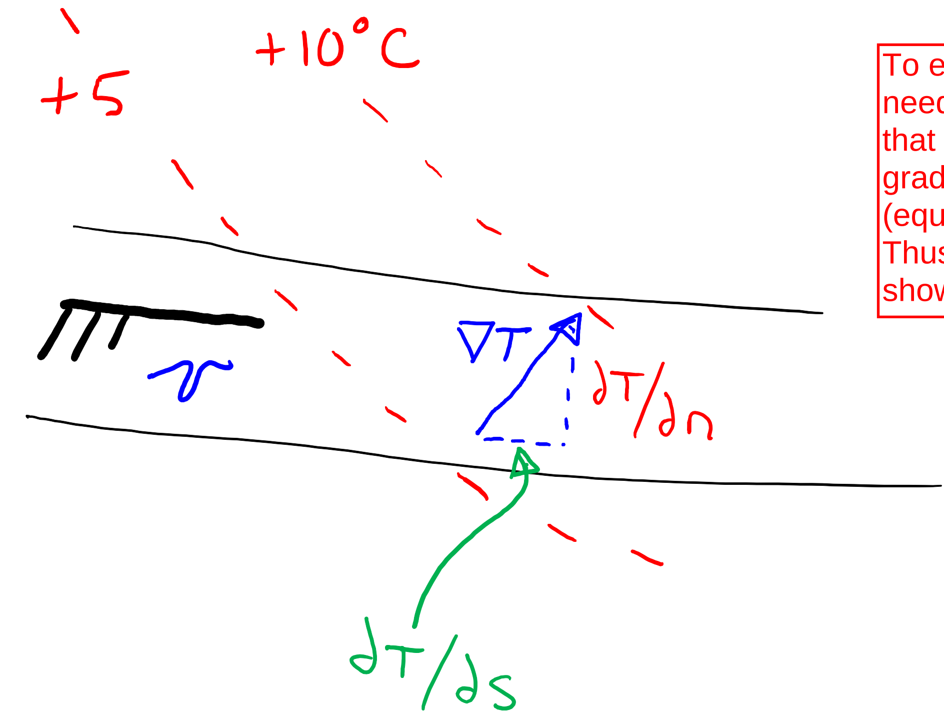
a constant

This means that in the context of the 1000 to 500 hPa thickness, T_v(500 hPa) is weighted <u>twice as strongly</u> in $\overline{T_v}$ as is T_v(1000 hPa).

IN CONNECTION WITH TODAY'S EXERCISE

"grad T"

∇T is a vector that is $\perp$ to isotherms, $\nabla T = \left(\frac{\partial T}{\partial s}, \frac{\partial T}{\partial n} \right)$



To evaluate the rate of temperature advection, we need to "project" grad T onto the velocity vector, that is, we want the component of the temperature gradient that is parallel to the velocity vector (equivalently, parallel to the height contours). Thus, we want the "partial T/partial s" component, shown in green.