A. Some named transport equations

Heat eqn. $\dfrac{\partial T}{\partial t} = \kappa \nabla^2 T$ $\kappa\,[m^2 s^{-1}]$ thermal diffusivity.

in 1D $\dfrac{\partial T}{\partial t} = \kappa \dfrac{\partial^2 T}{\partial x^2}$

dimensional homogeneity requires this have unit $m^2 s^{-1}$

$= -\dfrac{\partial}{\partial x}\left[-\kappa \dfrac{\partial T}{\partial x}\right]$ with $\kappa = $ const.

What mechanisms are at work?

Hint: $\dfrac{\partial \phi}{\partial t} = \underline{-\nabla \cdot \vec{F}} + Q$

flux divergence. "transport term" as has ∇ out front

where $-\kappa \dfrac{\partial T}{\partial x} = F_x$ "kinematic" heat flux along x axis

Recall Fourier Law $-\rho c_p \kappa \dfrac{\partial T}{\partial x} =$ conductive heat flux $[W\,m^{-2}]$

Alternative form of the heat equation $\rho c_p \dfrac{\partial T}{\partial t} = k \nabla^2 T$, $k = \rho c_p \kappa$ thermal conductivity $W m^{-1} K^{-1}$

What if problem is steady state? $\frac{\partial T}{\partial t} = 0 = \kappa \nabla^2 T$

$\nabla^2 T = 0$ Whose equation? Laplace's equation.

$\nabla . \nabla T = 0$ we now understand the "agency" at work.

$0 = \kappa \nabla^2 \phi + Q$ $\nabla^2 \phi = -Q/\kappa$ Poisson's eqn.

B. ADVECTION EQN

$$\frac{\partial T}{\partial t} = -\nabla . \vec{u} T$$

kinematic convective flux of sensible heat

In 1D $\frac{\partial T}{\partial t} = -\frac{\partial}{\partial x}(uT) \longrightarrow -u\frac{\partial T}{\partial x}$ if u indep of x

$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = 0$ 1D advection eqn with const. advective velocity.

$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$ non-linear 1D advection eq^n

C. ADVECTION - DIFFUSION EQN

$$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = \nabla \cdot \left(K \nabla \phi \right) + Q$$

advection

diffusion (K may not be const.)

In atmos. sci typically the K is a molecular diffusivity and is treated as const. and the molec. term may be neglected. An exception is when the diffusion term is retained to represent "turbulent (eddy) convection".

Note that $\vec{u} \cdot \nabla \phi = \nabla \cdot \vec{u} \phi - \phi \, \nabla \cdot \vec{u}$

velocity divergence, zero in some systems

It's often preferable to write the transport eqns in "flux form" ("transport form")

WATER VAPOUR VARIABLES

T_d dewpoint, e [Pa] vapour pressure (partial pressure of water vapour)

ρ_v absolute humidity [$kg\ m^{-3}$]

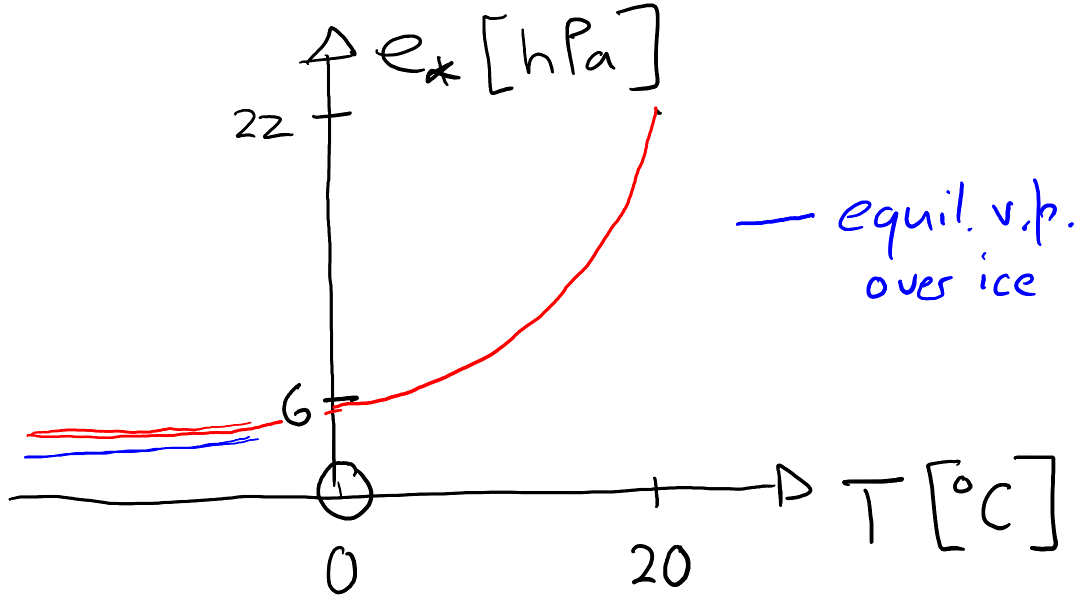
q specific humidity $\quad q = \rho_v / (\rho_d + \rho_v)$

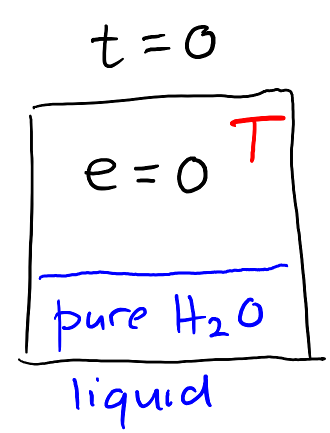
r mixing ratio $\quad r = \rho_v / \rho_d$

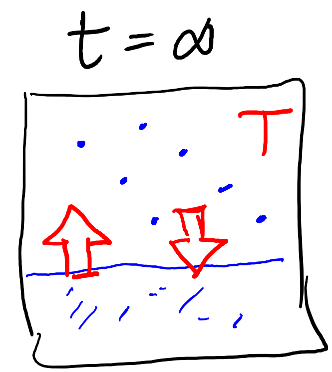
insignificantly different for most purposes

$$RH = e / e_*(T)$$

Benchmark for vapour pressure e_*







equil happens when $e = e_*$, and, $e_* = e_*(T)$

e and T_d are not indep. pieces of info:

$$e = e_*(T_d) \qquad RH = \frac{e_*(T_d)}{e_*(T)} \qquad T \geq T_d$$

$$T_d = e_*^{-1}(e)$$

Ideal gas law for w.v.

$$\boxed{e = \rho_v R_v T}$$

$$R_v = 462 \text{ J kg}^{-1} \text{ K}^{-1}$$

<u>Virtual temperature</u>

$$T_v = T(1 + 0.61 r)$$

$$p = \rho R_d T_v$$

total pressure

total density

$$p - e = \rho_d R_d T$$