ISOBARIC COORDINATE SYSTEM

For any field or function $\phi = \phi(x,y,z,t) \longrightarrow \phi(x,y,p,t)$

Vertical velocity is a diagnostic variable in a hydrostatic atmosphere, as by definition

$$\frac{Dw}{Dt} = 0 = \frac{-1}{\rho}\frac{\partial p}{\partial z} - g$$

But $w = \dfrac{Dz}{Dt} = \dfrac{Dp}{Dt} \Big/ \dfrac{\partial p}{\partial z} = \dfrac{-1}{\rho g}\,\omega$

$$\boxed{\omega = -\rho g w}$$

$\mathrm{Pa\ s^{-1}}$ $\mathrm{m\ s^{-1}}$

Horizontal Mtm Eqns

$$\frac{D}{Dt} = \left(\frac{\partial}{\partial t}\right)_{x,y,p} + \vec{V}_p \cdot \nabla_p + \omega\frac{\partial}{\partial p}$$

$$\vec{V}_p = (u,v)$$

$$\nabla_p = \left(\partial/\partial x, \partial/\partial y\right)$$

$$\boxed{\frac{D}{Dt}(u,v) = -g\left(\partial/\partial x, \partial/\partial y\right)Z - f(-v,u)}$$

neglecting friction

Continuity Eqn in the isobaric coord. system

$$\nabla_p \cdot \vec{V}_p + \frac{\partial \omega}{\partial p} = 0^{*} \quad (\text{a "diagnostic eqn"})$$

horizontal divergence

Let $p = p_\infty = 0$ at top of atmos. Then $\omega(p_\infty) = \omega_\infty = 0$

$$\int_p^{p_\infty} \frac{\partial \omega}{\partial p}\, dp = \left[\omega\right]_p^{p_\infty} = 0 - \omega(p) = -\int_p^{p_\infty} \underbrace{\nabla_p \cdot \vec{V}_p}_{D_p}\, dp$$

$$\therefore \quad \omega(p) = \int_p^0 D_p\, dp$$

At any level p, vertical velocity $\omega(p)$ is given by the height integral of the horizontal divergence up the overlying column

$\boxed{* \text{ If } \vec{U} = (u, v, \omega) \text{ and } \nabla = \left(\partial/\partial x, \partial/\partial y, \partial/\partial p\right) \text{ then } \nabla \cdot \vec{U} = 0}$

By the 1st mean value theorem $\omega(p) = \left(\int_p^0 dp \right) \times \overline{D}_p$

average value in column from p to top of atmos.

$$\therefore \boxed{\omega(p) = -p\,\overline{D}_p}$$

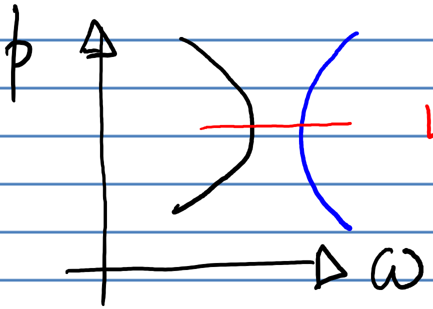
At ground, the "no slip, no leak" condition implies

$u = v = w = \omega = 0$

ⓐ Since ω is zero at the surface and at the top of the atmosphere, divergence is either zero everywhere in the column, or, it changes sign at least once. ("Dines compensation")

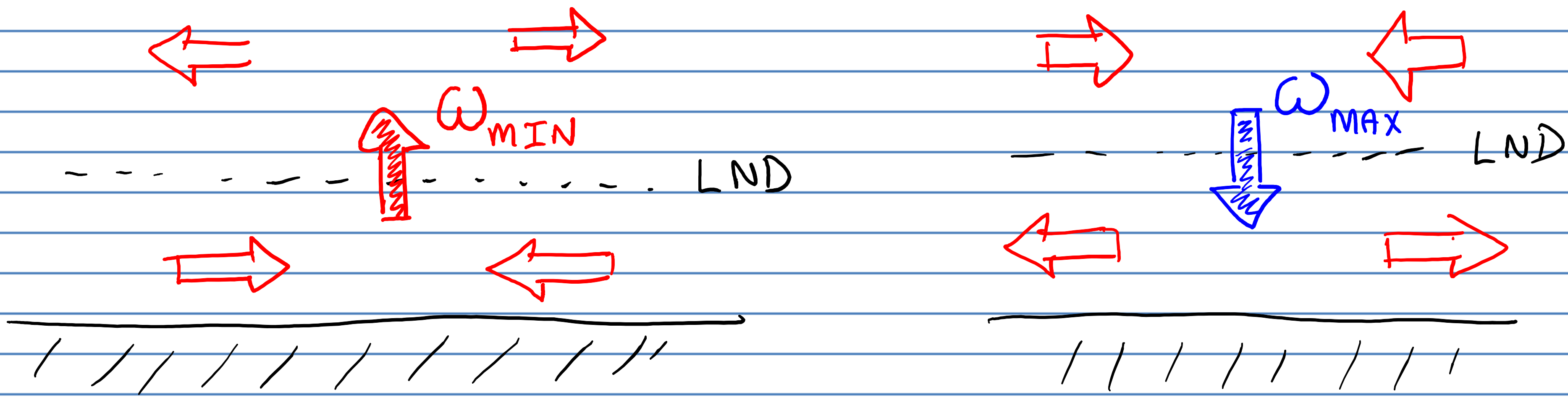
ⓑ at any "level of non-divergence" (LND), $\frac{\partial \omega}{\partial p} = 0$

(anywhere that D_p changes sign, we have an LND)



So ω is an "extremum" at any LND $\Rightarrow$ locally it takes on a max or min value.

SIMPLEST PARADIGM



Nominally, the LND is considered to be at ~ 500 hPa

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