The idealized (horizontally-homogeneous) atmospheric surface layer ("hh_ASL")

- shallow ground-based layer, order 100 m deep
- important because entire atmosphere interacts with surface through this layer, i.e. the ASL is a "valve" through which pass exchange fluxes of heat, water vapour, momentum
- horizontally-homogeneous: statistical properties (excluding mean pressure) indep. of (*x*,*y*)
- stationary: stat. properties indep. of *t* (pragmatically: ∂/∂t terms "small" in gov. eqs.)
- therefore statistical properties depend only on height, e.g. $\bar{u}=\bar{u}(z),\ \sigma_w=\sigma_w(z)$
- vertical fluxes of momentum and heat across the layer regarded as height-independent, i.e.

$$\overline{u'w'}=\text{const.},\quad \overline{v'w'}=\text{const.},\quad \overline{w'T'}=\text{const.}$$

- define friction velocity $u_*=\left[\left(\overline{u'w'}\right)^2+\left(\overline{v'w'}\right)^2\right]^{1/4}$
- drag of the wind on the surface has magnitude $\tau=\rho\, u_*^2$

$N\,m^{-2}$

- vertical profiles of statistics are given by the empirical "Monin-Obukhov similarity theory"

$\bar{w}=0$. Proof:

① $\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}+\dfrac{\partial w}{\partial z}=0$

② average

③ $\partial/\partial x=\partial/\partial y=0$

∴ ④ $\dfrac{\partial \bar{w}}{\partial z}=0$

⑤ $\bar{w}=$ const.

But $\bar{w}(0)=0$

eas372_horizunifASL.odp
JDW, EAS U. Alberta
Last. modif: 9 March 2017

Setting the ASL within the (fairweather) atmospheric boundary layer (ABL)

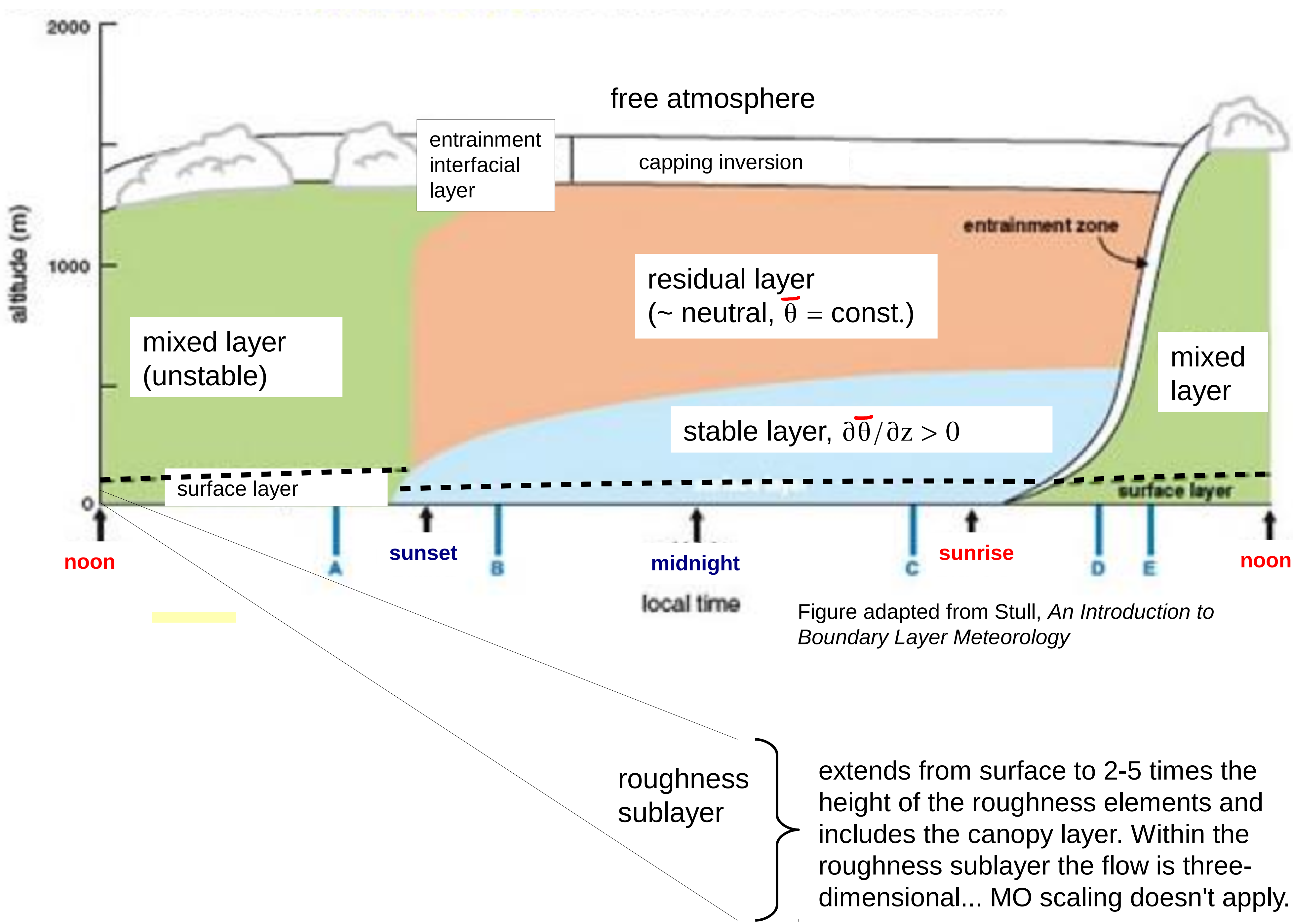

Figure adapted from Stull, *An Introduction to Boundary Layer Meteorology*

extends from surface to 2-5 times the height of the roughness elements and includes the canopy layer. Within the roughness sublayer the flow is three-dimensional... MO scaling doesn't apply.

INTRODUCTION TO OBUKHOV'S PAPER ON 'TURBULENCE IN AN ATMOSPHERE WITH A NON-UNIFORM TEMPERATURE'*

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and

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(Received 1 April, 1971)

In 1943 at the age of 25, A. M. Obukhov finished the remarkable paper that follows this brief introduction. Because World War II ravaged around the globe at that time this manuscript was not published until 1946. Unfortunately, however, it appeared in the very limited first issue of the Journal: *Trudy Instituta Teoreticheskio Geofiziki AN SSSR* (Works of the Institute of Theoretical Geophysics, Acad. Sci. USSR, No. 1). So without the aid of modern duplicating techniques, the paper was doomed to obscurity right from the start. Very few scientists outside the Soviet Union were aware of it and even in the U.S.S.R. not many realized its real importance.

At present, nearly 25 yr after its publication, most of the information it contains is known to the scientific community, partly from the original source and partly by independent rediscovery. Nevertheless, it seems fully justified to give this paper new exposure because it is a truly classical contribution and every serious student of the atmospheric boundary layer should have the opportunity to study it.

Probably the major contribution of the paper is the introduction of the 'length scale of the dynamic turbulence sublayer', L_1. This length scale was later introduced independently by Lettau (1949), and at present it is commonly known as the Monin-Obukhov length. Its fundamental role in the whole field of boundary-layer meteorology was most clearly explained in the well-known paper by Monin and Obukhov (1954). In this last paper the presentation is based on purely dimensional considerations which imply that the mean wind and temperature profiles must be determined by some universal function of the uniquely defined (to within a numerical factor) dimensionless height $\zeta = z/L_1$. (Let us note that Obukhov included the numerical factor $1/k$, where k is the von Karman constant, in the definition of L_1 which does not follow from dimensional considerations. The same factor was also included in the definition of length scale by Monin and Obukhov in 1954 and is used in practically all subsequent works although its presence produces some difficulties in comparison of the data

* Contribution No. 241, University of Washington, Dept. of Atmospheric Science.

Boundary-Layer Meteorology **2** (1971) 3–6.

<u>Monin-Obukhov similarity theory</u> (MOST)

- applies in horiz. homog. ASL, at heights
    - not too close to ground*
    - not too far from ground
- empirical

* above the "roughness sublayer"

- basis for scaling observed statistics; provides better ordering than any earlier suggestions
- any statistics with units of velocity is proportional to ("scales on") the friction velocity u_*
- height scales on the Obukhov length L
- "extended MOST" adds ABL depth δ as an additional length scale

The Obukhov length

$$L = \frac{-u_*^{\,3}}{k_v \dfrac{g}{T_K} \overline{w'T'}} \qquad \xrightarrow[0]{\left|\overline{w'T'}\right|} \qquad \pm\infty$$

k_v (=0.4) the von Karman constant

← decreasingly unstable ← → decreasingly stable → neutral

negative | 0 | positive | L

increasingly unstable ← neutral → increasingly stable

negative | 0 | positive | $1/L$

(or, z / L)

Dugway Proving Grounds, Utah, 2005 – 18 sonics, 50 km fetch

Boundary-Layer Meteorol (2008) 129:353–369
DOI 10.1007/s10546-008-9319-5

ORIGINAL PAPER

Monin-Obukhov Functions for Standard Deviations of Velocity

J. D. Wilson

Boundary-Layer Meteorol (2009) 132:193–204
DOI 10.1007/s10546-009-9399-x

ARTICLE

Turbulent Kinetic Energy Dissipation in the Surface Layer

D. Charuchittipan · J. D. Wilson

Boundary-Layer Meteorol (2013) 146:149–160
DOI 10.1007/s10546-012-9763-0

RESEARCH NOTE

Statistics of the Wind-Speed Difference Between Points with Cross-Wind Separation

J. D. Wilson

STOPPED HERE 9 MAR/2017

Monin-Obukhov theory arose from need to organize (and generalize from) observations. Based on "dimensional analysis", and amounts to introduction of "natural scales", e.g. u_* for velocity statistics, L for lengths

14 March/2017

MO theory posits that suitably non-dimensionalized quantities are universal functions of z/L

e.g. $\dfrac{\sigma_w}{u_*} = f\left(\dfrac{z}{L}\right)$, then experiment determines the function.

The Neutral* Wind Profile

* ie. $\partial\bar{\theta}/\partial z = 0$, $\partial\bar{T}/\partial z = \gamma_d = -g/c_p$, $\overline{w'\theta'} = 0$

the eddy diffusion model for the (vertical convective) eddy flux of sensible heat is

$$\overline{w'\theta'} = -K_h \, \partial\bar{\theta}/\partial z = -K_h\left(\frac{\partial\bar{T}}{\partial z} - \gamma_d\right)$$

↳ Align coord system with mean wind, $(\bar{u}, 0, 0)$ mean veloc. vector. Want to know $\bar{u}(z)$, and "no slip condition" tells us $\bar{u}(z_0) = 0$ (this <u>defines</u> the surface roughness length z_0).

$$\frac{z}{u_*} \frac{\partial \bar{u}}{\partial z} = \hat{\phi}_m\left(\frac{z}{L}\right) \text{ in general. But } |L| \xrightarrow[0]{|\overline{w'\theta'}|} \infty$$

$$= \text{const. in neutral limit} = 1/k_v$$

$$\boxed{\frac{k_v z}{u_*} \frac{\partial \bar{u}}{\partial z} = \phi_m\left(\frac{z}{L}\right)}$$

$$\frac{k_v}{u_*} \frac{\partial \bar{u}}{\partial \ln z} = 1 \text{ (in the neutral limit)}$$

$$\int_{\ln z_0}^{\ln z} \frac{\partial \bar{u}}{\partial \ln z} \, d\ln z = \bar{u}(z) - \underbrace{\bar{u}(z_0)}_{0} = \frac{u_*}{k_v} \int_{\ln z_0}^{\ln z} 1 \, d\ln z$$

$$\bar{u}(z) = \frac{u_*}{k_v}\left[\ln z - \ln z_0\right] = \frac{u_*}{k_v} \ln \frac{z}{z_0}$$

Eddy viscosity closure for the eddy momentum fluxes.

$$\overline{u'w'} = -u_*^2 = -K_m \left(\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x} \right)$$

K_m: eddy viscosity, $m^2 s^{-1}$

$\frac{\partial \bar{w}}{\partial x}$: vanishes in ideal ASL

May write K_m = velocity × length $= u_* \lambda$, $\lambda \propto z$

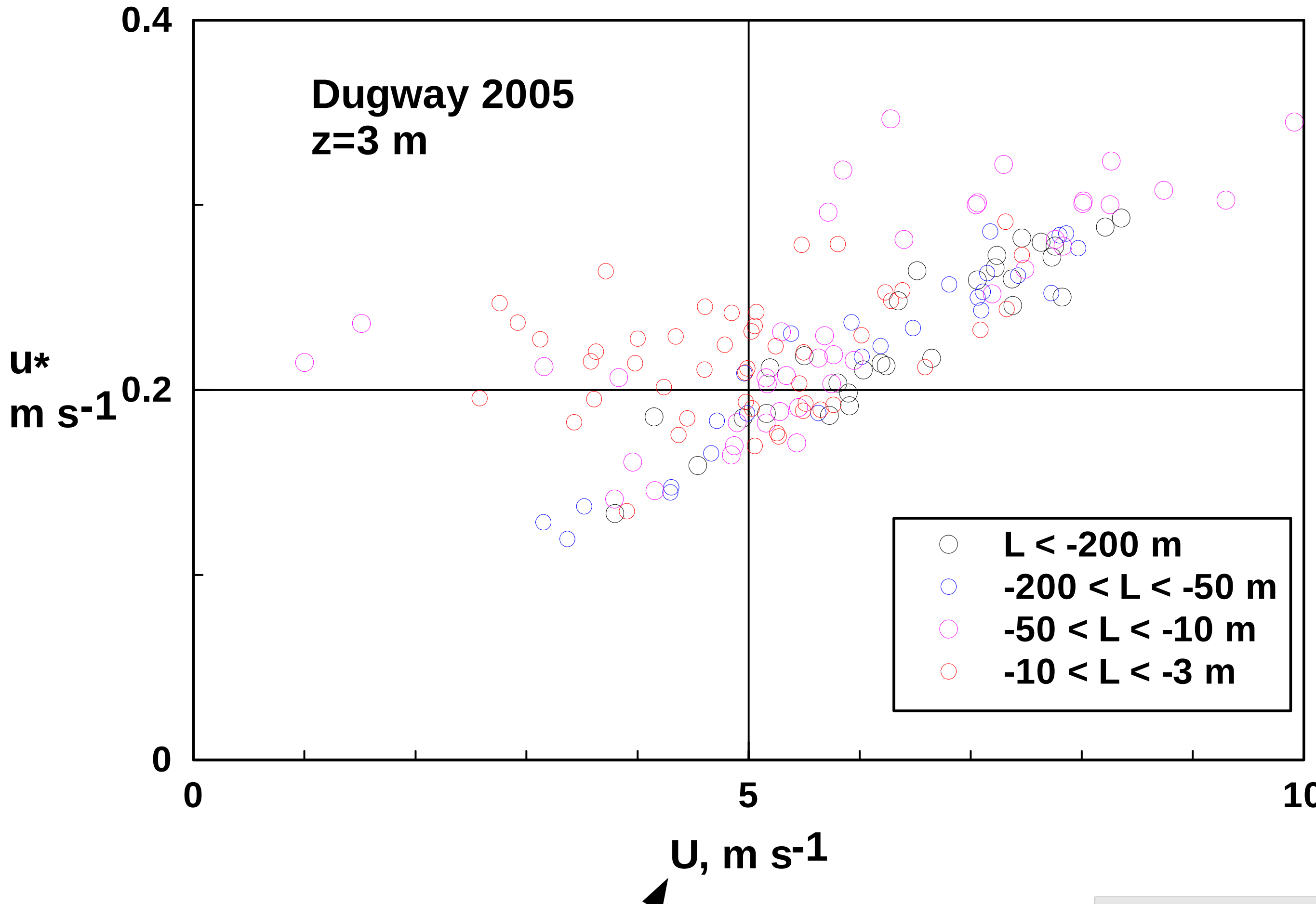
Lets write $\lambda = k_v z$ ‡

$$-u_*^2 = -k_v u_* z \frac{\partial \bar{u}}{\partial z} \longrightarrow \bar{u}(z) = \frac{u_*}{k_v} \ln \frac{z}{z_0}$$

‡ in the general case $\left(\frac{1}{L} \neq 0 \right)$ a more complex formulation is needed, to accommodate the influence of stratification on eddy size.

Typical values of the friction velocity

- assuming a wind strong enough to feel (but not a gale), about 0.1-0.5 m s^{-1}



i.e. $\overline{u}$

$$L = \frac{-u_*^{\,3}}{k_v \, \frac{g}{T_K} \, \overline{w'T'}}$$

Neutral wind profile ($|z / L| << 1$) – three near-neutral, 30 min runs, Dugway

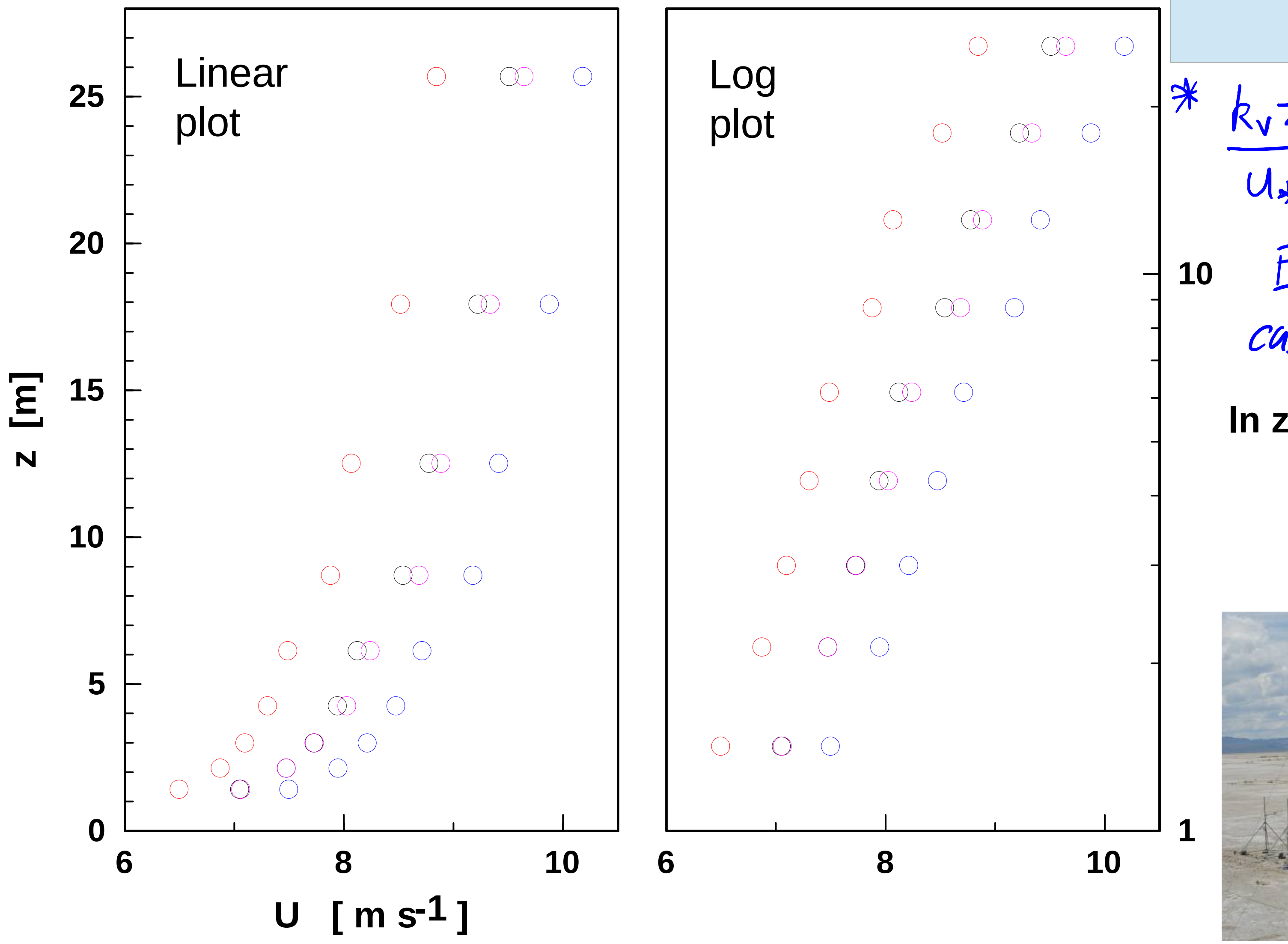
$$\frac{\bar{u}}{u_*} = \frac{1}{k_v} \ln \frac{z}{z_0}$$

z_0 the "surface roughness length", k_v (=0.4) the von Karman constant

Stable stratification (L > 0): *

$$\frac{\bar{u}}{u_*} = \frac{1}{k_v}\left[\ln\frac{z}{z_0} + \beta\,\frac{z - z_0}{L}\right]$$

$$(\beta \approx 4.7 - 5)$$



* $\frac{k_v z}{u_*} \frac{\partial \bar{u}}{\partial z} = \phi_m\left(\frac{z}{L}\right)$

Expts. show in stable case

$\phi_m = 1 + \beta \frac{z}{L}$

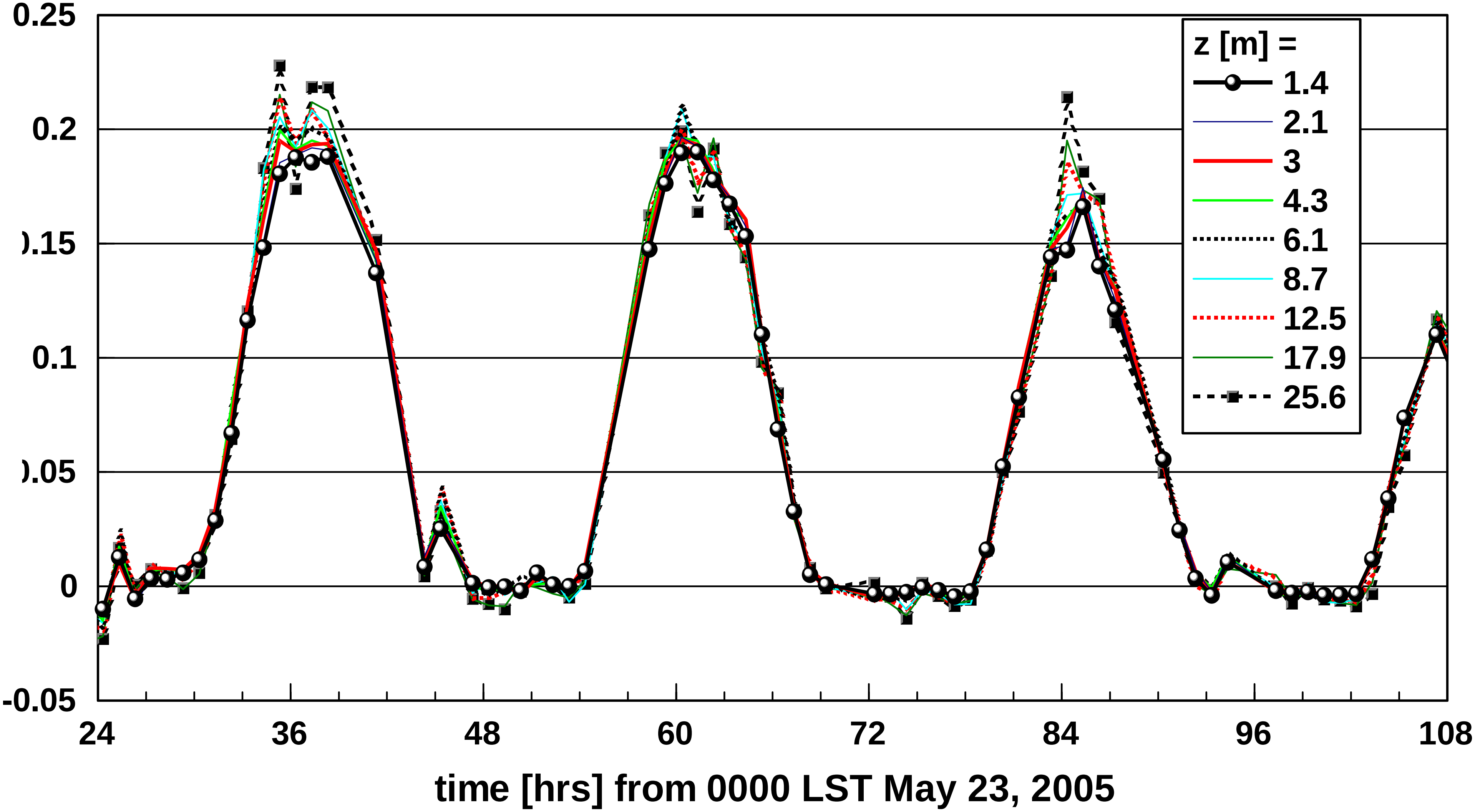
$\beta \simeq 5$

Fractional height-variation of the (kinematic, vertical, convective, eddy) heat flux (density) is *small*

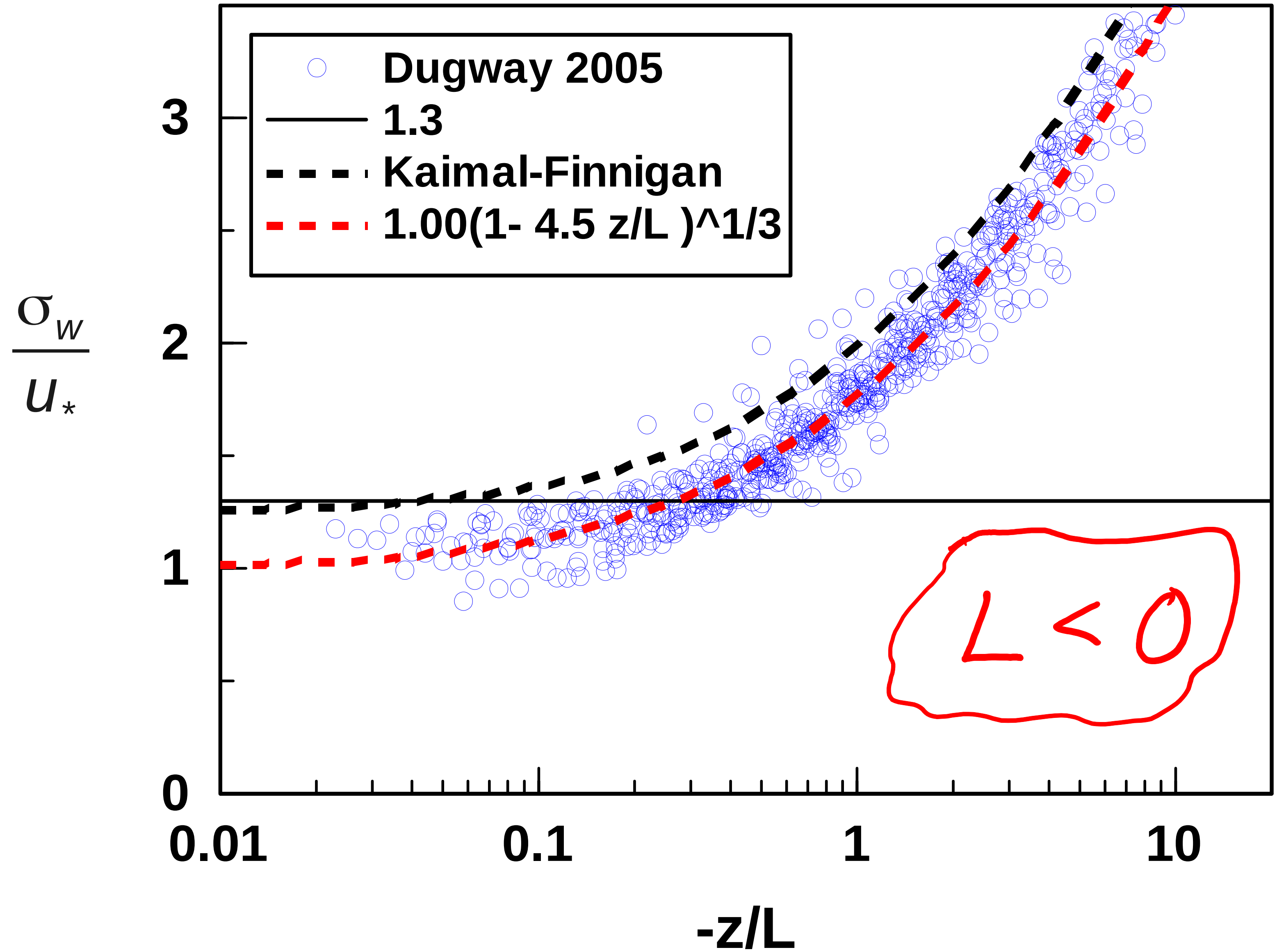
Consistent with MO treatment of ideal ASL as a "constant flux layer"

$\overline{w'T'}$, $\overline{u'w'}$, $\overline{v'w'}$

$\overline{w'T'}$ $\mathrm{m\,s^{-1}\,K}$



Demonstration of Monin-Obukhov scaling – "collapses" data from all 18 anemometers



$$\frac{\sigma_w}{u_*} = 1.0 \left(1 - 4.5 \frac{z}{L}\right)^{1/3}$$

Quart. J. R. Met. Soc. (1971), **97**, pp. 168-180

551.507.7 : 551.508 : 551.510.522 : 551.551.8

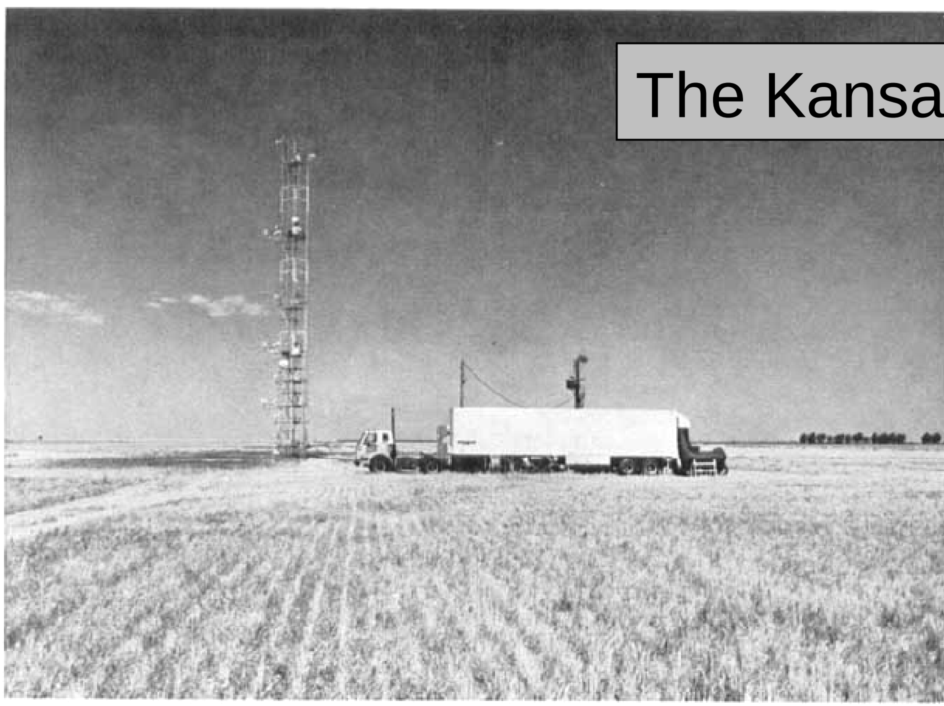
An experimental study of Reynolds stress and heat flux in the atmospheric surface layer

By D. A. HAUGEN, J. C. KAIMAL and E. F. BRADLEY

Air Force Cambridge Research Laboratories *CSIRO, Canberra, Australia*

The paper describes an experimental programme to study the characteristics of momentum and heat transport in the first 22·6 m of the atmosphere. Sonic anemometers and fine platinum wire thermometers were used for flux computations by the eddy correlation technique. Two drag plates were used to measure the surface stress.

The Kansas Experiment



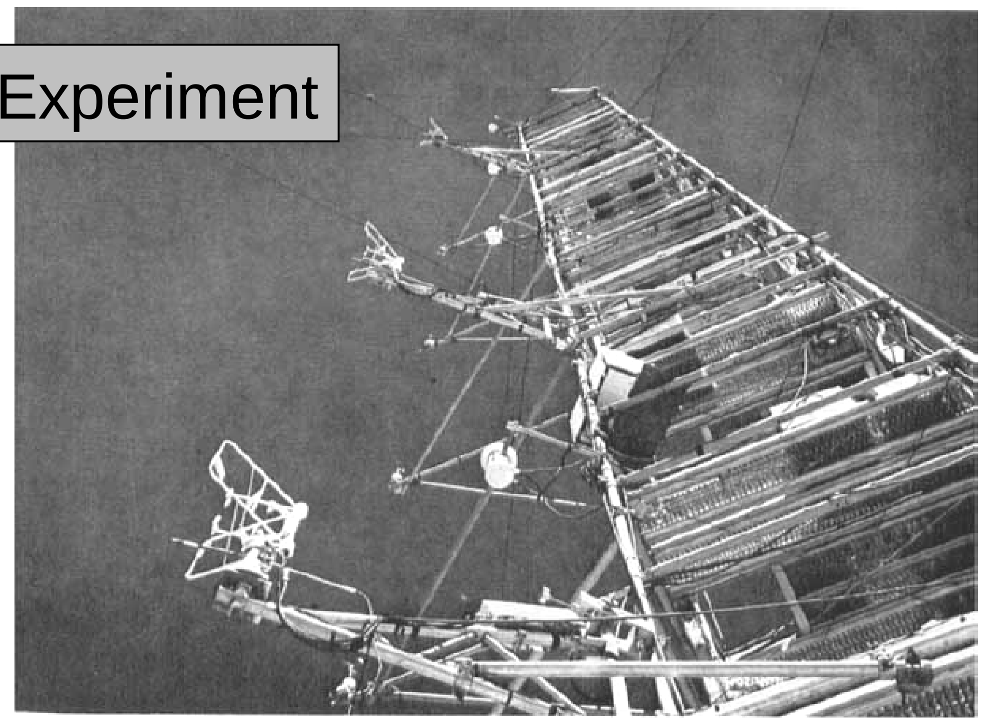


Figure 1. View of experiment site in Kansas showing instrumented van and tower.

The Kansas Experiment

Flux-Profile Relationships in the Atmospheric Surface Layer

J. A. Businger,[1] J. C. Wyngaard,[2] Y. Izumi[2] and E. F. Bradley[3]

(Manuscript received 27 July 1970)

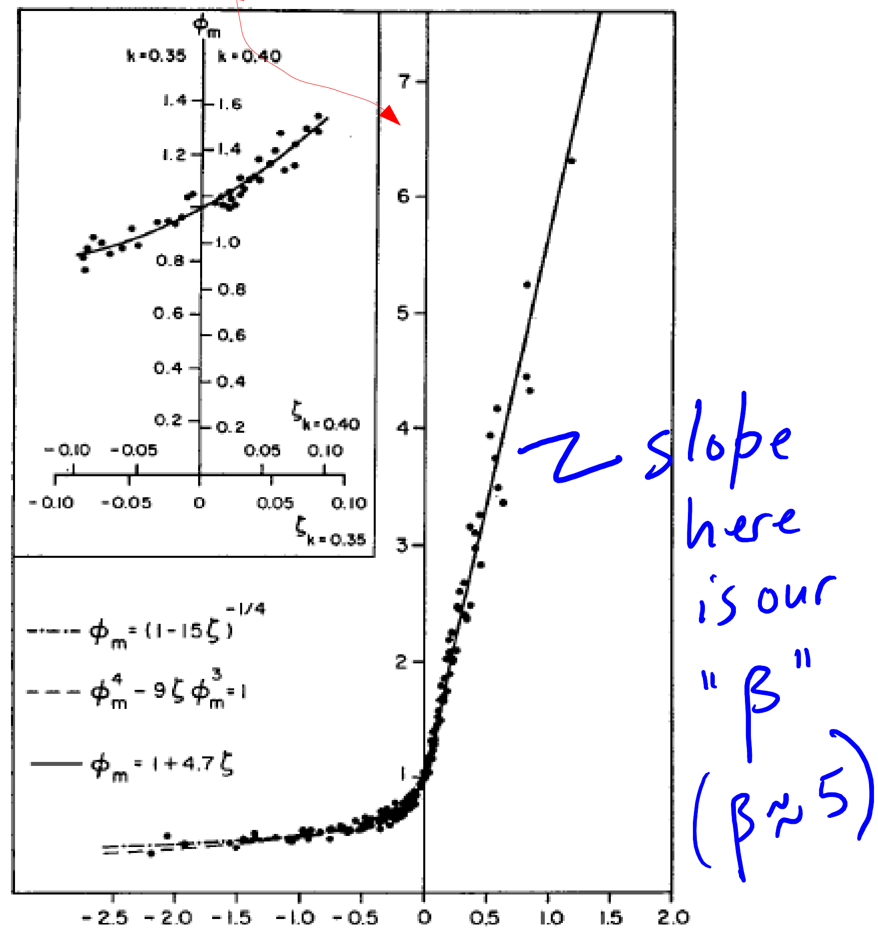
ABSTRACT **(J. Atmos. Sci., Vol. 28, 1971)**

Wind and temperature profiles for a wide range of stability conditions have been analyzed in the context of Monin-Obukhov similarity theory. Direct measurements of heat and momentum fluxes enabled determination of the Obukhov length L, a key independent variable in the steady-state, horizontally homogeneous, atmospheric surface layer. The free constants in several interpolation formulas can be adjusted to give excellent fits to the wind and temperature gradient data. The behavior of the gradients under neutral con-

$\mathrm{Ri}=\dfrac{g\partial\bar{\theta}/\partial z}{\bar{\theta}(\partial\bar{U}/\partial z)^2}$	Richardson number, a stability parameter
$\phi_m=\dfrac{kz}{u_*}\dfrac{\partial\bar{U}}{\partial z}$	A dimensionless wind shear
$\phi_h=\dfrac{z}{\theta_*}\dfrac{\partial\bar{\theta}}{\partial z}$	A dimensionless temperature gradient
$\alpha=\dfrac{K_h}{K_m}=\dfrac{\overline{w'\theta'}}{\overline{u'w'}}\dfrac{\partial\bar{U}/\partial z}{\partial\bar{\theta}/\partial z}$	Ratio of the eddy transfer coefficients
$\zeta=\dfrac{z}{L}=-\dfrac{kg\overline{w'\theta'}z}{\bar{\theta}u_*^3}$	A dimensionless height

MO theory postulates these are universal functions of z/L, but experiments are needed to determine their form

$$\phi_m=\frac{k_v\,z}{u_*}\,\frac{\partial U}{\partial z}$$



$\zeta=z/L$

MOST was found to organize the observations very well. However there was later a controversy regarding possible flow distortion by boxes on the tower. The Kansas expts. gave what is now regarded as an anomalously small von Karman const.

AN INTERNATIONAL TURBULENCE COMPARISON EXPERIMENT (ITCE 1976)

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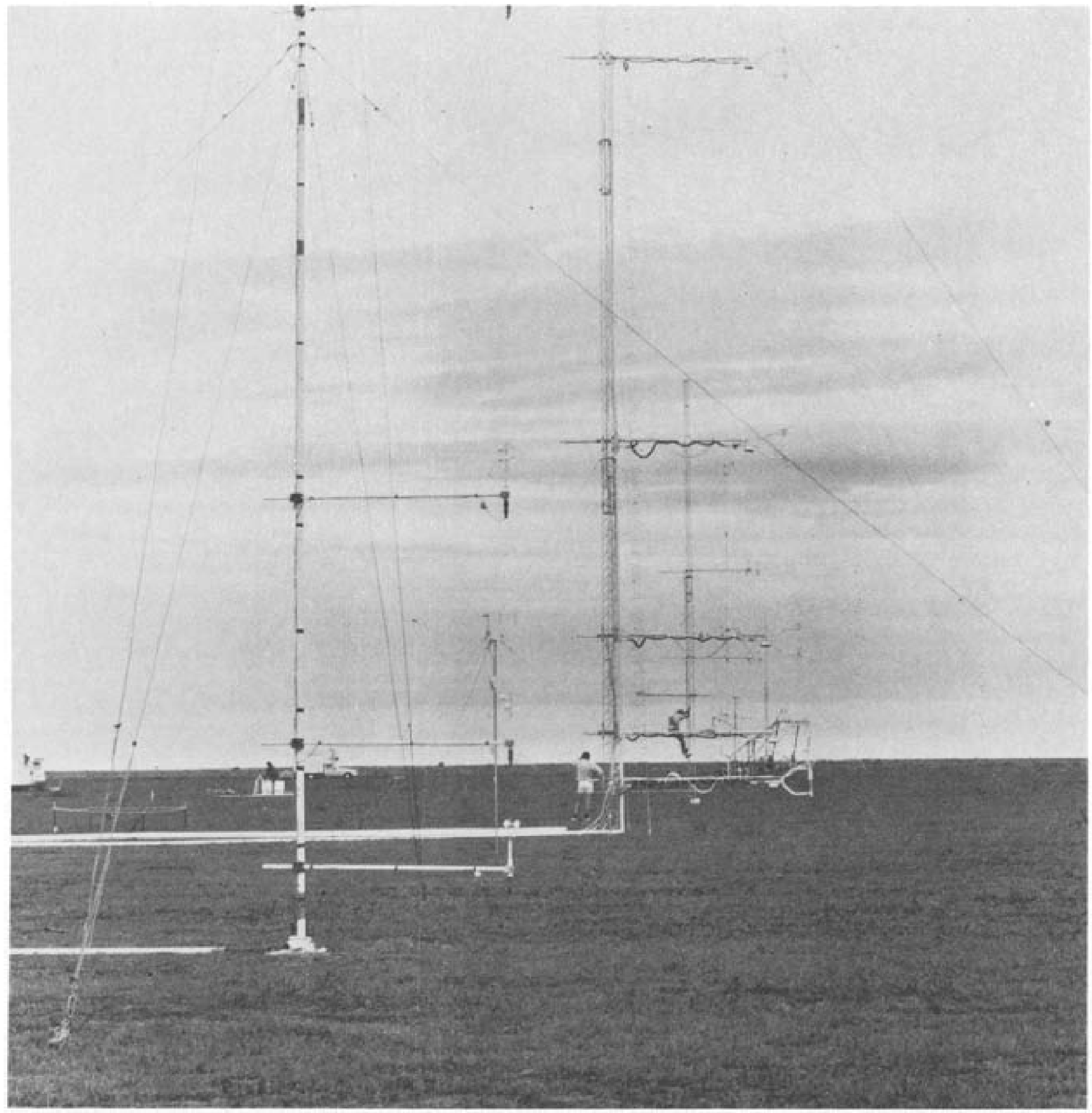
(Received 14 July, 1982)



AN ALTERNATIVE ANALYSIS OF FLUX-GRADIENT RELATIONSHIPS AT THE 1976 ITCE

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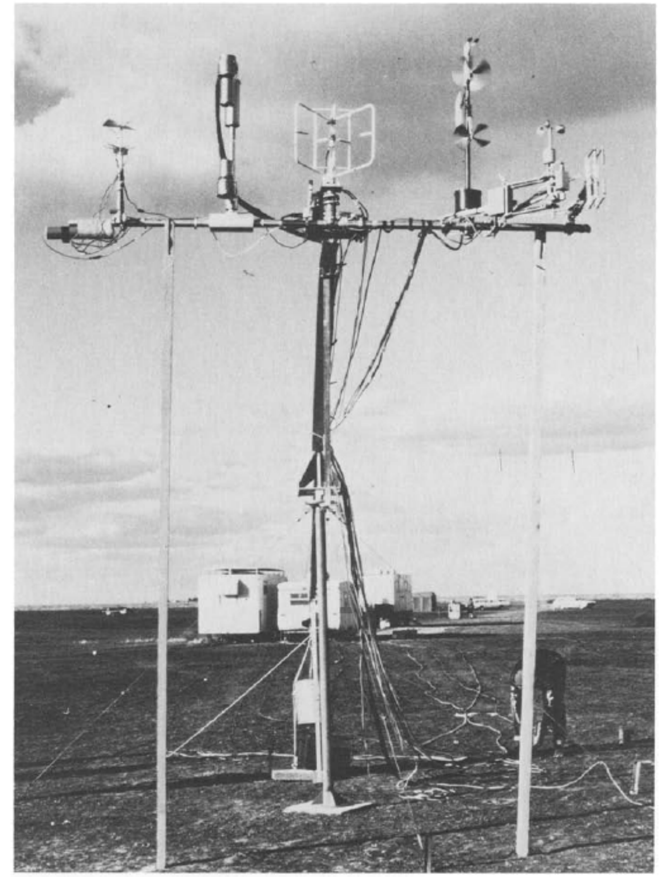
(1982; Bound. Layer Meteorol., Vol. 22)



Assumed:

$$\frac{k_{vm} z}{u_*}\,\frac{\partial \overline{u}}{\partial z} = \phi_m\left(\frac{z}{L}\right)$$

$$\frac{k_{vh} z}{T_*}\,\frac{\partial \overline{\theta}}{\partial z} = \phi_h\left(\frac{z}{L}\right)$$

$$\frac{k_{vw} z}{\rho_{v*}}\,\frac{\partial \overline{\rho}_v}{\partial z} = \phi_w\left(\frac{z}{L}\right)$$

and that all phi’s tend to unity in neutral limit (allowing possib. of distinct von Karman consts).

Concluded:

$$k_{vm} = k_{vh} = k_{vw} = 0.4$$

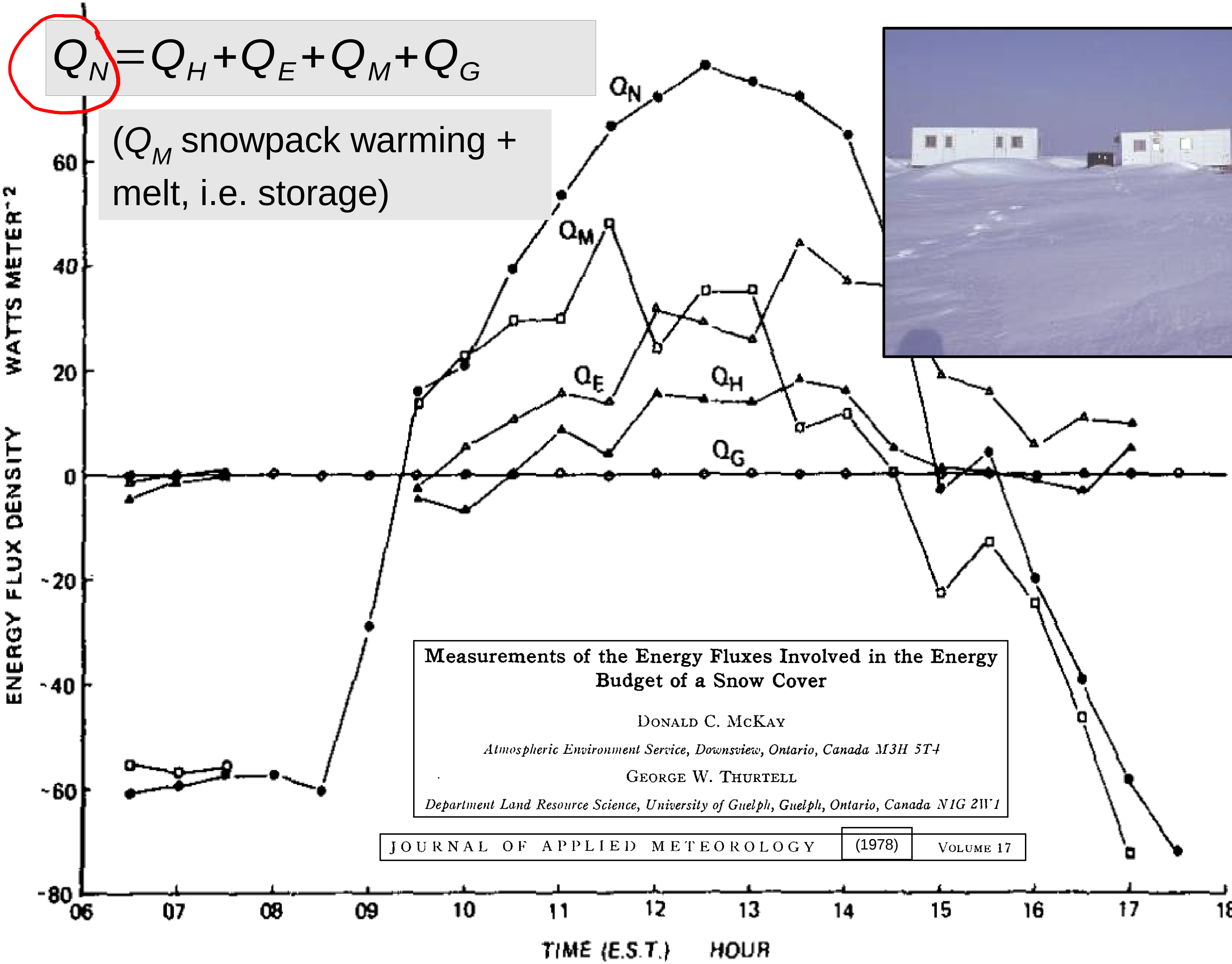
$$\left.\begin{aligned} \phi_m &= \left(1 - 28\,\frac{z}{L}\right)^{-\frac{1}{4}} \\ \phi_h = \phi_w &= \left(1 - 14\frac{z}{L}\right)^{-\frac{1}{2}} \end{aligned}\right\} \quad -4 \le z/L \le -0.004$$

Surface Energy Balance. Winter diurnal cycle measured at Elora, Ontario (Review)

Q^*

$$Q_N = Q_H + Q_E + Q_M + Q_G$$

(Q_M snowpack warming + melt, i.e. storage)



Measurements of the Energy Fluxes Involved in the Energy Budget of a Snow Cover

Donald C. McKay

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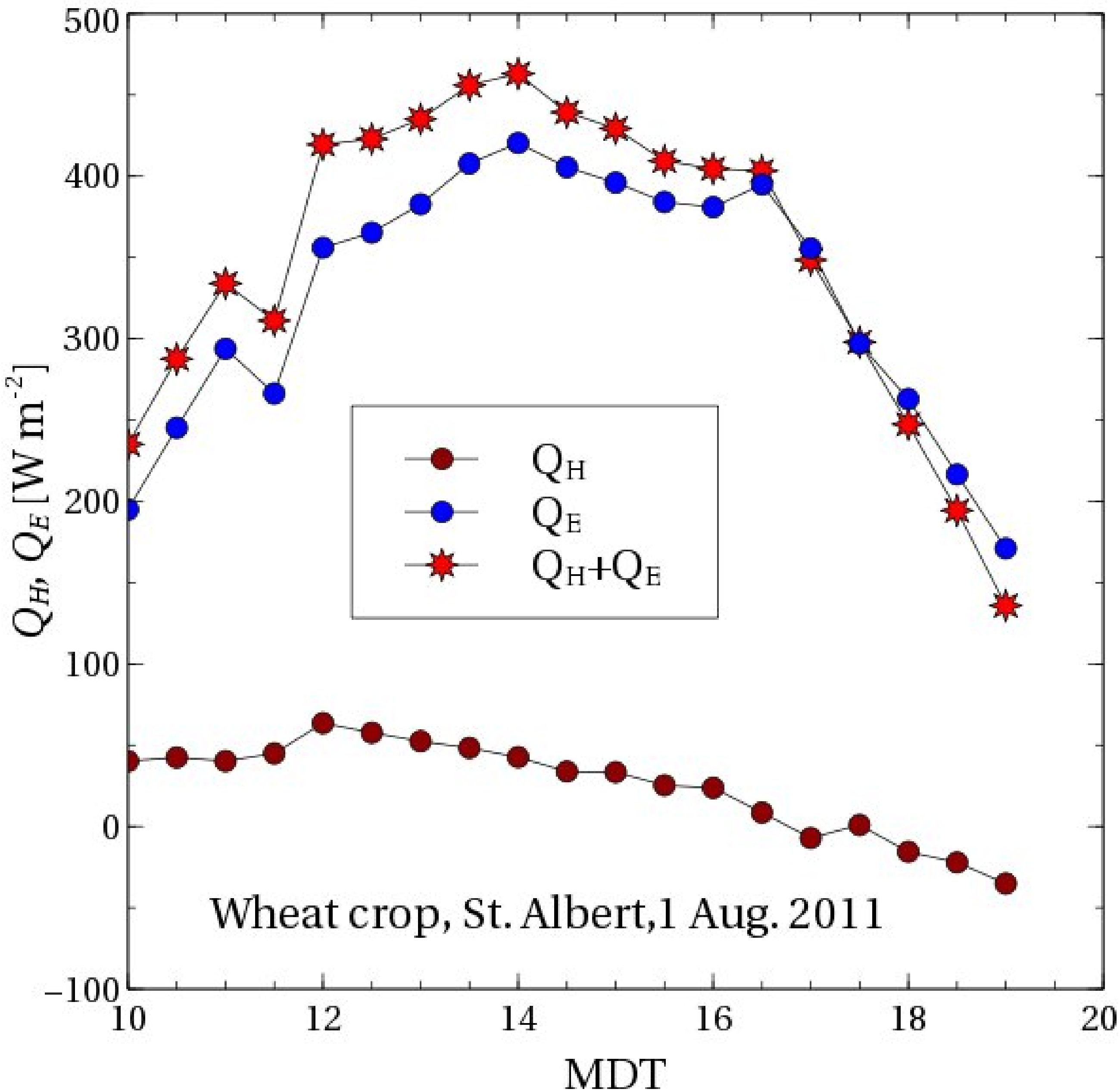
George W. Thurtell

Department Land Resource Science, University of Guelph, Guelph, Ontario, Canada N1G 2W1

JOURNAL OF APPLIED METEOROLOGY (1978) VOLUME 17

Sensible and latent heat flux densities (1 Aug. 2011, St. Albert)



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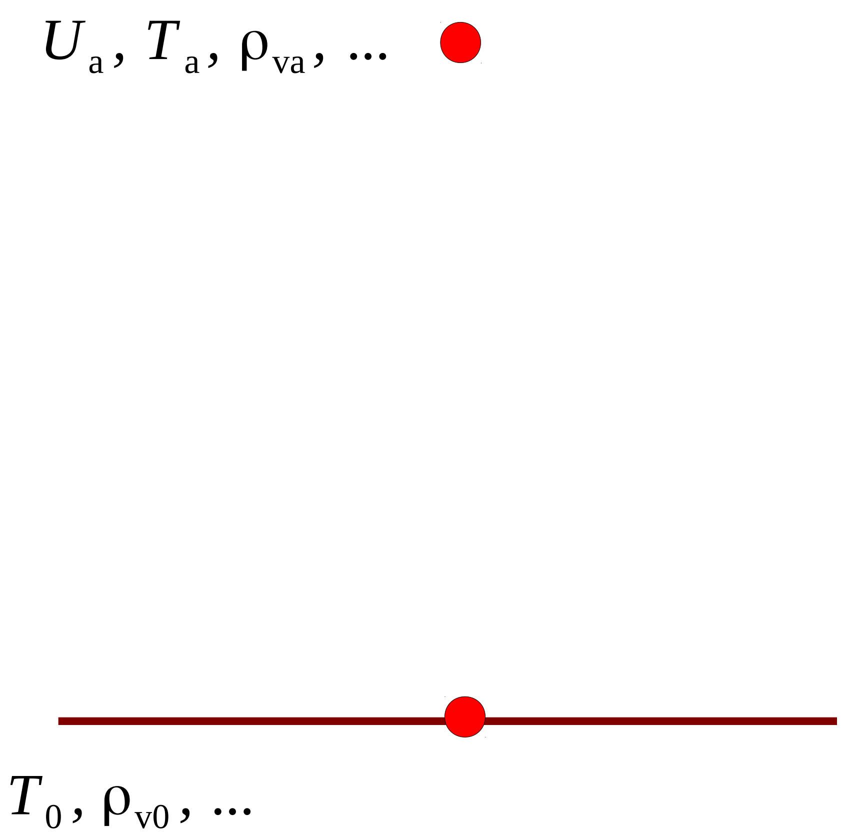
"Bulk transfer" formulae for surface-atmosphere exchange

- let "a" designate values at the lowest model level, the "anemometer height" (or in other contexts, a measurement level)
- let "0" designate surface values

$$\tau = \rho\, C_{\mathrm{M}}\, U_{\mathrm{a}}^2$$

$$Q_H = \rho\, c_p\, C_{\mathrm{H}}\, U_{\mathrm{a}} \left[T_0 - T_{\mathrm{a}} \right]$$

$$E = \rho\, C_{\mathrm{W}}\, U_{\mathrm{a}} \left[q_0 - q_{\mathrm{a}} \right]$$



The dimensionless constants (C's) are calibrated by requiring consistency with MO similarity theory**

** clearly $C_{\mathrm{M}} = u_*^2 / U_{\mathrm{a}}^2$

for a melting snow pack $q_0 = q_* \left(0^{\mathrm{o}}\, \mathrm{C} \right)$

What about the "disturbed" ASL, i.e. the general case?

- statistics (typically 30 min averaging interval) now depend on (*x,y,z*) instead of *z* alone – and vary slowly from one interval to the next
- Reynolds' momentum equations (seen earlier) govern the 3D field of mean velocities
- Reynolds averaging provides us governing equations for "higher order" statistics, like (e.g.) velocity variance, but these contain further unknowns (the "turbulence closure problem")
- surface layer models are able to compute spatial distribution of windspeed, temperature etc. over non-uniform surfaces (NWP on the microscale)
- this subject leads beyond the scope of EAS 372