Slope of dewpoint lapse rate lines

We would like to know how the dewpoint of an unsaturated parcel changes if it undergoes adiabatic vertical motion, i.e. $\partial T_d/\partial p$ or $\partial T_d/\partial \ln p$.

$e = e_*(T_d)$ v.p. and dewpoint are in 1:1 relationship

$$\frac{\partial T_d}{\partial \ln p} = \frac{\partial T_d}{\partial \ln e}\frac{\partial \ln e}{\partial \ln p}$$ unity

$$e = \rho_v R_v T$$

$$p - e = \rho_d R_d T$$

$$p = \rho R_d T_v$$

$$p = \rho_d + \rho_v$$

$$\frac{e}{p-e} = \frac{\rho_v}{\rho_d}\left(\frac{R_v}{R_d}\right)^{\varepsilon^{-1}} = r/\varepsilon$$

r = mixing ratio, constant under the envisaged motion

$$\frac{e/p}{1-e/p} = r/\varepsilon = c \text{ (const.)}$$

$$e/p = c - c\,e/p \quad \text{or} \quad \frac{e}{p}(1+c) = c$$

$$\text{or} \quad \frac{e}{p} = \frac{c}{1+c} = c_1 \quad (\text{const.})$$

$$de = c_1\, dp \quad \Rightarrow \quad \frac{de}{e} = c_1 \frac{dp}{e} = c_1 \frac{dp}{p}\frac{p}{e}$$

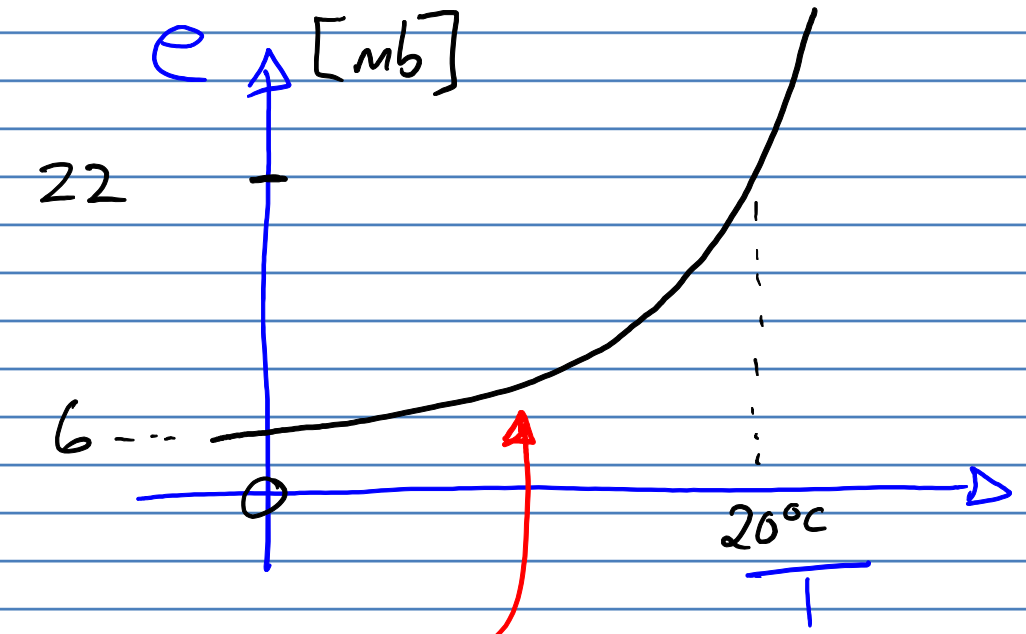
$1/c_1$

$$d\ln e = d\ln p$$

$$\frac{d\ln e}{d\ln p} = 1$$

Need $\dfrac{\partial T_d}{\partial \ln e}$

i.e. slope of the Clausius - Clapeyron curve



Djuric (1994) $\quad e(T_d) = 610.78 \exp \dfrac{22.5\,T_d}{273 + T_d}$ $\quad \left\{ \begin{array}{l} T_d \text{ in } ^\circ C \\ e \text{ in Pa} \end{array} \right.$

$$\ln e = \ln 610.78 + \ln \left[\exp \frac{22.5\,T_d}{273 + T_d} \right]$$

$$\ln e = \frac{22.5\,T_d}{273 + T_d}$$

$\ln ab = \ln a + \ln b$

$\ln e^a \equiv a$

$$\frac{\partial \ln e}{\partial T_d} = \frac{22.5}{273 + T_d} + 22.5\,T_d \frac{-1}{(273+T_d)^2}$$

$$= \frac{22.5\,(273+T_d) - 22.5\,T_d}{(273 + T_d)^2}$$

$$\frac{\partial T_d}{\partial \ln e} = \frac{(273 + T_d)^2}{273 \times 22.5} \equiv \frac{\partial T_d}{\partial \ln p}$$

slope of isohumes

e.g. if $T_d = -20^\circ C$ $\quad \dfrac{\partial T_d}{\partial \ln e} = 10.4\ K$