EAS372 Open Book Final Exam 11 April, 2013

Professor: J.D. Wilson Time available: 2 hours Value: 30%

***Please check the Terminology, Equations and Data section before beginning your responses. Answer all questions in the Examination Booklet.***

# A. Multi-choice (11 x 1% → 11 %)

1. Which of the following statements is **false** for a barotropic region of the atmosphere?
   (a) $p = p(\rho)$
   (b) $\nabla p$ is parallel to $\nabla \rho$, i.e. $\nabla p \cdot \nabla \rho = |\nabla p|\,|\nabla \rho|$
   (c) $\nabla p \times \nabla \rho = 0$
   (d) $\vec{V}_g \cdot \nabla T = 0$
   (e) $\nabla \theta \times \nabla T \neq 0$ ✓

2. According to the hypsometric equation (given as data), a 1 K increase in the mean temperature of the 1000-500 hPa layer results in that layer's thickness changing by
   (a) 0.5 dam
   (b) 1 dam
   (c) 2 dam ✓
   (d) -1 dam
   (e) -2 dam

3. Treating the temperature and pressure as $T = 258$ K and $p = 650$ hPa, calculate the static stability of the 700-600 hPa layer if $\theta_{700} = 288$ K and $\theta_{600} = 296$ K. The correct value (in MKS units) is:
   (a) $+2.4 \times 10^{-4}$
   (b) $-2.4 \times 10^{-4}$
   (c) $+3.1 \times 10^{-6}$ ✓
   (d) $-3.1 \times 10^{-6}$
   (e) $-3.1 \times 10^{-2}$

The vertical distribution of winds depicted in the figure implies

4. (a) warm advection ✓
   (b) cold advection
   (c) isotherms are perpendicular to $\mathbf{V_T}$
   (d) thickness contours are perpendicular to $\mathbf{V_T}$
   (e) horizontal divergence

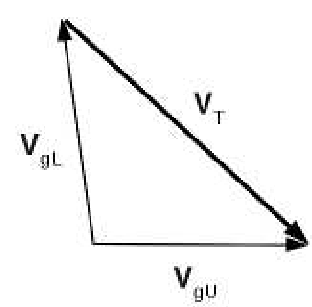


**Figure 1:** Wind vector at lower (L) and upper (U) levels.

**The next few questions pertain to the quasi-geostophic (QG) model**. The QG vorticity equation may be written in several equivalent forms, namely

$$\frac{d_g\eta}{dt} = f_0\,\frac{\partial\omega}{\partial p}\,, \tag{1}$$

$$\frac{\partial\eta}{\partial t} + \vec{V}_g\cdot\nabla_h\,\eta = f_0\,\frac{\partial\omega}{\partial p}\,, \tag{2}$$

$$\frac{\partial\zeta_g}{\partial t} + \vec{V}_g\cdot\nabla_h\,\eta = f_0\,\frac{\partial\omega}{\partial p}\,, \tag{3}$$

$$\frac{\partial\zeta_g}{\partial t} + \vec{V}_g\cdot\nabla_h\,\zeta_g + v_g\beta = f_0\,\frac{\partial\omega}{\partial p}\,, \tag{4}$$

$$\frac{d_g\zeta_g}{dt} + v_g\beta = f_0\,\frac{\partial\omega}{\partial p}\,, \tag{5}$$

where $d_g/dt$ is the derivative following the geostrophic wind (recall that $f$ does not vary in time). The QG omega equation is

$$\left(\nabla_h^2 + \frac{f_0^2}{\sigma}\,\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\,\frac{\partial}{\partial p}\left(\vec{V}_g\cdot\nabla_h\,\eta\right) + \frac{R}{\sigma\,p}\,\nabla_h^2\left(\vec{V}_g\cdot\nabla_h T\right)\,, \tag{6}$$

where $\vec{V}_g = (u_g, v_g)$ is the geostrophic wind, $\eta = \zeta_g + f$ is the absolute vorticity, $\zeta_g$ is the geostrophic relative vorticity, $\sigma$ is the static stability, $R$ the specific gas constant for air and (recall) $\omega = -\rho g w$ where $w$ is the true vertical velocity.

5. Which of the following restrictions or approximations regarding the quasi-geostrophic (QG) model (neglecting diabatic heating) is **false**?
   (a) frictionless, adiabatic, hydrostatic, extra-tropical motion
   (b) linear variation of Coriolis parameter with north-south coordinate $y$, viz. $f = f_0 + \beta y$
   (c) neglect vertical advection
   (d) neglect horizontal advection of and by the ageostrophic wind
   (e) divergence ($D = \nabla_h\cdot\vec{V} = -\partial\omega/\partial p$) evaluated using the geostrophic wind ✓

6. Suppose $\partial\omega/\partial p = 0$ (no stretching): which of the statements below is **false**?
   (a) following $\vec{V}_g$, absolute vorticity is conserved
   (b) following $\vec{V}_g$, relative vorticity is conserved *only* on the condition that the meridional component $v_g \neq 0$
   (c) neither PVA (positive vorticity advection) nor NVA can occur ✓
   (d) if the alongstream derivative in the natural coordinate system $\partial\eta/\partial s < 0$, then $\partial\eta/\partial t > 0$

7. If PVA (advection of positive absolute vorticity) is increasing with increasing height $z$, the first term on the right hand side of the $\omega$-eqn is
   (a) negative
   (b) zero
   (c) positive ✓

8. On a region of an isobaric surface where a local maximum of warm advection is occurring (i.e. $\vec{V}_g \cdot \nabla_h T$ negative with large magnitude relative to the surroundings), the second term on the right hand side of the $\omega$-eqn is
   (a) negative
   (b) zero
   (c) positive ✓

9. Qualitatively, the left hand side of the $\omega$-eqn can be interpreted as the 3D Laplacian of $\omega$, and accordingly it can be interpreted by replacing it with the expression
   (a) $\alpha\,\omega$, with proportionality constant $\alpha > 0$
   (b) $\alpha\,\omega$, with proportionality constant $\alpha < 0$ ✓

10. Suppose $(q, \omega)$ are respectively the specific humidity [kg/kg] and the vertical velocity [Pa/s] at the 700 hPa level (where pressure $p = 7 \times 10^4$ Pa). Which formula for the resolved vertical flux $E$ of water vapour [$\text{kg m}^{-2}\,\text{s}^{-1}$] is correct?
   (a) $E = -\omega\, q$
   (b) $E = -\omega\, q/g$ ✓
   (c) $E = \omega\, q$
   (d) $E = \omega\, q/g$

11. Referring to Figure (3), what label [hPa] would correctly identify the isobar at the north-east corner of the map?
   (a) 1020
   (b) 1022
   (c) 1024 ✓
   (d) 1028
   (e) 1032

# B. "Live" web weather data (5 x 1 → 5%)

1. Give the sign and magnitude [$\text{s}^{-1}$] of the strongest feature in the absolute vorticity field of the GEM-regional 0h prog that was initialized at 00Z today. [**Over Alaska,** $+26\times10^{-5}\ \text{s}^{-1}$ **or** $+28 \times 10^{-5}\ \text{s}^{-1}$]

2. What was the temperature at Coronation, Alberta at 06 MDT today? [**-1°C**]

3. To the nearest metre, what was the 850-700 hPa thickness at Churchill, Manitoba (YYQ) at 00Z today? [**1493 m**]

4. Retrieve and write down the METAR for Toronto Pearson (CYYZ) for 06Z today. [**CYYZ 110600Z 01007KT 15SM OVC100 04/02 A3004 RMK AC8 SLP177=**]

5. What was the potential temperature of the air at the 500 hPa level over Edmonton as of 00Z today. [**304.8 K, taken directly from the sounding data; or about** 36°**C, by following a dry adiabat to the surface on the skew-T chart**]

# C. Short answer (2 x 3 % → 6 %)

Please answer **two** of the following questions.

1. Neglecting some small terms, the instantaneous horizontal momentum equation may be written in Cartesian coordinates as

$$\left(\frac{\partial}{\partial t} + \vec{U}\cdot\nabla\right)\vec{V} = \frac{-1}{\rho}\nabla_h P - \ f\,\hat{k}\times\vec{V} \tag{7}$$

   where $\hat{k}$ is the unit vector along the vertical. Evaluate the cross product and give the implied equations for the two horizontal components $(u, v)$.

   We know that upon Reynolds averaging, extra terms arise, and that they represent momentum transport by unresolved scales of motion, i.e. "friction." Which term in Eq. 7 gives rise to the friction terms?

2. Provide an overview of the dynamics and physics of Canada's NWP model "GEM"

3. In an unsaturated and horizontally-homogeneous atmospheric boundary layer the Reynolds-averaged thermodynamic equation simplifies to

$$\frac{\partial \overline{T}}{\partial t} = -\ \frac{\partial\,\overline{w'T'}}{\partial z} \tag{8}$$

   where $\overline{w'T'}$ is the (kinematic) sensible heat flux density, and any contribution due to radiative flux divergence has been neglected. Compute the rate of warming [K hr$^{-1}$] assuming the ABL is 1000 m deep and the surface heat flux density $\left(\overline{w'T'}\right)_0 = 0.2\,\mathrm{K\,m\,s^{-1}}$.

4. In the isobaric coordinate system the continuity equation is

$$\nabla_h\cdot\vec{V} + \frac{\partial\omega}{\partial p} = 0\ . \tag{9}$$

   With reference to this equation, explain the statement that "the vertical velocity takes on a local maximum or minimum value at a level of non-divergence."

# D. Interpretation of weather situation. (1 x 8→ 8%)

On 7 March 2007 Edmonton experienced rapid warming (Figure 2). Give your own interpretation of the weather situation over Western Canada around that time, as conveyed by Figures (3-7).

**If an instructor was going to give a mark for each true statement one could make in regard to these charts, infinity would have been a common score. The challenge was to recognize a *pattern* underlying all the details, and stitch together a connected (coherent) depiction of that pattern.**

- recognize the lee trough in Alberta, visible esp. at sfc. & 850 hPa, and also (as kink in height contours near Rockies) at 700 hPa
- identify as ultimate cause of this the upper trough offshore, resulting in strong SW upper flow
- this flow resulted in thermal ridge in Alberta and western Sask., as evidenced most dramatically at 850 hPa level. Front in Sask.
- warm, dry air, descending off the Rockies, has pushed over Alberta towards the NE, witness the dry slot in the GOES IR image, and the sounding winds
- sounding shows that the cold surface air in Edmonton is shallow — strong inversion, much warmer aloft

## Terminology, Equations and Data.

- Nomenclature: $\vec{U}$ is a 3D velocity, $\vec{V}$ is the "horizontal" velocity (strictly, the components lying in a constant pressure surface), and $V_g = (u_g, v_g)$ is the geostrophic velocity. Actual and potential temperature, $T$ and $\theta$. Air density and pressure $\rho$, $p$.
- Unless otherwise stated, $\nabla$ is to be interpreted as $\nabla_h$ (sometimes also designated $\nabla_p$), the gradient on a constant pressure surface.
- $A_f = -\, v \, \frac{\partial f}{\partial s}$

  Rate of horizontal advection of the property $f$, expressed in natural coordinates. The unit vector $\hat{s}$ for the $s$-axis points downstream and parallel to the flow contours (eg. height contours), and $v$ is the wind *speed*.
- $\Delta Z = Z_2 - Z_1 = \frac{R}{g} \, \overline{T} \, \ln \frac{p_1}{p_2}$

  The hypsometric equation (where $Z_2 > Z_1$). The left hand side is the ($Z_2$ to $Z_1$ hPa) thickness expressed in [dam], and $\overline{T}$ is the weighted mean temperature of the layer (weighting factor is $p^{-1}$). $R = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$ is the specific gas constant for dry air, and $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational acceleration.
- $\sigma = \frac{-R\,T}{p} \, \frac{\partial \ln \theta}{\partial p}$

  The static stability parameter [$\mathrm{Pa^{-2}\,s^{-2}}$] where $(p, T)$ are the pressure and temperature; $R$ is the specific gas constant; and $\theta$ is the potential temperature

Hourly Data Report for March 7, 2007

| Time | Temp °C | Dew Point Temp °C | Rel Hum % | Wind Dir 10s deg | Wind Spd km/h | Visibility km | Stn Press kPa | Hmdx | Wind Chill | Weather |
|---|---|---|---|---|---|---|---|---|---|---|
| 00:00 | -8.5 | -10.2 | 87 | 14 | 19 | 24.1 | 92.77 | | -16 | Mostly Cloudy |
| 01:00 | -8.1 | -9.9 | 87 | 15 | 17 | 24.1 | 92.65 | | -15 | Mostly Cloudy |
| 02:00 | -7.7 | -9.4 | 88 | 16 | 15 | 24.1 | 92.59 | | -14 | Mostly Cloudy |
| 03:00 | -8.4 | -10.0 | 88 | 14 | 11 | 24.1 | 92.53 | | -14 | Mostly Cloudy |
| 04:00 | -9.1 | -10.7 | 88 | 17 | 9 | 24.1 | 92.47 | | -14 | Mostly Cloudy |
| 05:00 | -9.6 | -10.9 | 90 | 20 | 13 | 19.3 | 92.38 | | -16 | Cloudy |
| 06:00 | -9.8 | -11.1 | 90 | 17 | 13 | 19.3 | 92.32 | | -16 | Cloudy |
| 07:00 | -8.6 | -10.4 | 87 | 17 | 11 | 16.1 | 92.19 | | -14 | Mostly Cloudy |
| 08:00 | -7.2 | -9.0 | 87 | 17 | 9 | 19.3 | 92.10 | | -12 | Mostly Cloudy |
| 09:00 | -4.9 | -8.0 | 79 | 16 | 7 | 24.1 | 92.05 | | -8 | Mostly Cloudy |
| 10:00 | -3.7 | -6.3 | 82 | 15 | 9 | 24.1 | 92.01 | | -7 | Mostly Cloudy |
| 11:00 | 0.0 | -4.3 | 73 | 14 | 11 | 24.1 | 91.93 | | -4 | Mainly Clear |
| 12:00 | 2.2 | -2.0 | 74 | 15 | 13 | 24.1 | 91.87 | | | Mainly Clear |
| 13:00 | 3.5 | -1.5 | 70 | 17 | 19 | 24.1 | 91.78 | | | Mainly Clear |
| 14:00 | 4.3 | -0.3 | 72 | 16 | 11 | 24.1 | 91.69 | | | Mainly Clear |
| 15:00 | 4.7 | -0.8 | 67 | 15 | 7 | 24.1 | 91.64 | | | Mainly Clear |
| 16:00 | 4.5 | -0.7 | 69 | 16 | 7 | 24.1 | 91.60 | | | Mainly Clear |
| 17:00 | 3.0 | 0.3 | 82 | 16 | 6 | 24.1 | 91.56 | | | Mainly Clear |
| 18:00 | 3.7 | -0.7 | 73 | 12 | 6 | 24.1 | 91.55 | | | Mostly Cloudy |
| 19:00 | 1.9 | -0.3 | 85 | 15 | 11 | 24.1 | 91.51 | | | Mostly Cloudy |
| 20:00 | 2.7 | -0.5 | 79 | 18 | 11 | 24.1 | 91.44 | | | Mostly Cloudy |
| 21:00 | 2.2 | -0.5 | 82 | 16 | 6 | 24.1 | 91.38 | | | Mostly Cloudy |
| 22:00 | 1.4 | -1.1 | 83 | | 0 | 24.1 | 91.38 | | | Mostly Cloudy |
| 23:00 | 1.0 | -1.7 | 82 | 18 | 11 | 24.1 | 91.34 | | | Mostly Cloudy |

Figure 2: Hourly observations at Edmonton International Airport, 7 March 2007.

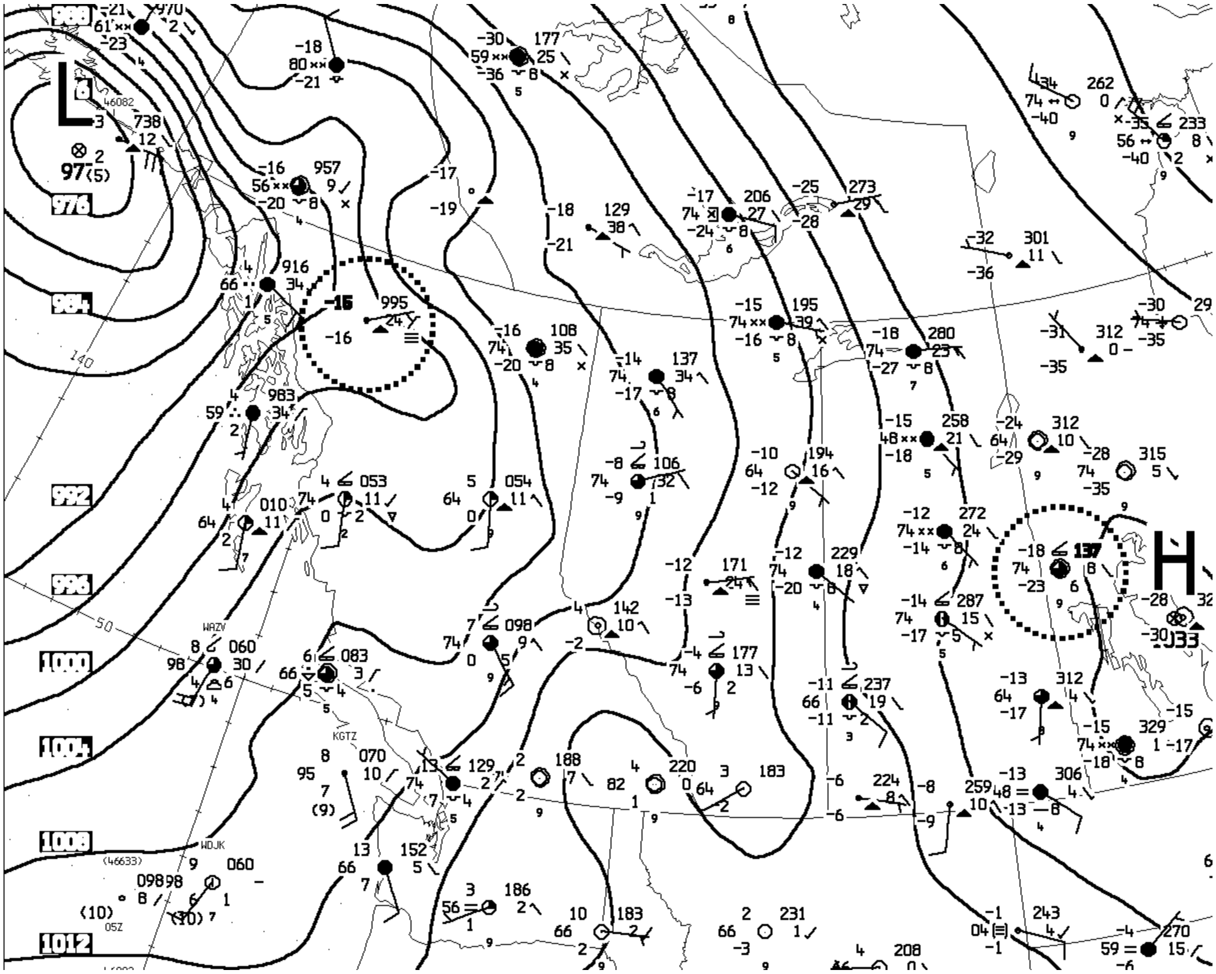


Figure 3: CMC surface analysis, 06Z, 7 March 2007.

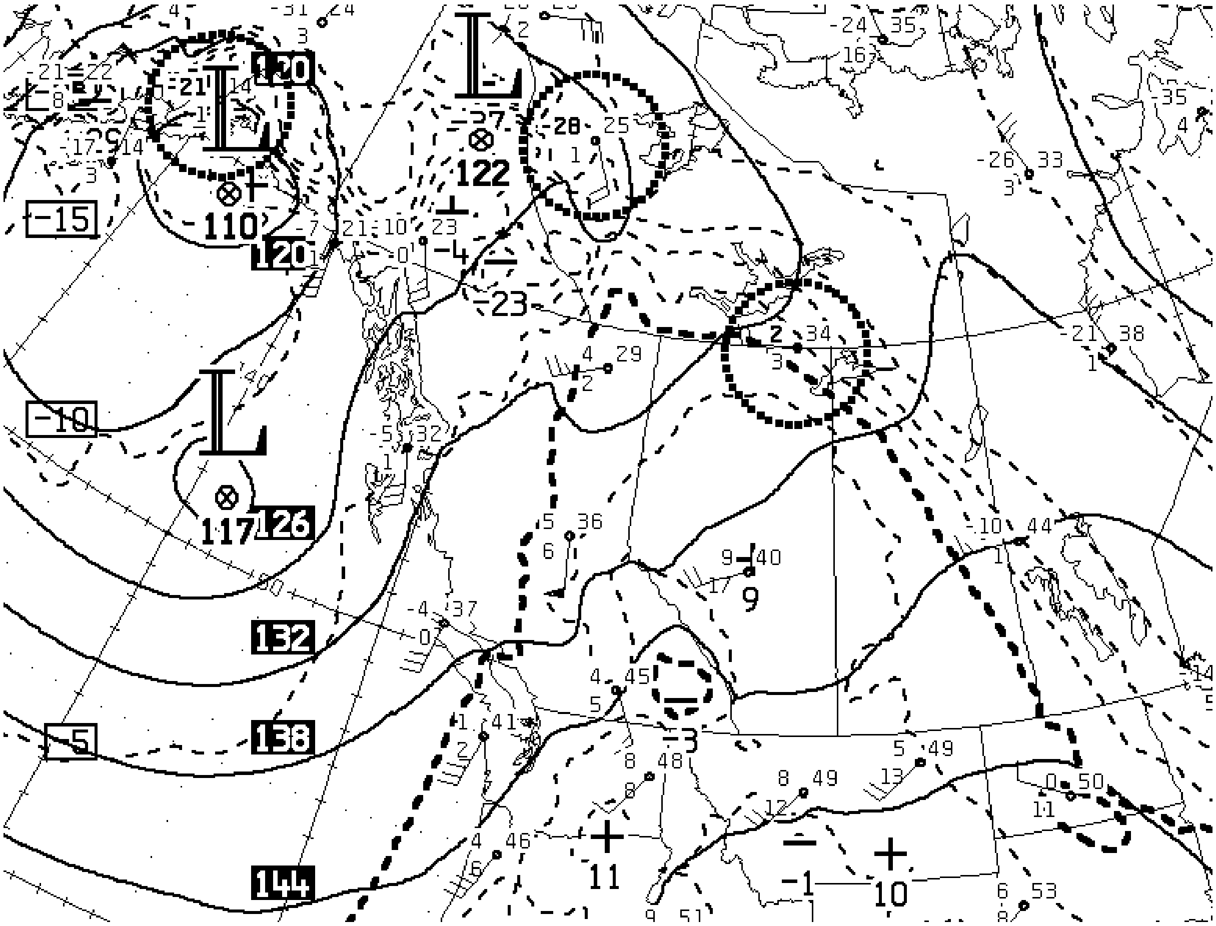


Figure 4: CMC 850 mb analysis, 12Z, 7 March 2007.

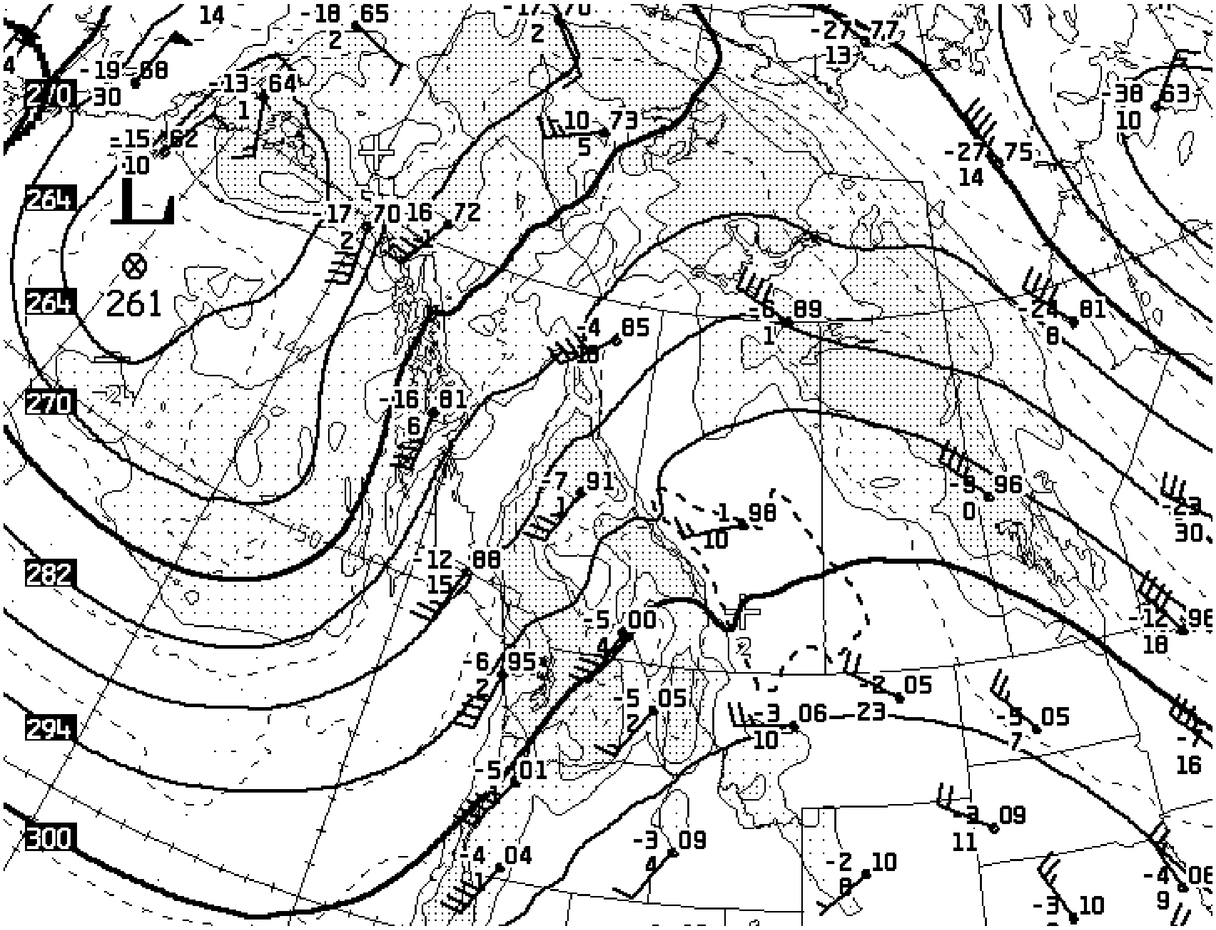


Figure 5: CMC 700 mb analysis, 12Z, 7 March 2007. Heavy (light) stippling, $T - T_d \leq 2^{\circ}$C ($5^{\circ}$C).

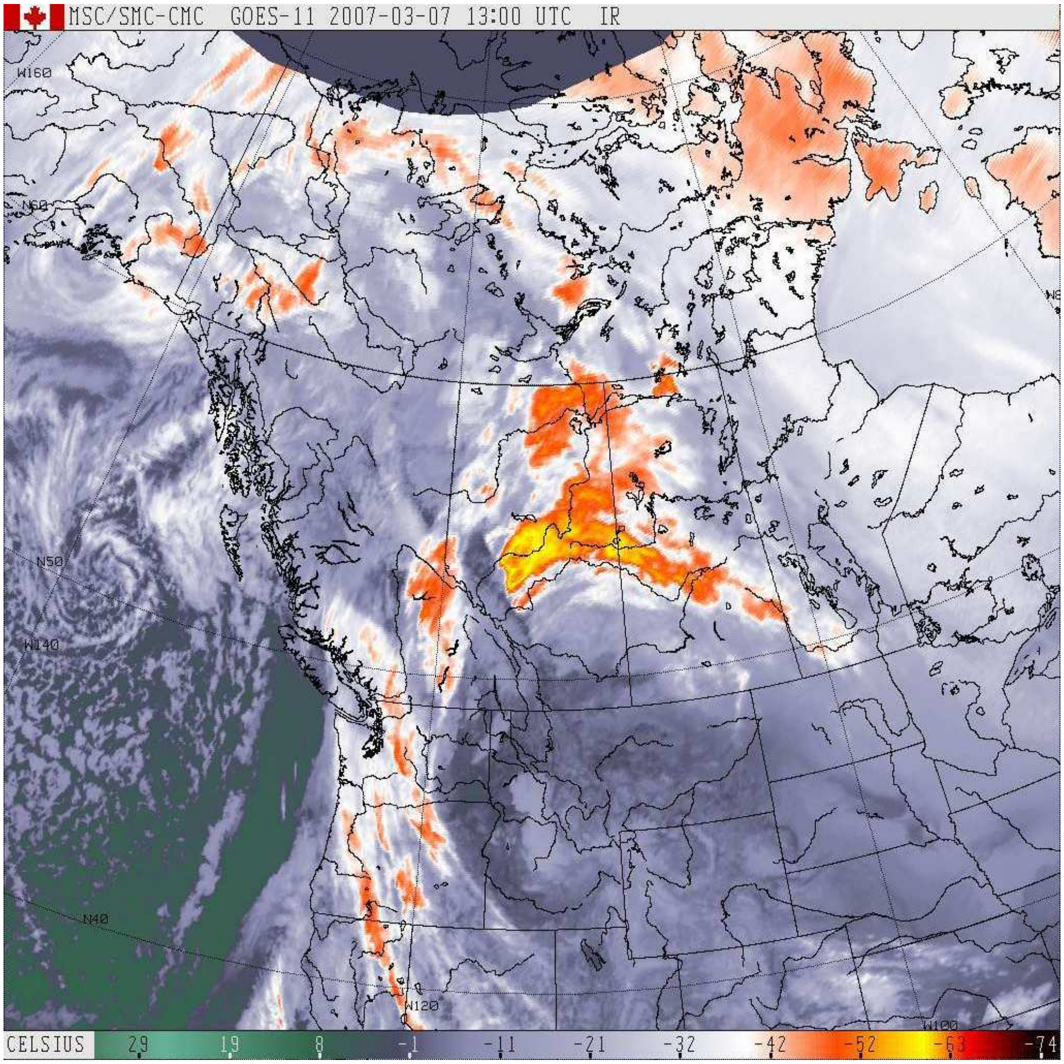


Figure 6: GOES west IR image, 12Z, 7 March 2007.

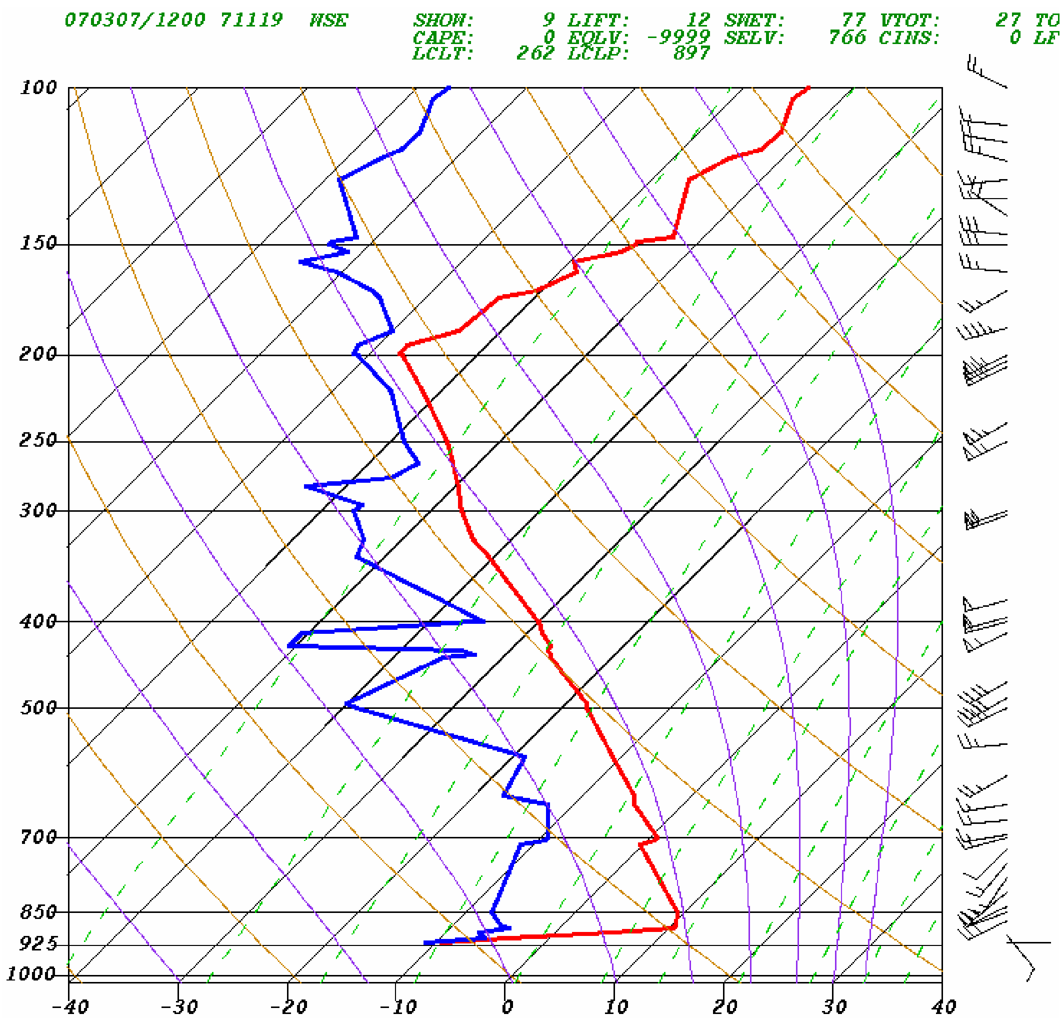


Figure 7: Edmonton (Stony Plain) sounding, 12Z, 7 March 2007.