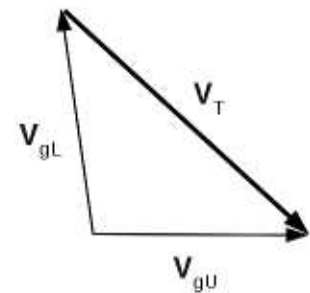


**A. Multi-choice (16 x 1/2% → 8 %)**

1. If  $D/Dt$  represents the Lagrangian derivative,  $\mathbf{u}$  is the 3D velocity vector and  $\phi$  is a conserved variable, then
  - (a)  $D\phi/Dt = \partial\phi/\partial t$
  - (b)  $\partial\phi/\partial t = -\mathbf{u} \cdot \nabla\phi$
  - (c)  $\partial\phi/\partial t = \mathbf{u} \cdot \nabla\phi$
  - (d)  $\partial\phi/\partial t = \partial\phi/\partial x = \partial\phi/\partial y = \partial\phi/\partial z = 0$
  - (e)  $\mathbf{u} \cdot \nabla\phi = 0$
  
2. If vectors  $\mathbf{P}$ ,  $\mathbf{Q}$  respectively have components  $(0, p, \alpha)$  and  $(\beta, q, 0)$  relative to Cartesian coordinates with unit vectors  $(\hat{i}, \hat{j}, \hat{k})$ , then the quantity  $\mathbf{P} \cdot \mathbf{Q}$  (ie. 'dot product' of the two vectors) is \_\_\_\_
  - (a) 0
  - (b)  $(0, pq, 0)$
  - (c)  $pq$
  - (d)  $(\beta, p + q, \alpha)$
  - (e)  $p + q + \alpha + \beta$
  
3. A vertical distribution of winds in the N. hemisphere free troposphere as depicted in the figure implies
  - (a) warm advection
  - (b) cold advection
  - (c) isotherms are perpendicular to  $\mathbf{V}_T$
  - (d) thickness contours are perpendicular to  $\mathbf{V}_T$
  - (e) horizontal divergence



**Figure 1:** Wind vector at lower (L) and upper (U) levels.

4. Which option is the **false** restatement of the continuity equation

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

( $\nabla$  here being the 3-D grad operator,  $\mathbf{u}$  the 3-D velocity, and  $D/Dt$  the Lagrangian derivative):

(a)  $\partial \rho / \partial t = -\rho \nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \rho$

(b)  $\partial \rho / \partial t + \mathbf{u} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{u}$

(c)  $D\rho/Dt = -\rho \nabla \cdot \mathbf{u}$

(d)  $\nabla \cdot \mathbf{u} = 0$

(e)  $D \ln \rho / Dt = -\nabla \cdot \mathbf{u}$

5. Which statement in regard to dry adiabats on a thermodynamic chart is **untrue** or **illogical**?

(a) every point along a dry adiabat has the same potential temperature

(b) if a parcel's ascent is such that the  $T(p)$  curve it traces on the chart is parallel to a dry adiabat, the environment has constant potential temperature

(c) if a parcel's ascent is such that the  $T(p)$  curve it traces on the chart is parallel to a dry adiabat, the parcel is cooling at a rate of  $g/c_p$  degrees Celcius per metre of ascent

(d) if a parcel's ascent is such that the  $T(p)$  curve it traces on the chart is parallel to a dry adiabat, the atmosphere may be said to be neutrally stratified (and, well mixed) with respect to unsaturated adiabatic motion

(e) in an unsaturated atmosphere dry adiabats coincide with moist adiabats

6. What is the spatial resolution of the GOES infra-red image at the equator (instantaneous geographic field of view at nadir)?

(a) 40m

(b) 400m

(c) 4km

(d) 40km

(e) 400km

7. The continuity equation in the  $x, y, p$  (“isobaric”) coordinate system reads

$$\nabla \cdot \mathbf{V} + \frac{\partial \omega}{\partial p} = 0$$

where  $\mathbf{V}$  represents the projection of the wind vector onto the isobaric surface (loosely, the “horizontal” wind),  $\nabla$  is the 2-D grad operator in the isobaric surface, and  $\omega$  [ $\text{Pa s}^{-1}$ ] is the vertical velocity. Wherever  $\nabla \cdot \mathbf{V} = 0$ ,

- (a)  $\omega = 0$
- (b)  $\omega > 0$
- (c)  $\omega < 0$
- (d) the  $\omega$  versus  $p$  profile exhibits a local maximum or a local minimum
- (e)  $\partial \omega / \partial t = 0$

8. The quasi-geostrophic (QG) vorticity equation can be written

$$\frac{\partial \eta}{\partial t} + \mathbf{V}_g \cdot \nabla \eta = -f_0 \nabla \cdot \mathbf{V}_{ag}$$

where  $\eta \equiv f_0 + \hat{k} \cdot (\nabla \times \mathbf{V}_g)$  is the vertical component of the absolute geostrophic vorticity,  $f_0$  is the Coriolis parameter at the reference latitude,  $\mathbf{V}_g$  is the geostrophic wind and  $\mathbf{V}_{ag}$  is the ageostrophic wind. Wherever  $D_g \eta / Dt = 0$  (with  $D_g / Dt$  representing the Lagrangian derivative following the geostrophic wind)

- (a)  $\nabla \cdot \mathbf{V}_{ag} > 0$
- (b)  $\nabla \cdot \mathbf{V}_{ag} = 0$
- (c)  $\nabla \cdot \mathbf{V}_{ag} < 0$
- (d)  $\mathbf{V}_{ag} = 0$
- (e)  $\mathbf{V}_g = 0$

9. The quasi-geostrophic (QG) vorticity equation can also be written

$$\frac{\partial \eta}{\partial t} + \mathbf{V}_g \cdot \nabla \eta = f_0 \frac{\partial \omega}{\partial p}.$$

On an isobaric surface that coincides with a level of non-divergence (LND),

- (a)  $\eta$  is constant along streamlines of the geostrophic wind
- (b)  $\nabla \eta = 0$
- (c)  $\mathbf{V}_g \cdot \nabla \eta = 0$
- (d)  $\partial \eta / \partial t = 0$
- (e)  $\omega = 0$

10. The  $\mathbf{Q}$ -vector formulation of the quasi-geostrophic omega ( $\omega$ ,  $\text{Pa s}^{-1}$ ) equation reads

$$\frac{\sigma}{2} \left( \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega = -\nabla \cdot \mathbf{Q}$$

where  $\nabla$  is the grad operator in a constant pressure surface,  $\sigma = -(R_d T/p) \partial \ln \theta / \partial p$  is the static stability,  $f_0$  is the Coriolis parameter evaluated at the reference latitude, and for the purposes of the question the LHS is to be interpreted as being proportional to the Laplacian of  $\omega$ . If, at a point C, the value  $\omega_C$  of  $\omega$  is a local maximum, the  $\mathbf{Q}$ -vectors at C

- (a) have a local maximum in magnitude
  - (b) have a local minimum in magnitude
  - (c) converge
  - (d) diverge
  - (e) are perpendicular to the isobaric surface through C
11. Referring to the quasi-geostrophic omega equation as given above, now consider a neutrally-stratified layer. The distribution of the  $\mathbf{Q}$ -vectors controls
- (a) the 2-D horizontal curvature ( $\nabla^2$ ) of the  $\omega$  field
  - (b) the 2-D horizontal curvature of the height field
  - (c) the 1-D vertical curvature ( $\partial^2/\partial p^2$ ) of the  $\omega$  field
  - (d) the 1-D vertical curvature of the height field

12. The quasi-geostrophic height tendency equation can be written

$$g \left[ \nabla^2 + \frac{\partial}{\partial p} \left( \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \right) \right] \frac{\partial Z}{\partial t} = -f_0 \mathbf{V}_g \cdot \nabla \eta - \frac{f_0^2 R_d}{\sigma p} \frac{\partial}{\partial p} [-\mathbf{V}_g \cdot \nabla T]$$

where  $\nabla$ ,  $\sigma$ ,  $f_0$  are defined above,  $\eta$  is the vertical component of the geostrophic absolute vorticity,  $\mathbf{V}_g$  is the geostrophic wind,  $Z$  is the isobaric height, and (again) the LHS is to be interpreted as a Laplacian (in this case, of  $g \partial Z / \partial t$ ). Consider the exit region of an upper trough and suppose that the second term on the right hand side (involving thermal advection) were zero, such that the first term acting alone were to control  $\partial Z / \partial t$ . Which statement is **false**?

- (a) under the stated assumptions, PVA results in a local maximum in  $\partial Z / \partial t$
- (b) the term ‘‘PVA’’ (positive vorticity advection) corresponds to  $-\mathbf{V}_g \cdot \nabla \eta > 0$
- (c) in an upper trough exit region PVA typically occurs
- (d) the  $Z$  field determines the geostrophic component of the total wind  $\mathbf{V}_g + \mathbf{V}_{ag}$

13. A simplified form of the conservation equation for the vertical eddy heat flux density  $\overline{w'\theta'}$  in a stationary, horizontally-homogeneous atmosphere reads

$$\frac{\partial \overline{w'\theta'}}{\partial t} = 0 = -\overline{w'^2} \frac{\partial \bar{\theta}}{\partial z} + \frac{\partial}{\partial z} \left( -\overline{w'^2} \tau \frac{\partial \overline{w'\theta'}}{\partial z} \right) - \frac{\overline{w'\theta'}}{\tau} + \frac{g}{\theta_0} \overline{\theta'^2},$$

where  $\tau$  is a time scale and  $\overline{w'^2} \tau$  is effectively an eddy diffusivity for heat. There are three types of term in any of the “transport equations” of fluid mechanics: storage, transport, and source-sink (also known as volumetric production/destruction) terms. In the above, which term(s) are source-sink terms?

- (a) the left hand side (LHS), i.e.  $\partial \overline{w'\theta'} / \partial t$
  - (b) the first, third, and final terms on the RHS
  - (c) the second term on the RHS
  - (d) the final term on the RHS
  - (e) there are no source-sink terms in this heat flux budget equation
14. Referring to the heat flux budget given above, which term(s) vanish in a neutrally-stratified layer? (You may assume the layer is perfectly well mixed, and that parcel motion is adiabatic).
- (a) all terms remain non-zero, with the exception of the left hand side (LHS)
  - (b) first, third, and final terms on the RHS
  - (c) second term on the RHS
  - (d) final term on the RHS
  - (e) all terms
15. Again referring to the heat flux budget given above, which pair of terms on the RHS suggests that the co-existence of unresolved (fluctuating) vertical motion and thermal stratification tends to result in an unresolved heat flux, symbolically

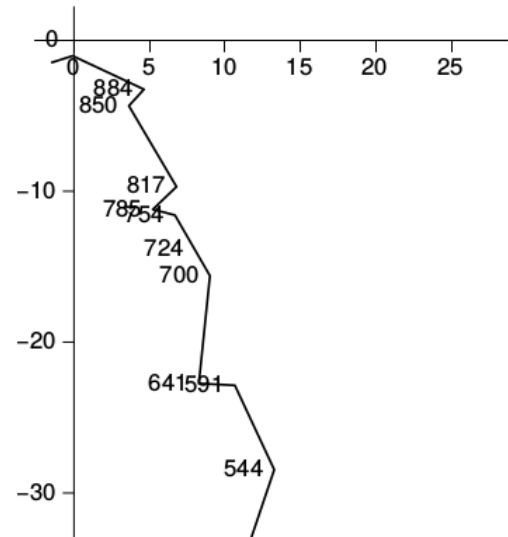
$$\overline{w'\theta'} \propto -\frac{\partial \bar{\theta}}{\partial z},$$

that is directed down the resolved potential temperature gradient?

- (a) first two term on the RHS
- (b) last two terms on the RHS
- (c) second and third terms on the RHS
- (d) first and last terms on the RHS
- (e) first and third terms on the RHS

16. Referring to Figure (2), the thermal wind for the 700 hPa to 544 hPa layer is a

- (a) NNW
- (b) SSE
- (c) NNE
- (d) E
- (e) W



**Figure 2:** Stony Plain (WSE) hodograph, 12Z Feb. 17, 2015.

### B. Short answer (4 x 3 % → 12 %)

Please answer any **four** of the following questions. Give sufficient detail that the basis for your answers can be understood.

- B1. Consider an unsaturated region of the atmosphere where the wind is a uniform westerly with  $u = 10 \text{ m s}^{-1}$ , i.e. constant in space and time, and where temperature is changing in time only due to horizontal advection. If the temperature trend is  $0.1 \text{ K hr}^{-1}$ , what is the magnitude of the horizontal temperature gradient? Give an equation (for  $\partial\bar{\theta}/\partial t$ ) illustrating the basis for your method.
- B2. Again consider an unsaturated region of the atmosphere, this time without any horizontal temperature gradient, and where temperature is changing in time only due to eddy heat flux convergence. If, again, the temperature trend is  $0.1 \text{ K hr}^{-1}$ , what is the magnitude of the eddy heat flux convergence? Again, give an equation illustrating the basis for your method.
- B3. If the components (west → east, south → north) of the Geostrophic wind at levels  $z_1$  and  $z_2$  ( $> z_1$ ) are respectively  $\vec{V}_1 = (3, 3) \text{ m s}^{-1}$  (i.e.  $\vec{V}_1$  is a southwester) and  $\vec{V}_2 = (3, -3) \text{ m s}^{-1}$  (a northwester), then what are the components of the thermal wind vector  $\vec{V}_T = \vec{V}_2 - \vec{V}_1$  and what is the orientation of  $\vec{V}_T$ . Describe the orientation of thickness isolines and the orientation of the thickness gradient. Give a diagram clarifying your answer.

B4. Suppose rain were falling steadily at rate  $P = 2 \text{ mm hr}^{-1}$  over a wide region such that as a first approximation the situation could be treated as uniform both in the horizontal and in time. If, furthermore, all rain mass were being created in a 1 km deep layer aloft (whose mean density  $\rho = 0.85 \text{ kg m}^{-3}$ ) and evaporation below that layer were negligible, what is the temperature trend of the rain producing layer (assume no heating or cooling processes are at work, other than condensation of water vapour within the source layer).

B5. In the context of the quasi-geostrophic paradigm the geostrophic wind vector is

$$\mathbf{V}_g = \frac{g}{f_0} \hat{k} \times \nabla Z$$

where  $Z$  is the height of an isobaric surface,  $\nabla$  is the 2-D grad operator and  $f_0$  is the Coriolis parameter at the reference latitude. Carry out the needed manipulation to obtain an expression for the geostrophic relative vorticity  $\zeta_g = \hat{k} \cdot (\nabla \times \mathbf{V}_g)$  that involves the Laplacian  $\nabla^2 Z$  of the height field.

B6. Suppose the temperature profile above height  $z = 500 \text{ m}$  did not change between 00 UTC and 12 UTC, while in the layer from ground ( $z = 0$ ) to  $z = z_T = 500 \text{ m}$  the temperature profiles (in  $^\circ\text{C}$ ) at 00Z and 12Z were

$$\begin{aligned} T_{00}(z) &= 0, \\ T_{12}(z) &= 5 (1 - z/z_T) . \end{aligned}$$

Treating the density as a constant  $\rho = 1 \text{ kg m}^{-3}$ , and assuming all heat loss from the layer below 500 m was by convective transfer to the surface, deduce the mean magnitude of the surface sensible heat flux density  $Q_H [\text{W m}^{-2}]$  over the 12 h interval. (Hint: on a  $T$  versus  $z$  plot the given temperature profiles form a right-angle triangle.)

### C. Weather chart interpretation. (→ 10%)

Figures (3–6) document conditions as of 12 UTC (i.e. 07 EST, Eastern Standard Time) on Thursday 16 April 2015 in a region of central Canada centred around western Ontario and the radiosonde station (CYPL) at Pickle Lake, identified on the surface analysis by a dotted ring (added by JDW). The 12Z metar from Pickle Lake was:

```
CYPL 161200Z 24012G20KT 15SM FEW100 SCT240 10/M04 A2986 RMK AC1CI2  
SLP119
```

- C1. On the surface analysis (Fig. 3, upper panel) draw in any front(s) for which you find evidence (2 marks)
- C2. On the 850 hPa chart (Fig. 3, lower panel) shade in the zones of strongest cold and warm advection (1 mark)
- C3. In what direction is the surface low moving? What evidence determined your answer? (1 mark)
- C4. Can you identify a mechanism for the lift that has resulted in the high cloud tops seen on the IR image (Fig. 5)? (1 mark)
- C5. North-westward from Pickle lake, the 0h prog (Fig. 6b) shows a vort max. What is the associated aspect of the 500 hPa *velocity* field? (1 mark)
- C6. Comment on the consistency (or otherwise) of Figs.(6a,c) with the forecast of a band of *rain* (Fig. 6e) near Pickle Lake. (1 mark)
- C7. Can you identify a possible lifting mechanism for the forecast rain band near Pickle Lake? (1 mark)
- C8. Would you consider upper support for this surface low to be non-existent, weak or strong? (1/2 mark)
- C9. Give the temperature, dewpoint and MSLP identifying a surface station where reported conditions seem compatible with the paradigm of a cold conveyor belt? Note: at this time, Hudson Bay was ice-covered. (1/2 mark)
- C10. Is there any aspect of this weather situation, not hinted at above, that attracted your attention and seems worthy of being noted? (1 mark)



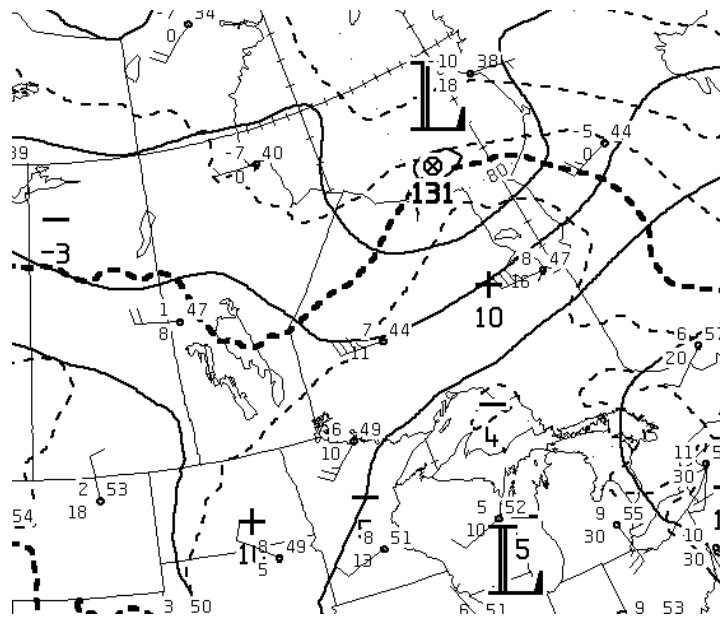
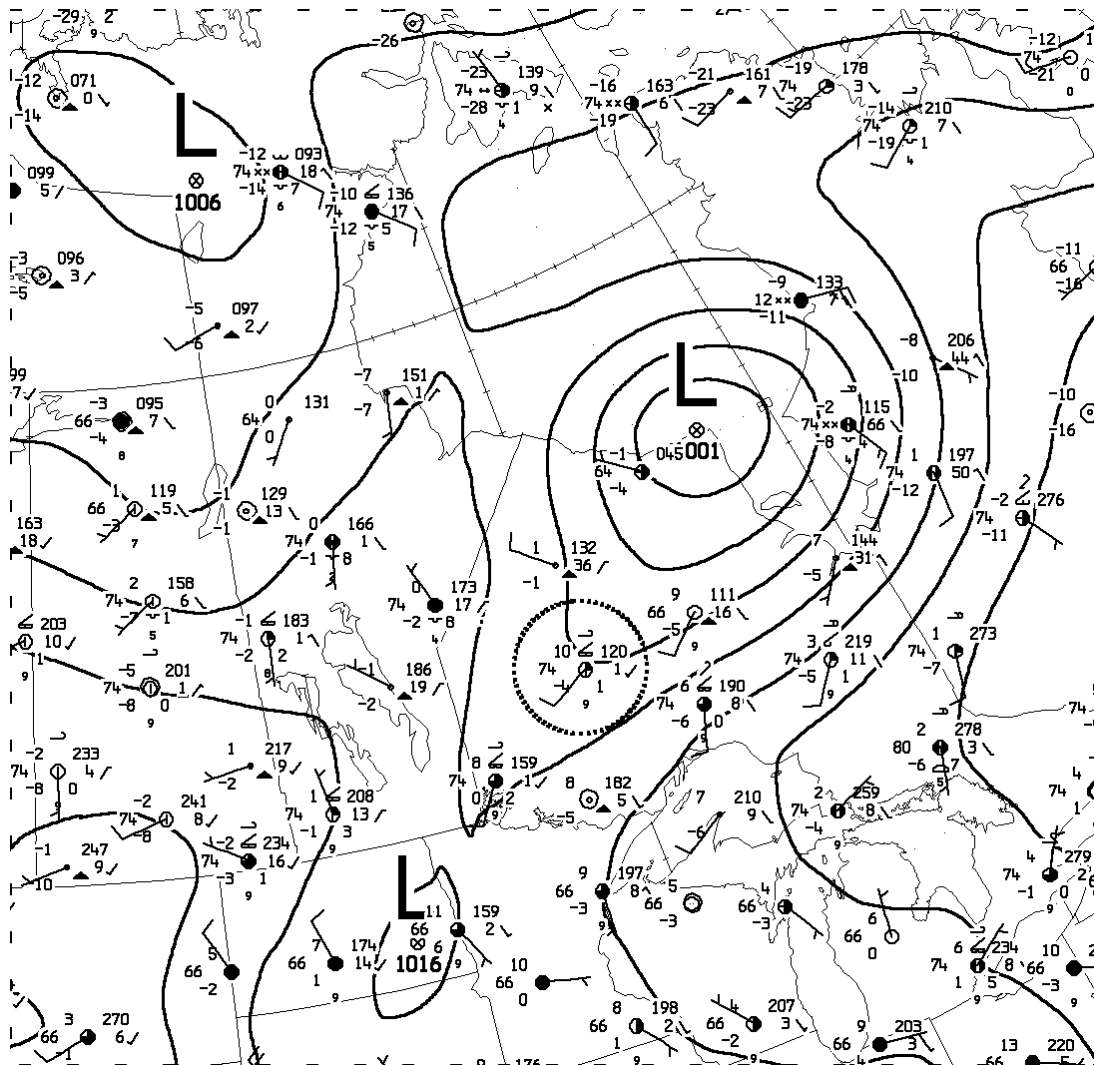
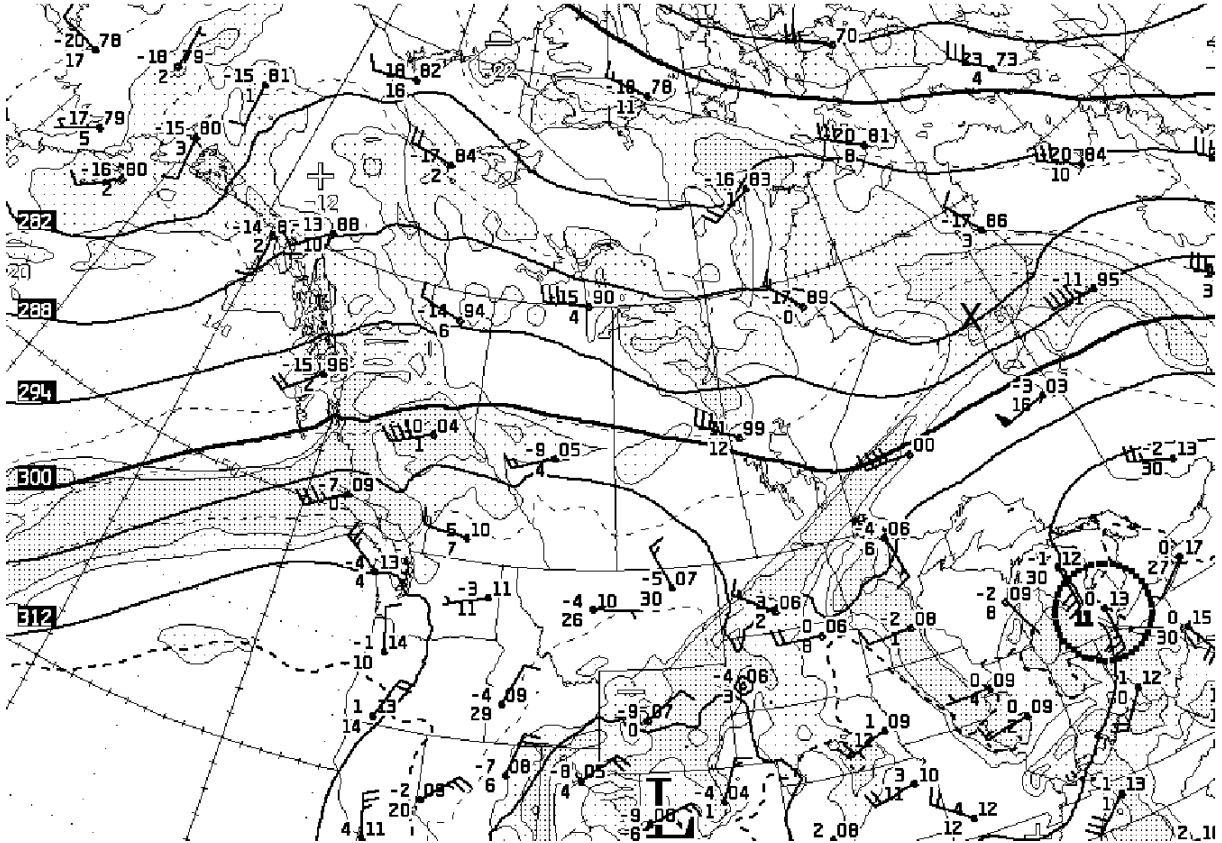


Figure 3: CMC preliminary surface and 850 hPa analyses for 12Z Thursday 16 April 2015.



### Pickle Lake Airport, Ontario

Past 24 Hour Conditions						
Date / Time (EST)	Conditions	Temp (°C)	Humidity (%)	Dew Point (°C)	Wind (km/h)	Pressure (kPa)
16 April 2015						
15:00	Partly Cloudy	10	45	-1	NNW 5	101.5
14:00	Partly Cloudy	9	53	0	W 9	101.5
13:00	Mostly Cloudy	7	62	0	W 7	101.5
12:00	Light Rainshower	5 ↓	71	0	NNW 5	101.5
11:00	Cloudy	7	61	0	WNW 13	101.5
10:00	Partly Cloudy	9	48	-1	NNW 11 gust 28	101.4
9:00	Mostly Cloudy	10	41	-2	WNW 18 gust 31	101.3
8:00	Mainly Sunny	11	38	-3	WSW 22 gust 34	101.2
7:00	Partly Cloudy	10	38	-4	WSW 21 gust 37	101.2
6:00	Partly Cloudy	9	37	-5	SW 23 gust 39	101.2
5:00	N/A	9	36	-5	SW 23 gust 33	101.2

Figure 4: CMC 700 hPa analysis for 12Z Thursday 16 April 2015 [X marks the approximate location of the surface low; Pickle lake is the station on the 300 dam contour for which  $(T, T - T_d)$  are missing]. Table gives observed conditions at Pickle Lake (EST = UTC-5).

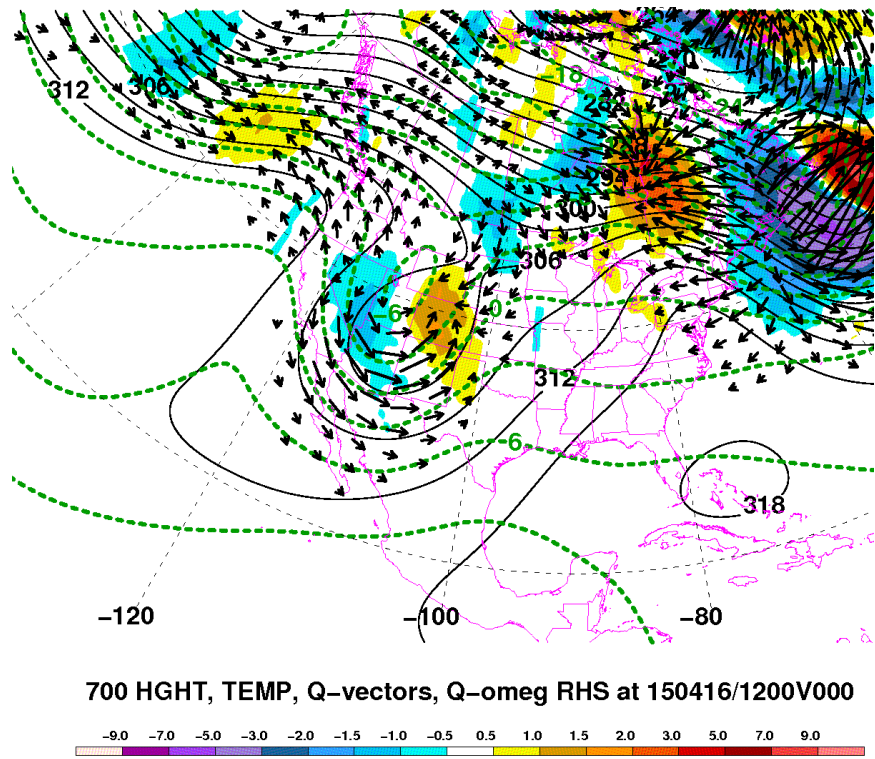
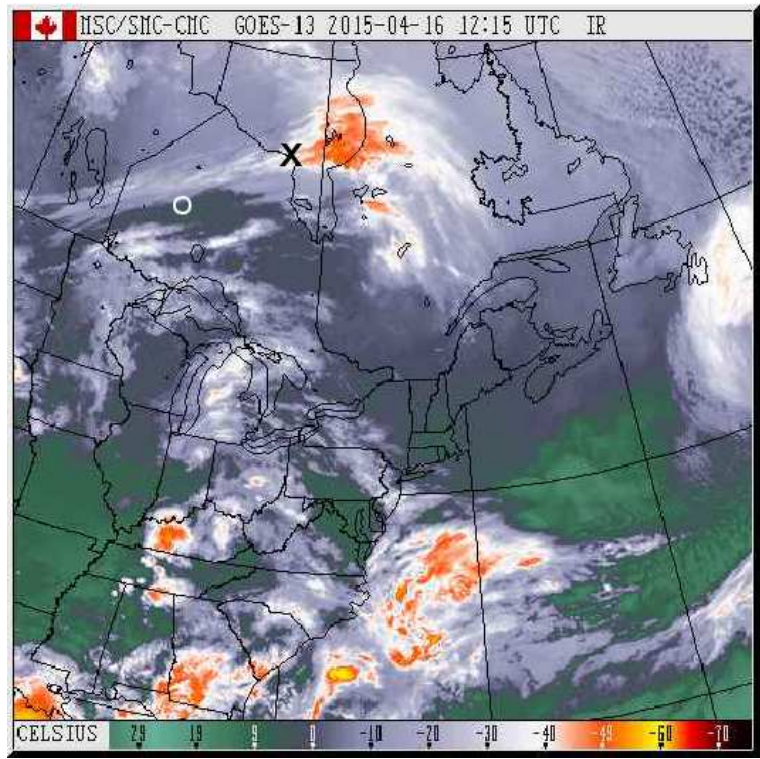
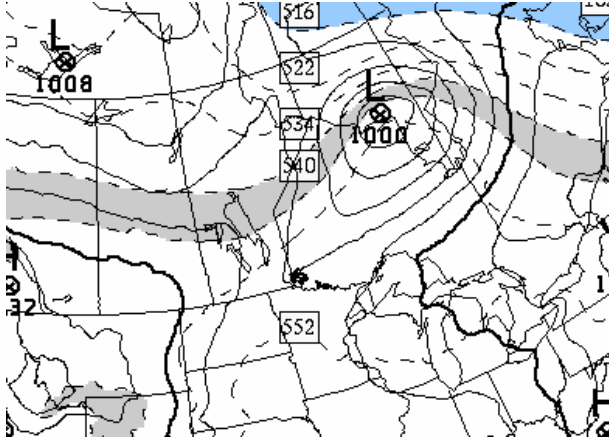
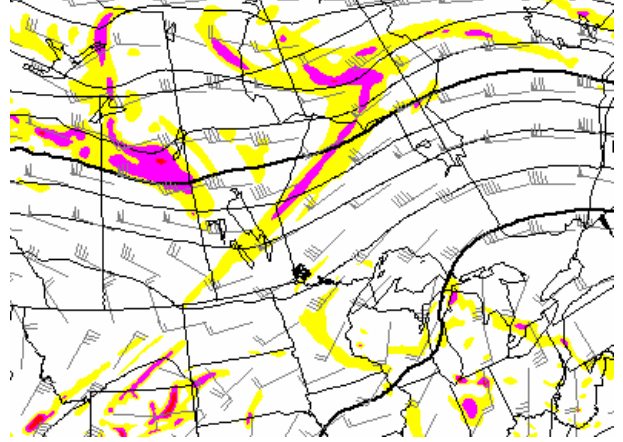


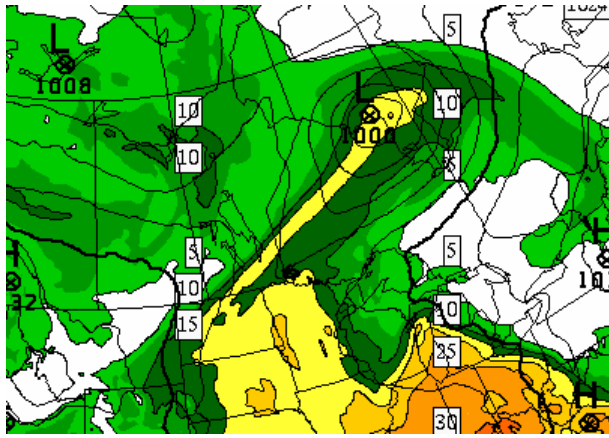
Figure 5: GOES East ir image 1215 Z Thursday 16 April 2015 (black cross, location of the surface low; white circle, the *approximate* location of Pickle Lake); and Q-vectors.



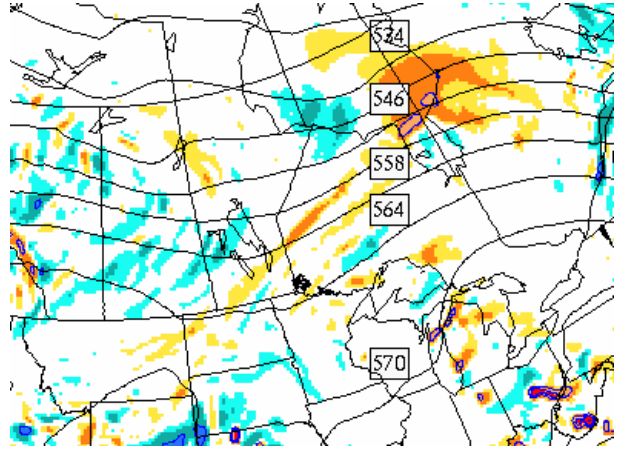
(a) MSLP & thickness



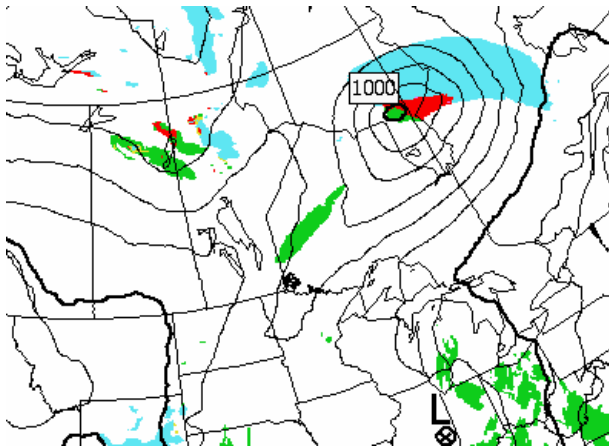
(b) 500 hPa height & vorticity (red=cyclonic vorticity)



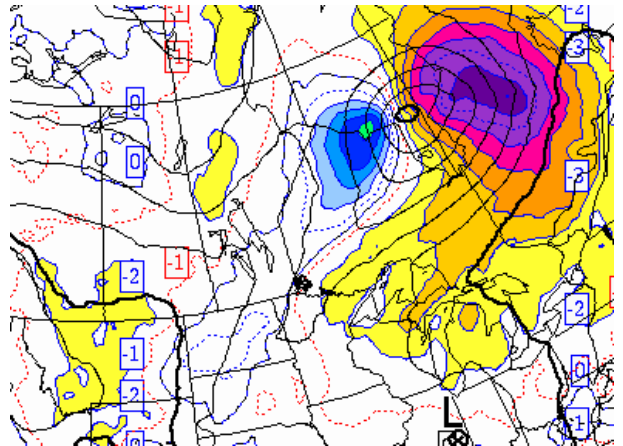
(c) Precipitable water (yellow 15-20 mm)



(d)  $\omega$  at 500 hPa (bright orange -0.5 to -1 Pa s<sup>-1</sup>)



(e) Dominant 3h precip type (green=rain)



(f)  $\partial P_{\text{sfc}}/\partial t$  over 3h (purple=rapid fall, blue=rise)

Figure 6: RDPS prog initialized 12Z Thursday 16 April 2015. Panels (a-d) 0h prog valid 12Z. Panels (e-f) 3h prog valid 15Z.