Modelling an idealized nocturnal (stable) boundary-layer ("NBL" or "SBL")

– *in what sense* idealized? Cloudless, unsaturated, horizontally homogeneous

From Stull (1988), *An Intro. To Boundary Layer Meteorology* (see also Garratt's Fig. 6.1)

eas471_SBL_Delage.odp
JD Wilson, EAS Ualberta
20 Mar. 2014

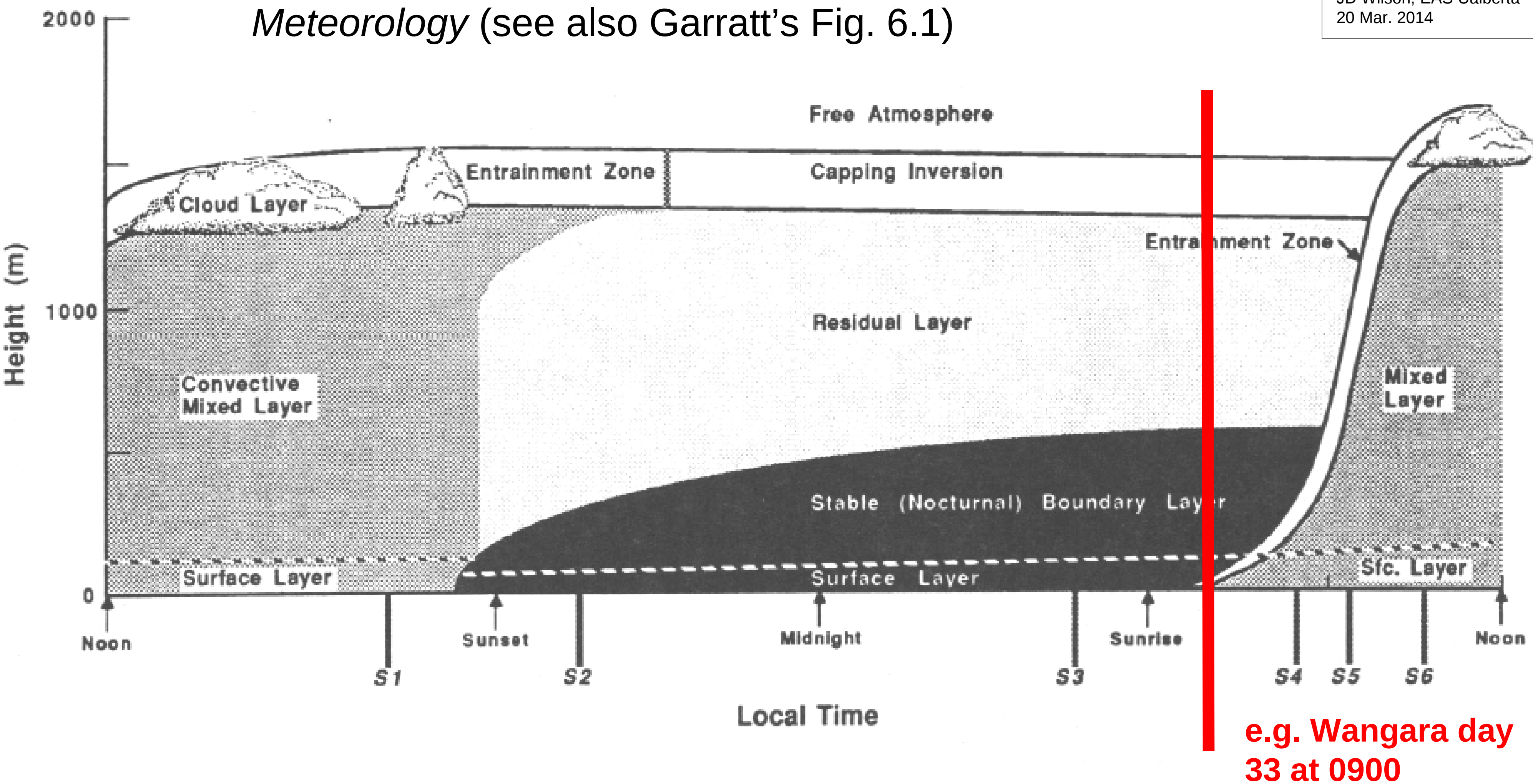
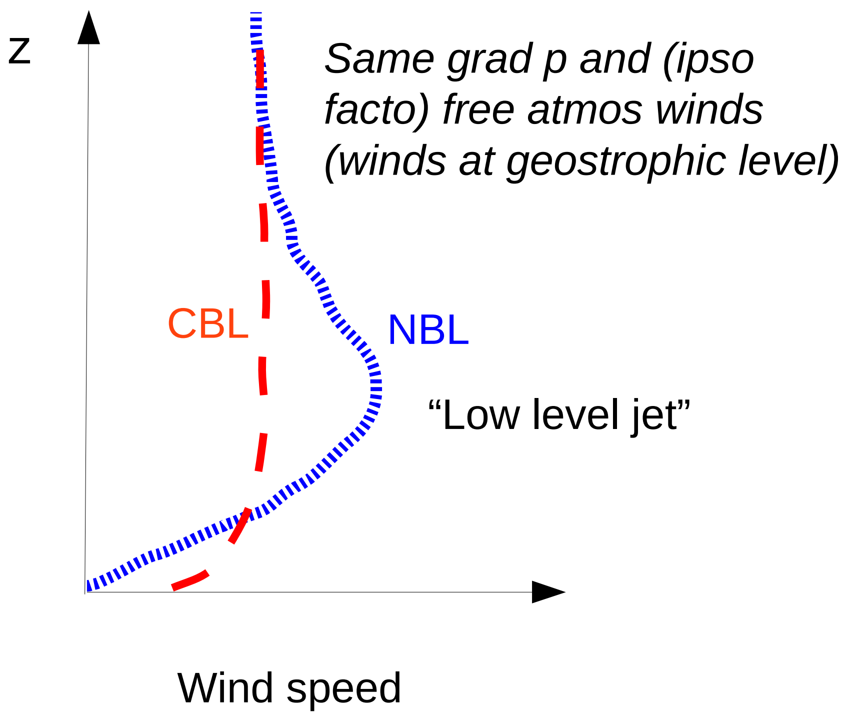
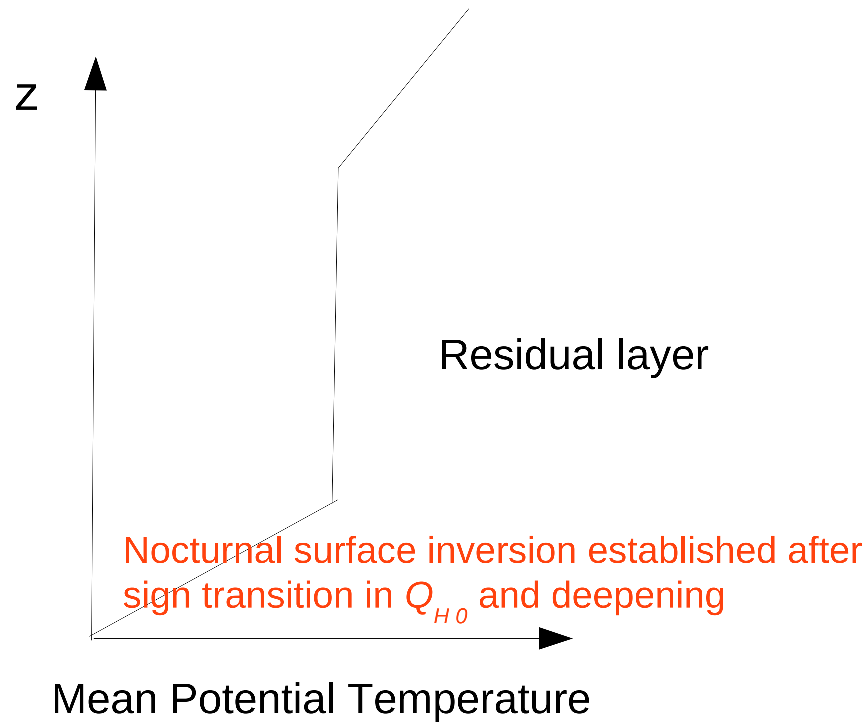


Fig. 1.7 The boundary layer in high pressure regions over land consists of three major parts: a very turbulent mixed layer; a less-turbulent residual layer containing former mixed-layer air; and a nocturnal stable boundary layer of sporadic turbulence. The mixed layer can be subdivided into a cloud layer and a subcloud layer. Time markers indicated by S1-S6 will be used in Fig. 1.12.

Inertial oscillation





Weakened friction hints at possibility of inertial oscillations in horizontal velocity

$$\frac{\partial U}{\partial t} = -\frac{\partial \overline{u'w'}}{\partial z} + f\,(V - V_G)$$

$$\frac{\partial V}{\partial t} = -\frac{\partial \overline{v'w'}}{\partial z} - f\,(U - U_G)$$

turbulence damped out?

$$\longrightarrow \quad \frac{\partial^2 U}{\partial t^2} = -f^2\,U + f^2 U_G$$

Energetics of the NBL – perspective of the TKE equation & surface energy budget

- surface energy budget results in surface cooling $Q^* \equiv K^* + L^* \;=\; Q_{H0} + Q_{E0} + Q_G < 0$

0 **L*<0**

- TKE budget:

$$\frac{\partial k}{\partial t} = \underbrace{-\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z}}_{\text{shear production}} + \underbrace{\frac{g}{\theta_0}\overline{w'\theta'}}_{\text{buoyant prodn}} \underbrace{- \frac{\partial}{\partial z}\overline{w'\left(\frac{p'}{\rho_0} + \frac{u'u'+v'v'+w'w'}{2}\right)}}_{\text{pressure transport + turbulent transport}} \underbrace{-\,\epsilon}_{\text{viscous dissip'n}}$$

- as daytime winds die down, shear production is reduced; and because the **layer is stably stratified buoyant production is negative**, offsetting what (little?) shear production continues

- thus turbulence dies down to low level – unless a strong free atmos. wind sustains shear production and overcomes buoyant destruction of TKE, sustaining the mixing and ensuring that a strong inversion does not develop

- and/or unless heavy cloud cover prevents rapid sfc cooling by longwave radiation

Energetics of the NBL – perspective of the velocity variance equations ($\overline{w'w'} \equiv \sigma_w^2$ etc.)

By manipulating the Navier-Stokes eqns. (under the Boussinesq approx.), the variance budget eqns for a <u>horizontally-homogeneous</u> layer are:

$$\frac{\partial \sigma_u^2}{\partial t} = -2\,\overline{u'w'}\,\frac{\partial U}{\partial z} - \frac{\partial}{\partial z}\,\overline{w'u'u'} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial u'}{\partial x}} - \epsilon_{uu}$$

$$\frac{\partial \sigma_v^2}{\partial t} = -2\,\overline{v'w'}\,\frac{\partial V}{\partial z} - \frac{\partial}{\partial z}\,\overline{w'v'v'} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial v'}{\partial y}} - \epsilon_{vv}$$

$$\frac{\partial \sigma_w^2}{\partial t} = \underbrace{2\,\frac{g}{\theta_0}\,\overline{w'\theta'}}_{\text{buoyant prodn}} - \underbrace{\frac{\partial}{\partial z}\,\overline{w'\left(\frac{2p'}{\rho_0} + w'w'\right)}}_{\text{turbulent (+ press.) transp.}} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial w'}{\partial z}} - \underbrace{\epsilon_{ww}}_{\text{viscous dissip'n}}$$

(small)

redistribution

$$\frac{\partial}{\partial t}\,\frac{\sigma_u^2+\sigma_v^2+\sigma_w^2}{2} = \ldots + \frac{2}{\rho_0}\,\overline{p'\underbrace{\left[\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}\right]}_{\text{redistribution terms sum to zero in TKE eqn}}} - \frac{\epsilon_{uu}+\epsilon_{vv}+\epsilon_{ww}}{2}$$

shear and buoyant production, turbulent and pressure transport

Energetics of the NBL – perspective of the velocity variance equations ($\overline{w'w'} \equiv \sigma_w^2$ etc.)

By manipulating the Navier-Stokes eqns. (under the Boussinesq approx.), the variance budget eqns for a <u>horizontally-homogeneous</u> layer are:

$$\frac{\partial \sigma_u^2}{\partial t} = -2\,\overline{u'w'}\,\frac{\partial U}{\partial z} - \frac{\partial}{\partial z}\,\overline{w'u'u'} + \frac{2}{\rho_0}\,\overline{p'\frac{\partial u'}{\partial x}} - \epsilon_{uu}$$

$$\frac{\partial \sigma_v^2}{\partial t} = -2\,\overline{v'w'}\,\frac{\partial V}{\partial z} - \frac{\partial}{\partial z}\,\overline{w'v'v'} + \frac{2}{\rho_0}\,\overline{p'\frac{\partial v'}{\partial y}} - \epsilon_{vv}$$

$$\frac{\partial \sigma_w^2}{\partial t} = \underbrace{2\,\frac{g}{\theta_0}\,\overline{w'\theta'}}_{\text{buoyant prodn}} - \underbrace{\frac{\partial}{\partial z}\,\overline{w'\left(\frac{2p'}{\rho_0} + w'w'\right)}}_{\text{turbulent (+ press.) transp.}} + \underbrace{\frac{2}{\rho_0}\,\overline{p'\frac{\partial w'}{\partial z}}}_{\text{redistribution}} - \underbrace{\epsilon_{ww}}_{\text{viscous dissip'n}}$$

(small)

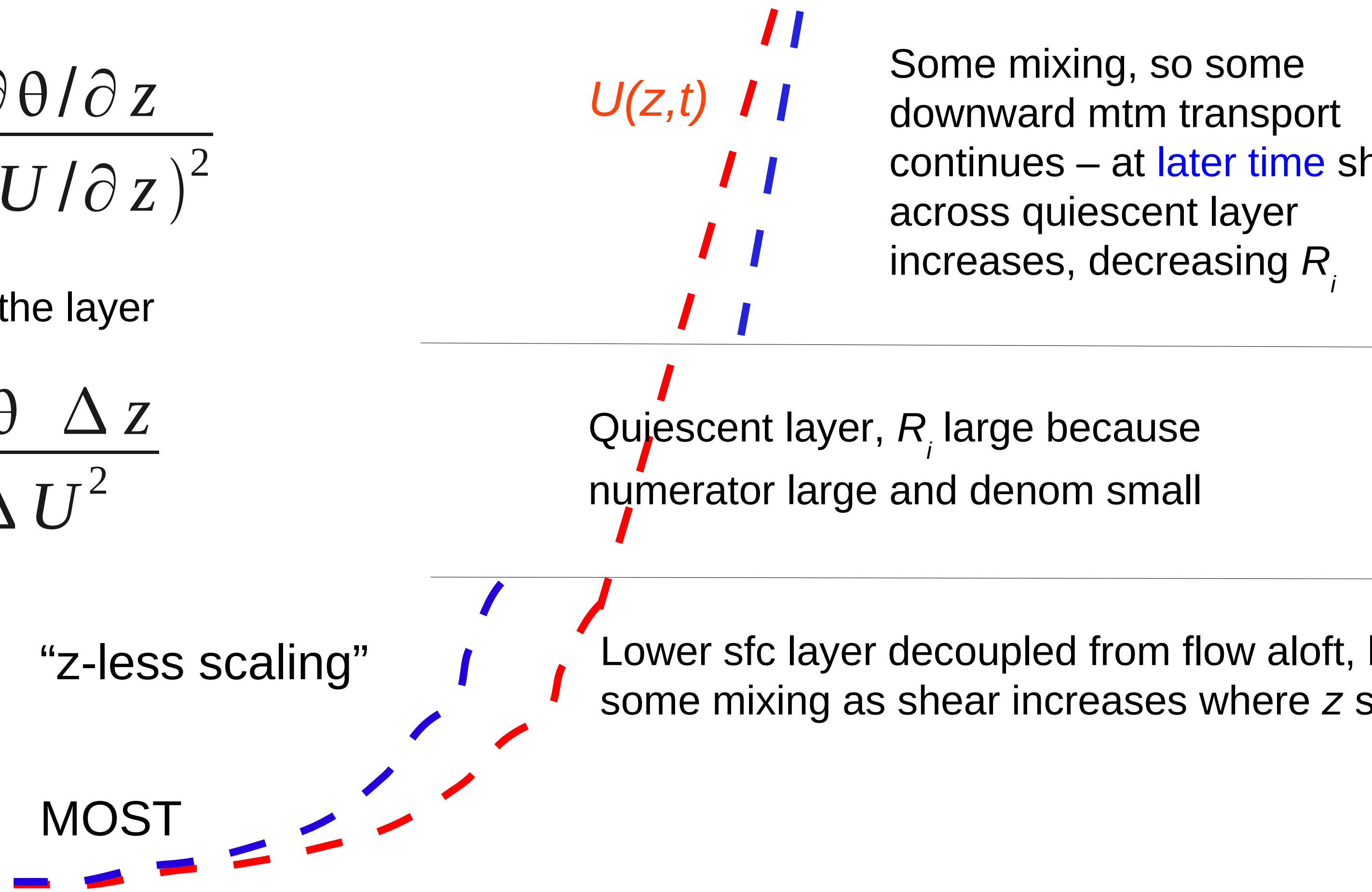
L*<0 → Q*<0 → Q_H<0 → buoyant suppression of the vertical motion (thus) TKE, effective in the energy-containing range of scales; *w'* fed by inter-component transfer (redistribution) alone; lack of energy in *w'* limits heat and (downward) momentum transport by turbulent convection; light winds (measurement challenge), turbulence may be intermittent; gravity waves; ratio of buoyancy (gT'/T_0) to inertial (u'^2/L) forces becomes large so slight topographic irregularities can result in drainage flows (three-dimensional and intermittent) – see Wyngaard's textbook Eqn. (12.20) where veloc. field parallel to gently sloping sfc contains buoyancy terms

An interesting cycle of intermittency can occur (Van de Wiel et al., J.Atmos.Sci. 67, 2010)

$$R_i = \frac{g}{\theta_0} \frac{\partial\theta/\partial z}{(\partial U/\partial z)^2}$$

or as bulk index for the layer

$$R_i = \frac{g}{\theta_0} \frac{\Delta\theta \ \Delta z}{\Delta U^2}$$



Critical value of Ri to suppress turbulence surely not universal, but order 0.1; textbooks cite obs. suggesting about 0.2

Delage** (1974; QJRMS Vol. 100) 1-D model of dry SBL – turbulent transport (only)

**formerly of CMC; Yves Delage had much to do with ABL parameterization in CMC's NWP models

$$\frac{\partial \overline{u}}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \overline{u}}{\partial z} \right) + f(\overline{v} - V_g)$$

$$\frac{\partial \overline{v}}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \overline{v}}{\partial z} \right) - f(\overline{u} - U_g)$$

$$\frac{\partial \overline{\theta}}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \overline{\theta}}{\partial z} \right)$$

(neglects radiative divergence)

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial k}{\partial z} \right) + K \left[\left(\frac{\partial \overline{u}}{\partial z} \right)^2 + \left(\frac{\partial \overline{v}}{\partial z} \right)^2 \right] - \frac{g}{\theta_{00}} K \frac{\partial \overline{\theta}}{\partial z} - \epsilon$$

<u>Closure</u>

$$K = \lambda(z) \sqrt{c_e\, k}$$

eddy diffusivity

effective velocity scale

$$\epsilon = (c_e\, k)^{3/2} / \lambda$$

TKE dissipation rate

$$\frac{1}{\lambda} = \frac{1}{k_v z} + \frac{1}{\lambda_\infty} + \frac{\beta}{k_v L}$$

length scale

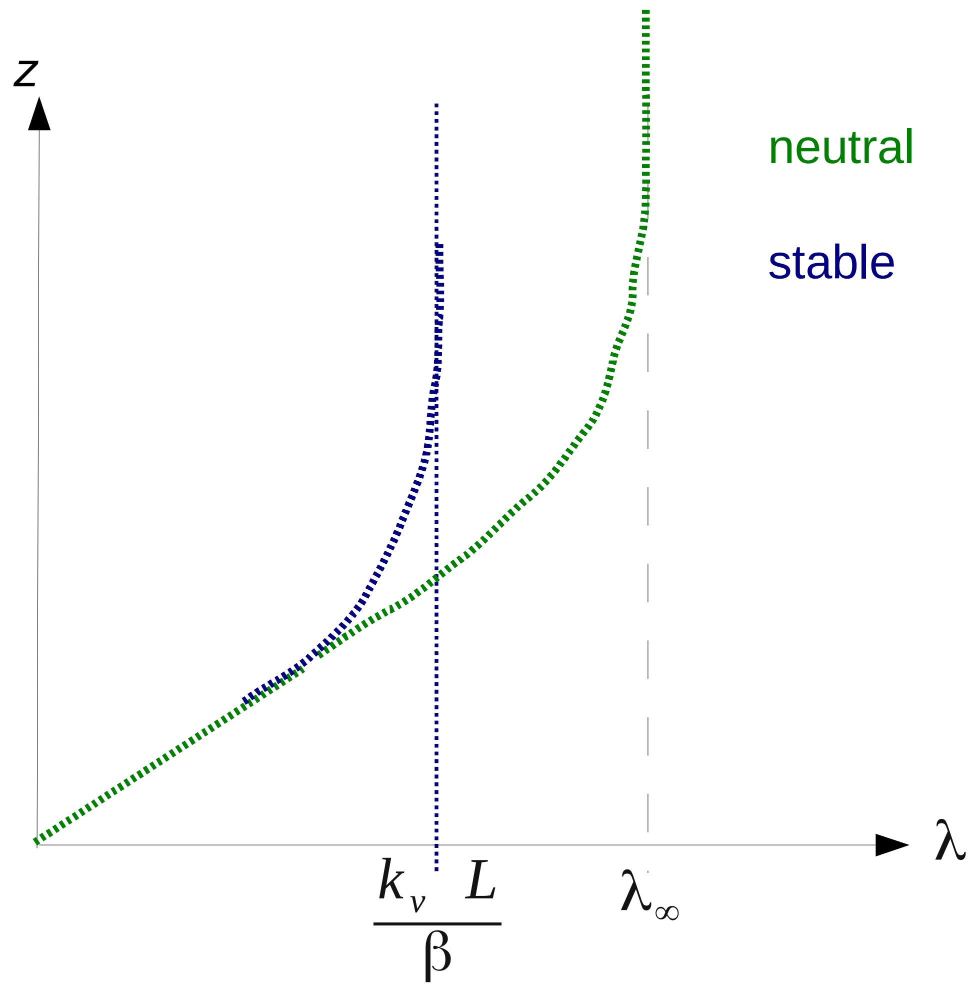
Recall that in context of Monin-Obukhov Similarity Theory (MOST): $K_{m,k,v} = \dfrac{k_v\, u_*\, z}{\varphi_{m,h,v}(z/L)}$

Delage's imposed algebraic length scale

$$\frac{1}{\lambda(z)} = \frac{1}{k_v \; z} + \frac{1}{\lambda_\infty} + \frac{\beta}{k_v \; L}$$

limits λ in neutral layer

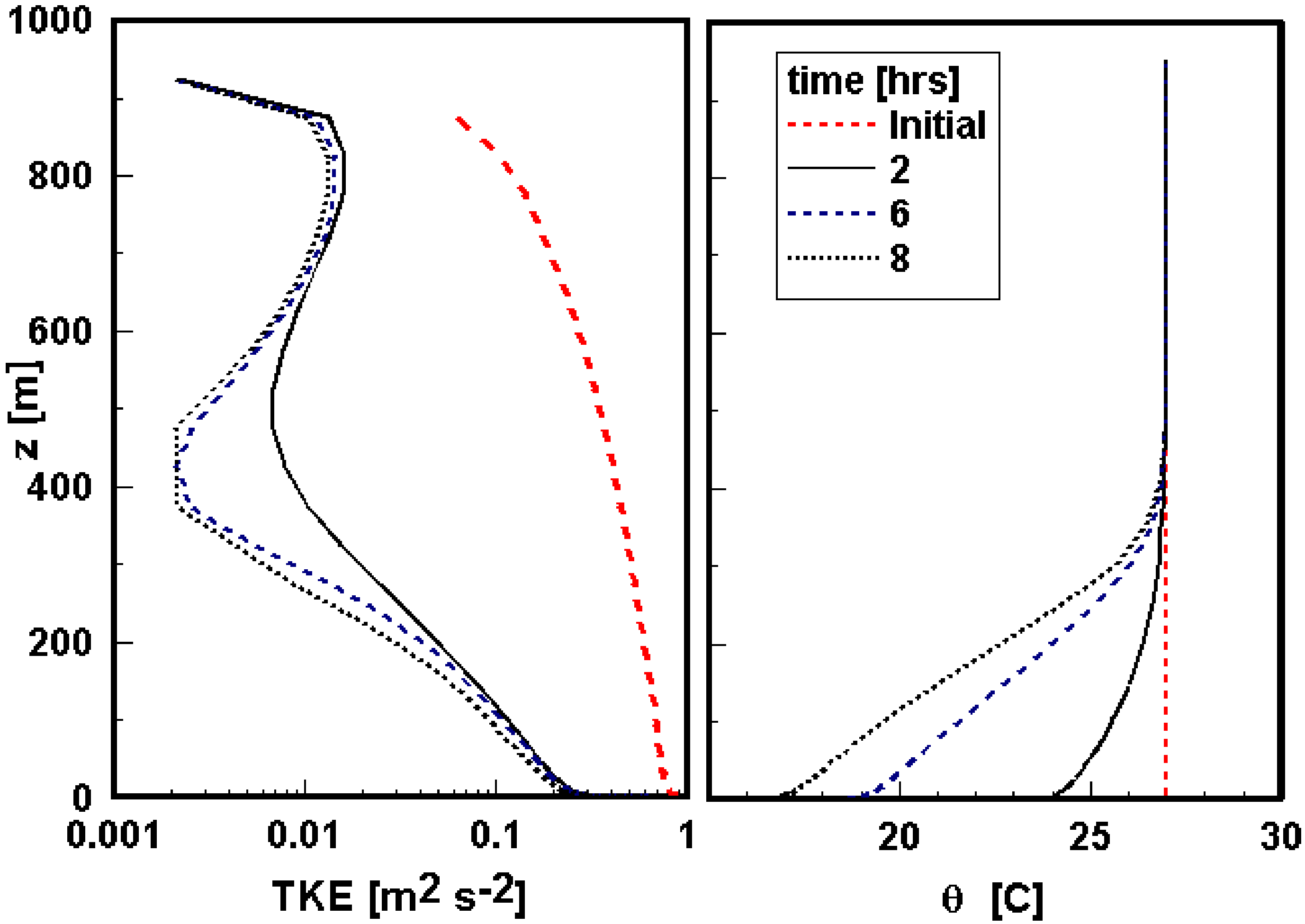
limits λ in stratified layer



Delage's numeric solution

Initial condition: $\theta(z,0) = \theta_{00}$ and corresponding steady-state wind and TKE profiles from solution of these equations for the neutral state.

Forcing: “driven” by an imposed cooling trend in surface temperature

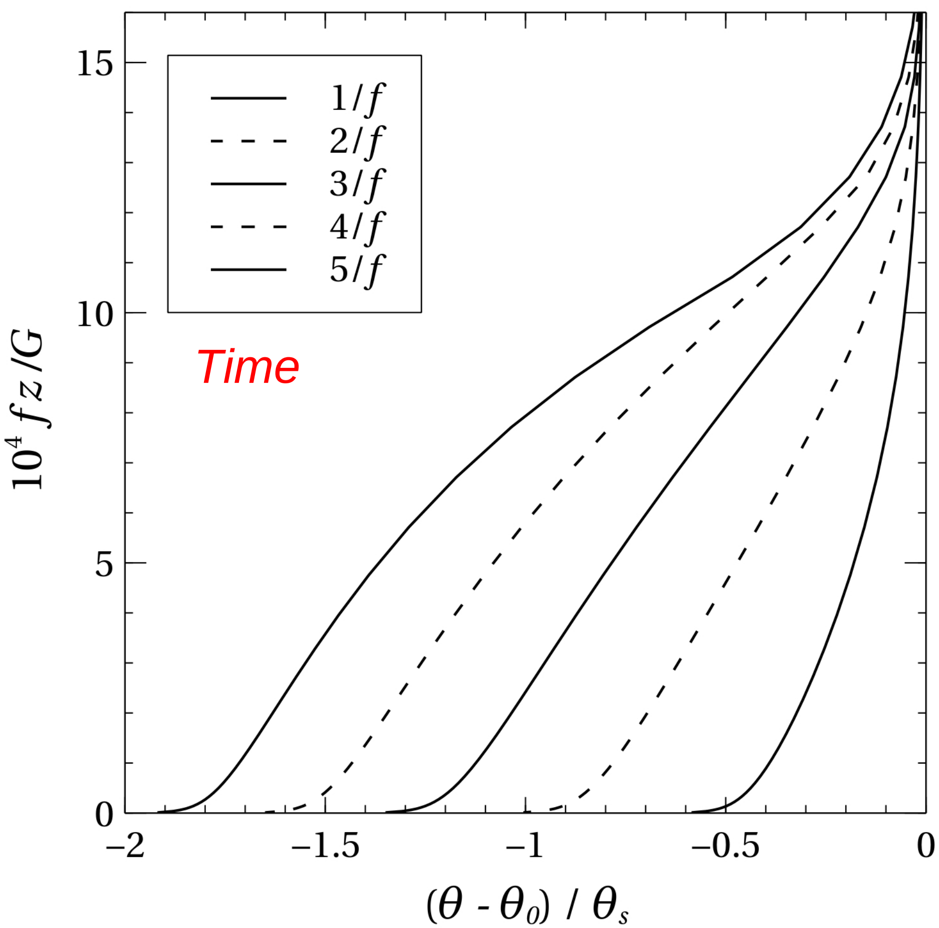


- intensifying surface-based inversion self-limits its own depth h_i
- depth h_t of surface-based mixing layer drops. Mixing continues in residual neutral layer aloft

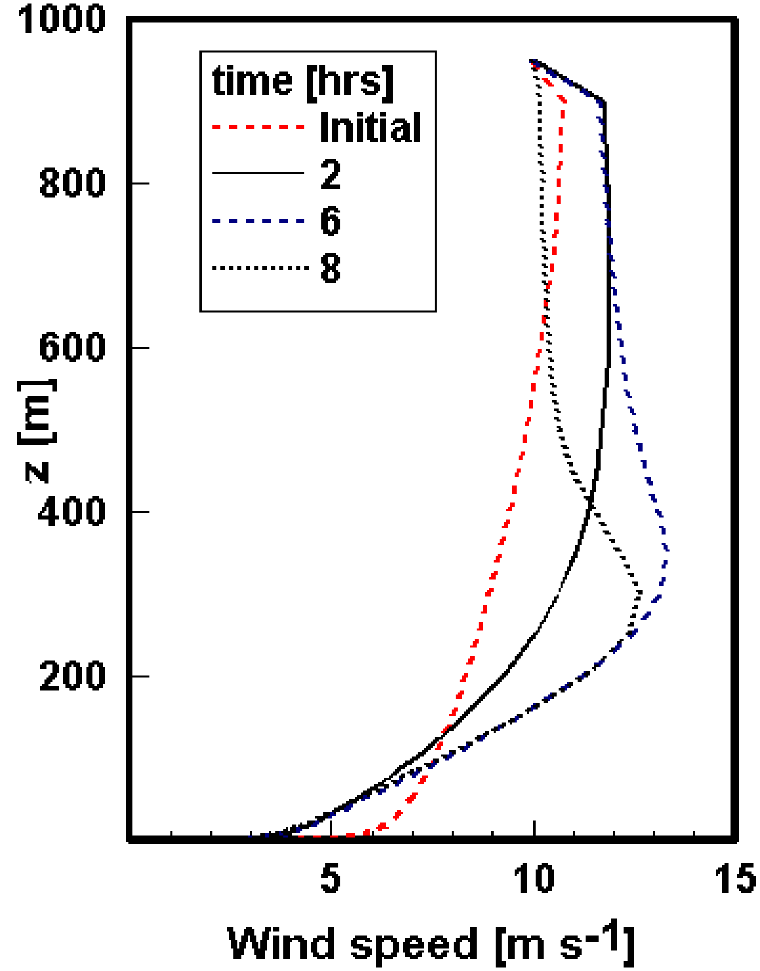
Delage's numeric solution

Delage's result for cooling rate, presented in dimensionless form. Case chosen corresponds to a strong geostrophic wind *G* such that the Rossby number

$$R_o = \frac{G}{z_0 \ f} = 10^7$$



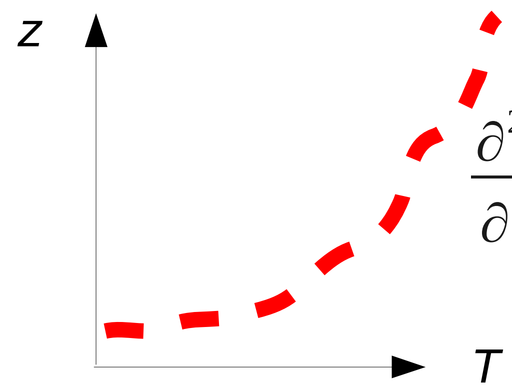
Low-level jet develops in Delage simulation



Neglected role of radiative divergence

André & Mahrt (1982, JAS Vol. 39) showed that the role of nocturnal longwave divergence can be to deepen the ground-based inversion h_i so that it reaches several times higher than the height h_t of the turbulent (ie. stirred) shear layer, at the same time moderating the stratification of that turbulent layer - whereas the convective flux divergence, acting alone, would progressively steepen the temperature gradient as the ground cools, without deepening the inversion.

Ha and Mahrt (2003, Tellus A Vol. 55) computed longwave radiative divergence from both idealized and measured profiles of temperature and humidity, and determined that “radiative cooling increases with the thermal stratification, moisture content, negative curvature of the temperature profile and temperature deficit of the ground surface”

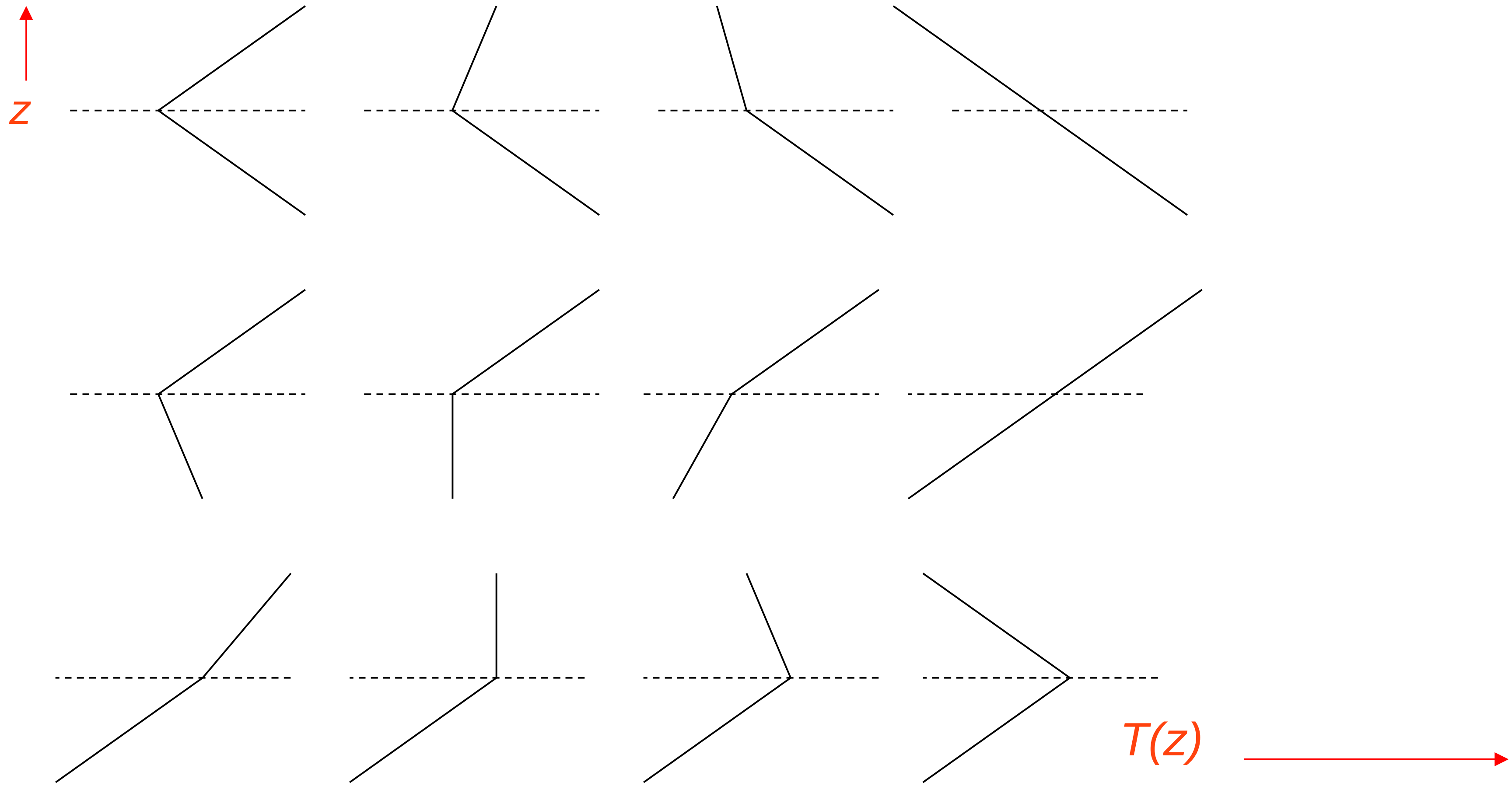


$$\frac{\partial^2 T}{\partial z^2} < 0$$

With horiz. homog. assumed, $$\rho\, c_p \frac{\partial \overline{\theta}}{\partial t} = -\frac{\partial Q_H}{\partial z} - \frac{\partial Q^*}{\partial z}$$ where $$\frac{\partial Q^*}{\partial z} \equiv \frac{\partial L^*}{\partial z}$$

Schaller (1977; BLM Vol. 11) observed** that “during the clear night radiative cooling exceeds the cooling caused by the sensible heat flux.” André et al. (1978; JAS Vol. 35) concluded longwave divergence is “more important than turbulent transport … except close to the ground”

**See also André et al. (1978; JAS Vol. 35), André & Mahrt (1982, JAS Vol. 39), Ha & Mahrt (2003, Tellus A Vol. 55: “Radiative and Turbulent Fluxes in the Nocturnal Boundary Layer”)



Simplified temperature profiles *T(z)*, in relation to the rate of heating due to longwave radiative flux divergence at a fixed height *z* . Heating rate depends on the *difference* in slope

$$\left(\frac{\partial T}{\partial z}\right)_{(z'>z)} - \left(\frac{\partial T}{\partial z}\right)_{(z'<z)}$$

which is related to the curvature $\partial^2 T / \partial z^2$ of the temperature profile. It can be shown that the rate of heating is determined non-locally, viz. by the difference between weighted height-integrals of the temperature-gradient above and below the observation level. The top two rows correspond to radiative heating (except the right-most profiles where heating vanishes), while the lowest row corresponds to radiative cooling.