Investigating an alternative formulation for the eddy viscosity in the ABL

$$K = q\,\lambda$$

q: Velocity scale

λ: Length scale

- Who uses eddy viscosity closure?
- Why?
  - Layers/structure of the ideal ABL
  - Governing eqns & the closure problem
  - Approaches to specifying the needed K 's
  - Elusive formulation for λ in specification $K = q\,\lambda$
- Implication of G.I. Taylor's Lagrangian analysis
- Numerical solutions using alternative closure
- Conclusion

$$K = -\frac{\text{mean vertical flux by unresolved eddies}}{\text{corresponding mean vertical gradient}}$$

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J.D. Wilson, EAS, U. Alberta
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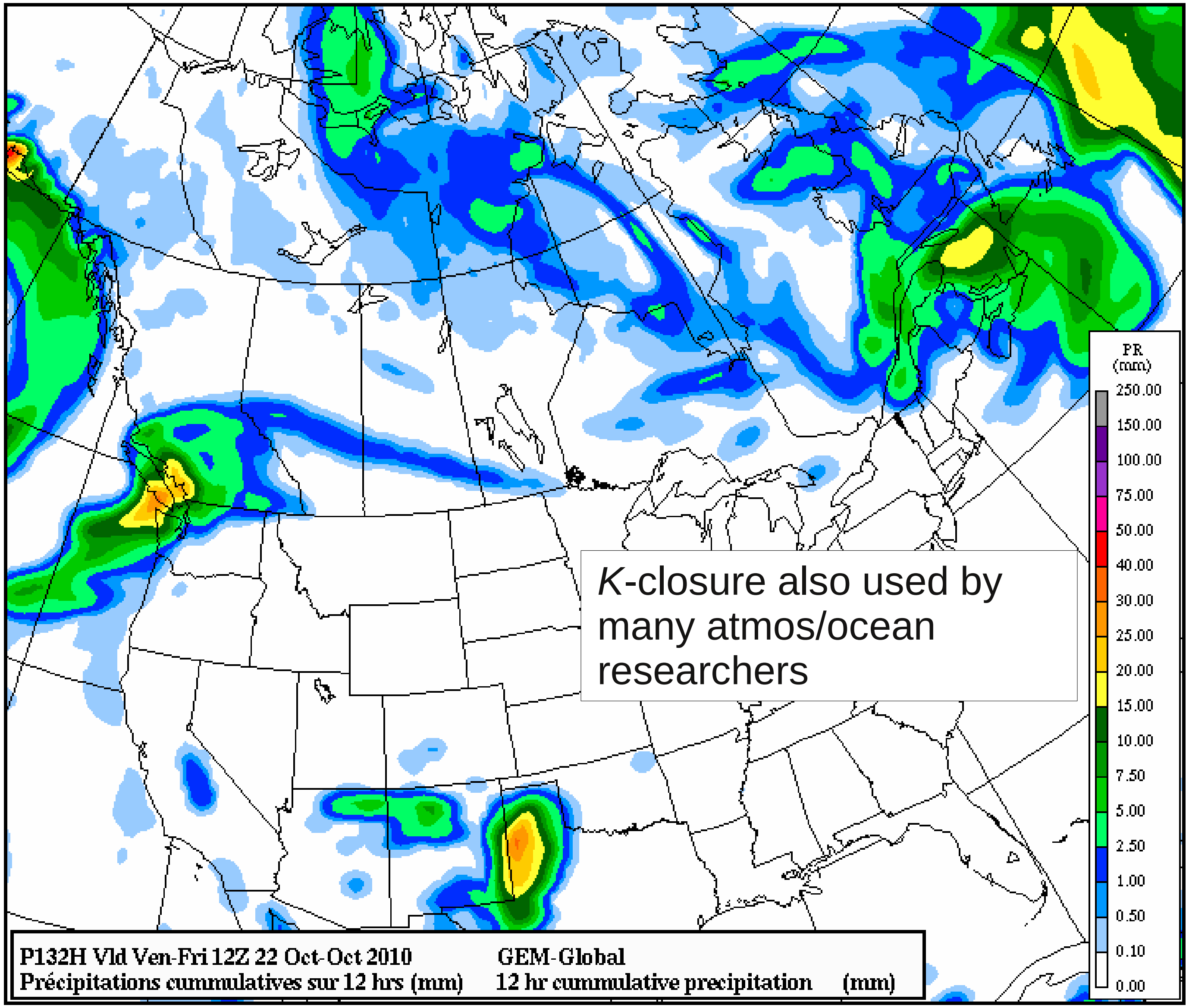
Many weather models, including GEM, use *K*-theory closure in “grid point computations”

Necessary in order to:

- adjust vertical distribution of heat & moisture in the ABL
- achieve realistic surface-air exchange rates
- Achieve realistic stratification in ABL and above

Thus critical for weather system development



K-closure also used by many atmos/ocean researchers

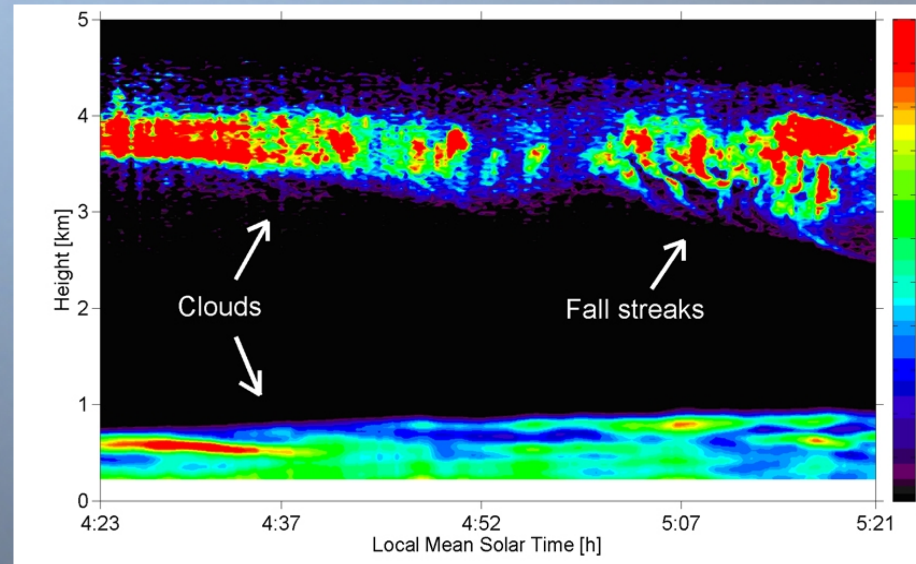
Daylength 24.66 hrs

Solar constant 591 W m^{-2}

Atmos. mostly CO_2

Surface pressure varies ~ 6-8 hPa

Sfc temperature varies about -100 to 0^oC



- Phoenix Lidar detected water ice clouds

- ABL modelling has indicated crucial role of dust in modulating distributed solar heating and (thus) stratification and (thus) a daily water cycle

- the models used are eddy viscosity models

- treatment of radiative divergence crucial

Sunset on Mars

$$\frac{\partial U}{\partial t} = -\frac{\partial \overline{u'w'}}{\partial z} + f(V - V_G)$$

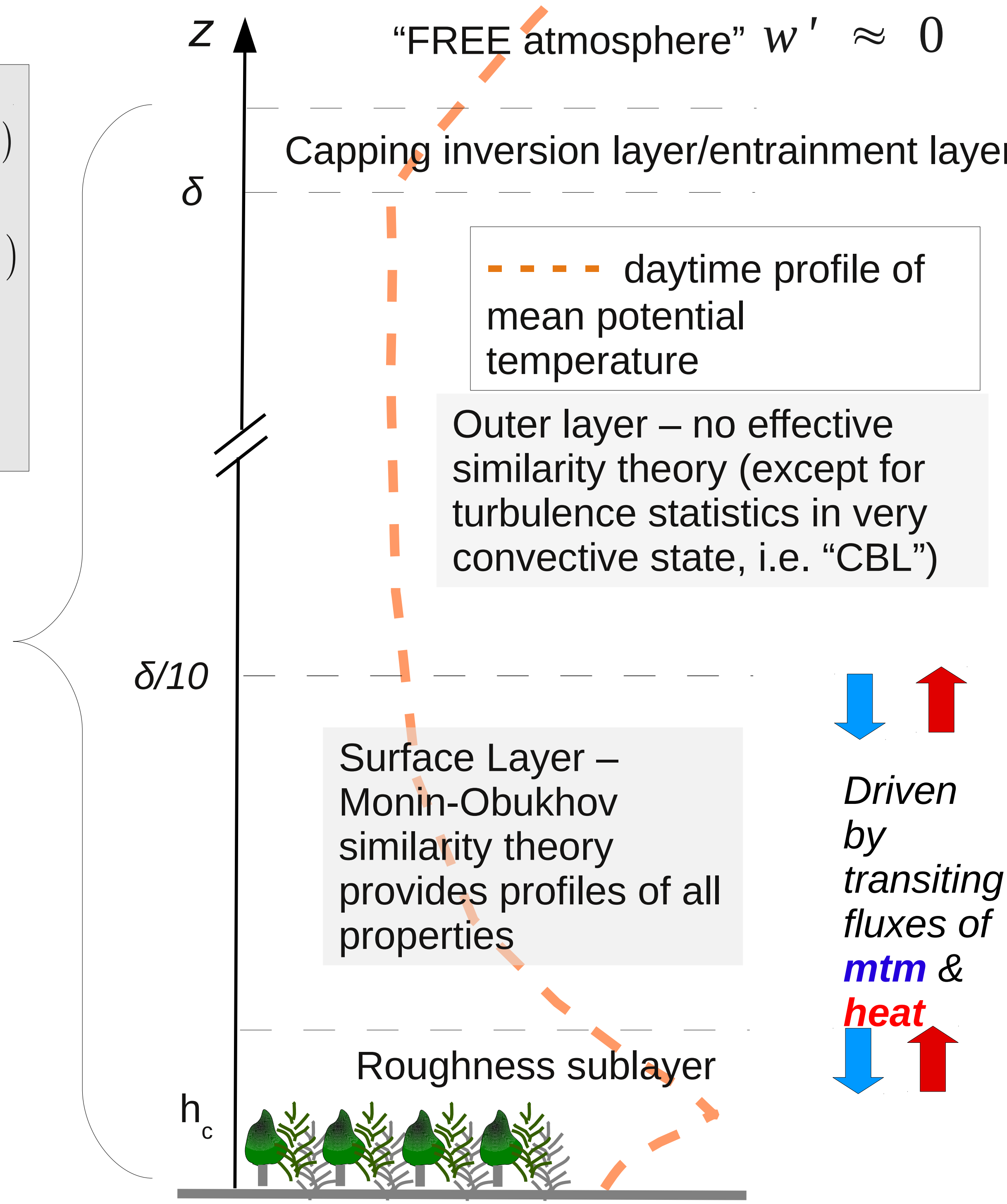
$$\frac{\partial V}{\partial t} = -\frac{\partial \overline{v'w'}}{\partial z} - f(U - U_G)$$

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial \overline{w'\theta'}}{\partial z}$$

Atmospheric boundary layer
(Friction layer)

Unresolved ("turbulent") velocity fluctuations

$$u' \approx v' \approx w' = O[\mathrm{m\ s^{-1}}]$$



Theoretical foundation for treating ABL – Reynolds' mean momentum eqn etc.

- Decompose every variable into sum of Mean + Fluctuation $w = W + w'$

- Apply the averaging operation to the Navier-Stokes equations → Reynolds' equations

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\left(U^2 + \overline{u'u'}\right) + \frac{\partial}{\partial y}\left(UV + \overline{u'v'}\right) + \frac{\partial}{\partial z}\left(UW + \overline{u'w'}\right) = \frac{-1}{\rho}\frac{\partial P}{\partial x} + fV + \nu\nabla^2 U$$

horiz. homogeneity

implied by hor. homogeneity
+ non-divergence of veloc.

Viscous
friction

Posit horizontal homogeneity of *statistics* – then indicated terms vanish, leaving:

$$\frac{\partial U}{\partial t} = -\frac{\partial \overline{u'w'}}{\partial z} + f\left(V - V_G\right)$$

$$\frac{\partial \overline{\theta}}{\partial t} = -\frac{\partial \overline{w'\theta'}}{\partial z}$$

(corresponding heat budget)

“turbulent friction” (divergence of Reynolds stress). Fluctuations *u′* and *w′* are (anti-) correlated – so $\overline{u'w'}$ is a downward flux of *u*-momentum

“Geostrophic deficit”
(balances to zero in free atmos)
Large scale pressure gradient defines the Geostrophic wind
(so $f\,V_G$ is just a substitution)

BIG PROBLEM – new unknowns – unclosed set of eqns

Assume the unresolved vertical fluxes of momentum and heat can be modelled as:

$$\overline{u'w'} = -K \frac{\partial U}{\partial z} \qquad \overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z} \qquad \left(\frac{K}{K_h} \text{ is the turbulent Prandtl number} \right)$$

The K 's are (dimensionally) the product of a velocity scale and a length scale.

$$K = q \lambda$$

The problem now is to model K in terms of “resolved” properties.

There is in fact no problem in the surface layer – where the K 's are determined by the Monin-Obukhov similarity theory (MOST)

$$K = \frac{k_v \, z \, u*}{\phi(z/L)}$$

Neutral length scale $k_v z$

Velocity scale $u*$ the friction velocity

Stability correction Φ (z/L)

How to prescribe *K* in the outer layer?

Hierarchy of approaches to specifying eddy viscosity/achieving closure

Constant *K* or *K*-profile – no interaction with computed (resolved) state/flow field

↓

Prandtl's method

Empirical algebraic length scale, e.g. $\frac{1}{\lambda} = \frac{1}{k_v \, z} + \frac{1}{\lambda_{max}}$

and velocity scale $q = \lambda \left| \frac{\partial \vec{U}}{\partial z} \right|$

↓

Prandtl-Kolmogorov method

$$\frac{1}{\lambda} = \frac{\phi(R_i)}{k_v \, z} + \frac{1}{\lambda_{max}}$$

and

$$q = \sqrt{k}$$

TKE computed from its (simplified) p.d.e.

used in CMC's models:

- Tune φ and λ_{max}

↓

k-ε method(s)

$$q = \sqrt{k}$$

and

$$\lambda = \frac{k^{3/2}}{\epsilon}$$

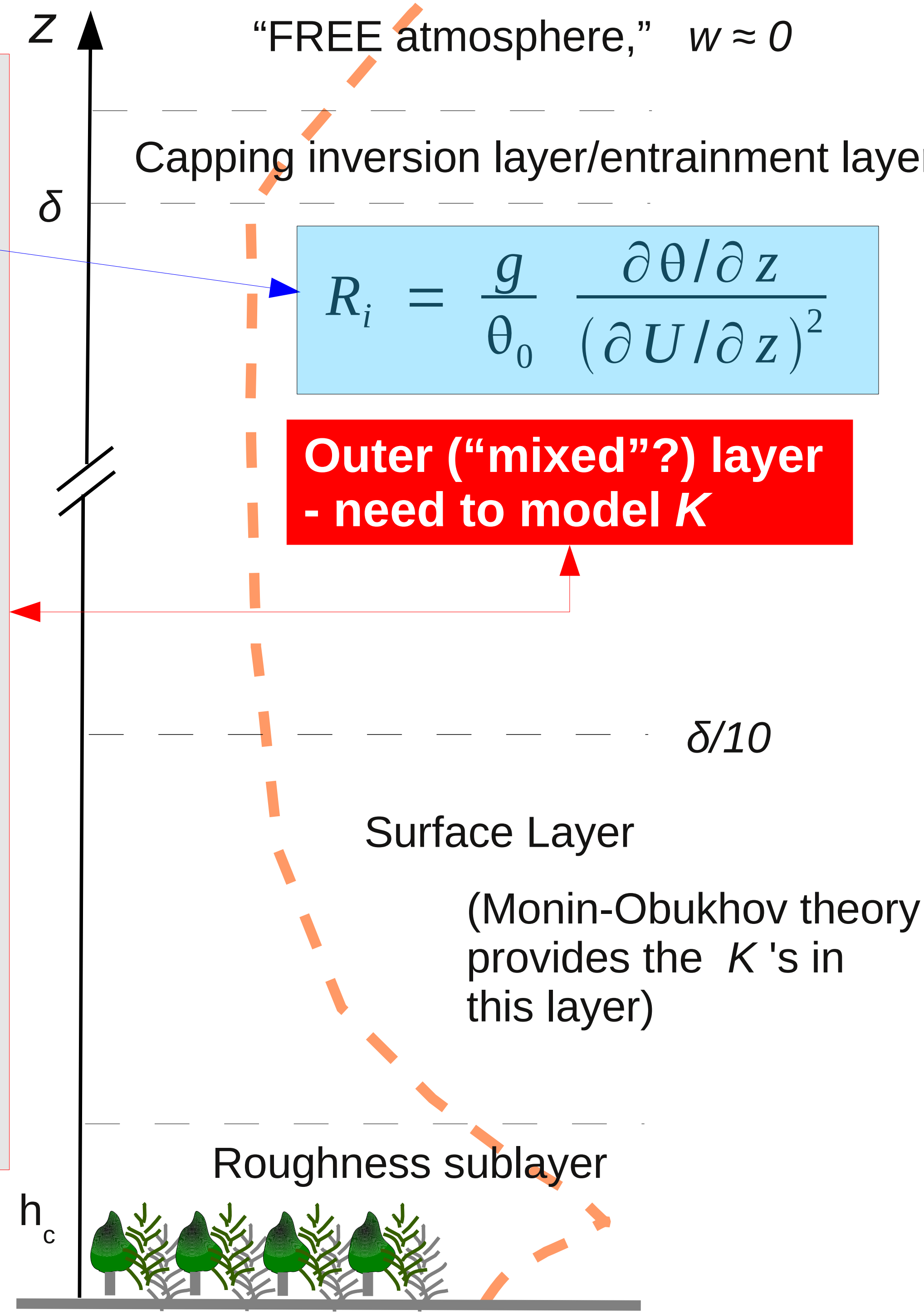
TKE *k* and dissipation rate *ε* both computed from p.d.e.'s (but utility of the *ε*-equation controversial)

popular in engineering and some atmos. apps

↓

Higher-order closure or LES

- no consensus on formulation for λ, usually made a function of the local Richardson number R_i
- i.e. in existing formulations both q and λ respond to wind (U,V) shear
- no universal form of U,V profiles
- models typically not grid-indep; tuned closures may not generalize as grid is refined
- Bougeault-Lacarrère scheme (works as well as any other) relates λ only to the profile of potential temperature (i.e. to stratification)
- is this a hint that $q=k^{1/2}$ is a false turn and a source of difficulty?



Motivation to use σ_w as veloc. scale for eddy viscosity– Taylor's Lagrangian theory of dispersion

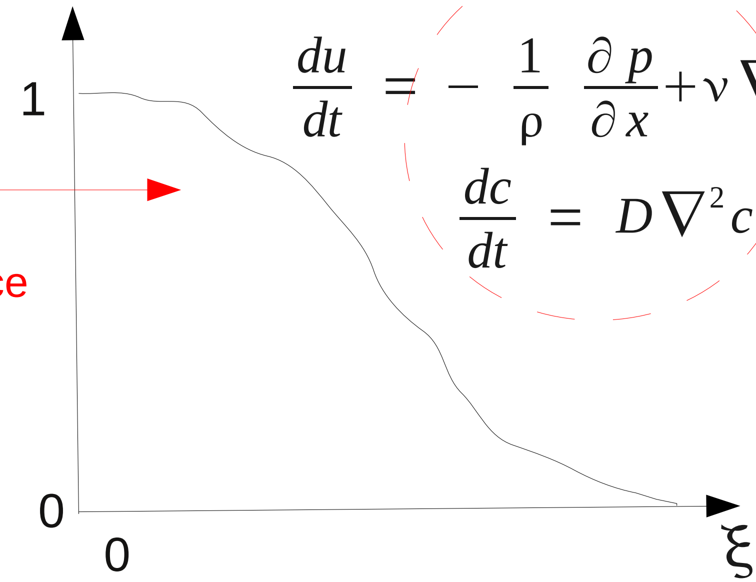
At high Peclet number *UL/D* concentration *c* conserved along trajectories – contrast *u*

$$K = \sigma_w^2 \, \tau_w \equiv \sigma_w \, \lambda_w$$

Lagrangian integral timescale (area under this curve of the auto-covariance in velocity versus time lag ξ)

Variance (i.e. square of the standard deviation) of the vertical velocity

$$\overline{w(t)\,w(t+\xi)}/\sigma_w^2$$



$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \nabla^2 u$$

$$\frac{dc}{dt} = D \nabla^2 c$$

This is Taylor's result for the “far field diffusivity”, i.e. valid at travel times (since release of the tracer) that satisfy $t \gg \tau_w$

Since we are quantifying *vertical* motion, correct velocity scale for the eddy viscosity is the standard deviation σ_w of vertical velocity – not $k^{1/2}$. The length scale λ (<u>also</u>) is a statistic of the *vertical* motion. Recall that in a horiz. homog. ABL the vert. velocity variance satisfies:

$$\frac{\partial \sigma_w^2}{\partial t} = 2\,\frac{g}{\theta_0}\,\overline{w'\theta'} - \frac{\partial}{\partial z}\,\overline{w'\left(\frac{2p'}{\rho_0} + w'w'\right)} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial w'}{\partial z}} - \epsilon_{ww}$$

Numerical solution for 1-D ABL (solutions depend on time and height alone) with $K = \sigma_w^2 \tau_w$

- Solve pde's for *U, V , θ, q* and *k*, σ_w^2 (using Fortran under Linux on PC)

- Unsaturated ABL; radiative heat transport not included

- Stretched, staggered grid $(0.1 \le \Delta z \le 50 \text{ m})$ extending to z=2000 m, $\Delta t = 5$ s

- Solutions are grid- and timestep- independent

- Crank-Nicholson discretization; coupling of eqns handled by iteration to convergence during each timestep

- Driven by measured (evolving) sfc flux of sensible heat & measured *U, V, θ, q* at 2000 m

- Initialized with analytical fit to measured profiles of 0900

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial U}{\partial z}\right) + f\left(V - V_G\right)$$

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial V}{\partial z}\right) - f\left(U - U_G\right)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} K_h \frac{\partial \theta}{\partial z}$$

and similar eqn for specific humidity

$$\frac{\partial \sigma_w^2}{\partial t} = -2 \frac{g}{\theta_R} K_h \frac{\partial \bar{\theta}}{\partial z} - C_1 \frac{\sigma_w^2}{\tau_w} + \frac{\partial}{\partial z}\left(K \frac{\partial \sigma_w^2}{\partial z}\right) + C_2 \frac{2k/3 - \sigma_w^2}{\tau_w}$$

These constants fixed algebraically by requiring the model equations to reproduce *analytically* the structure of a neutral surface layer

modelled form of the redistrib'n term

Parameterization of needed time scale – no reference to wind shear or to R_i

Construction by sum of reciprocals

$$\frac{1}{\tau_w} = \frac{1}{\tau_{w,SL}} + \frac{1}{\tau_{w,OL}}$$

Timescale of surface layer is known

$$\tau_{w,SL} = \frac{k_v u_* \ z}{\sigma_w^2 \ \phi(z/L)}$$

Timescale of outer layer for $\partial\theta / \partial z \geq 0$

$$\frac{1}{\tau_{w,OL}} = \frac{1}{\tau_\infty} + C_{BV}\left(\frac{g}{\theta_0}\frac{\partial \theta}{\partial z}\right)^{\frac{1}{2}}$$

Brunt-Vaisala frequency N_{BV}

Timescale of outer layer for $\partial\theta / \partial z < 0$

$$\frac{1}{\tau_{w,OL}} = \frac{1}{\tau_\infty}$$

arbitrary constant

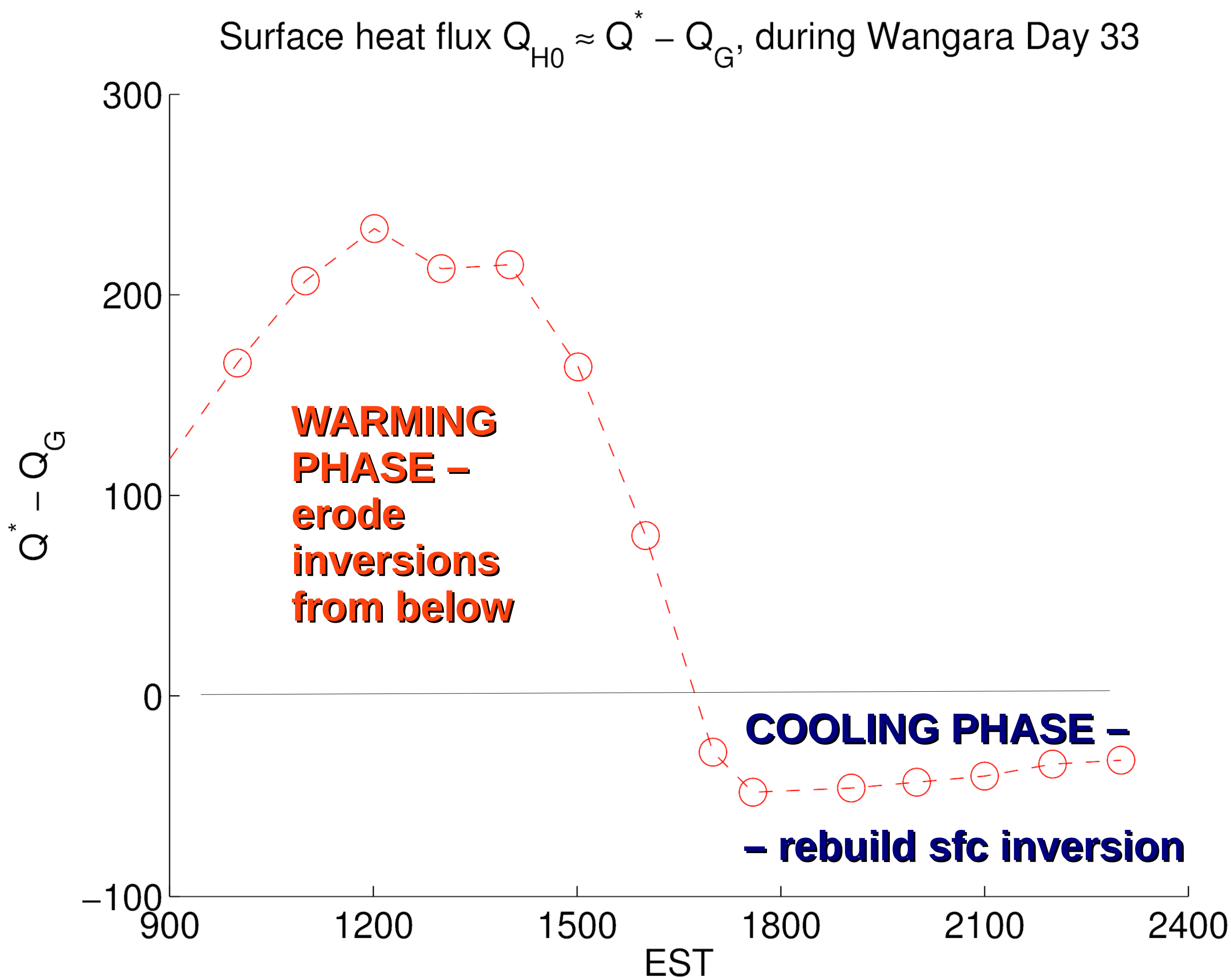
Brunt-Vaisala frequency provides a natural time scale (so does Coriolis parameter, but latter not found useful)

Wangara Day 33

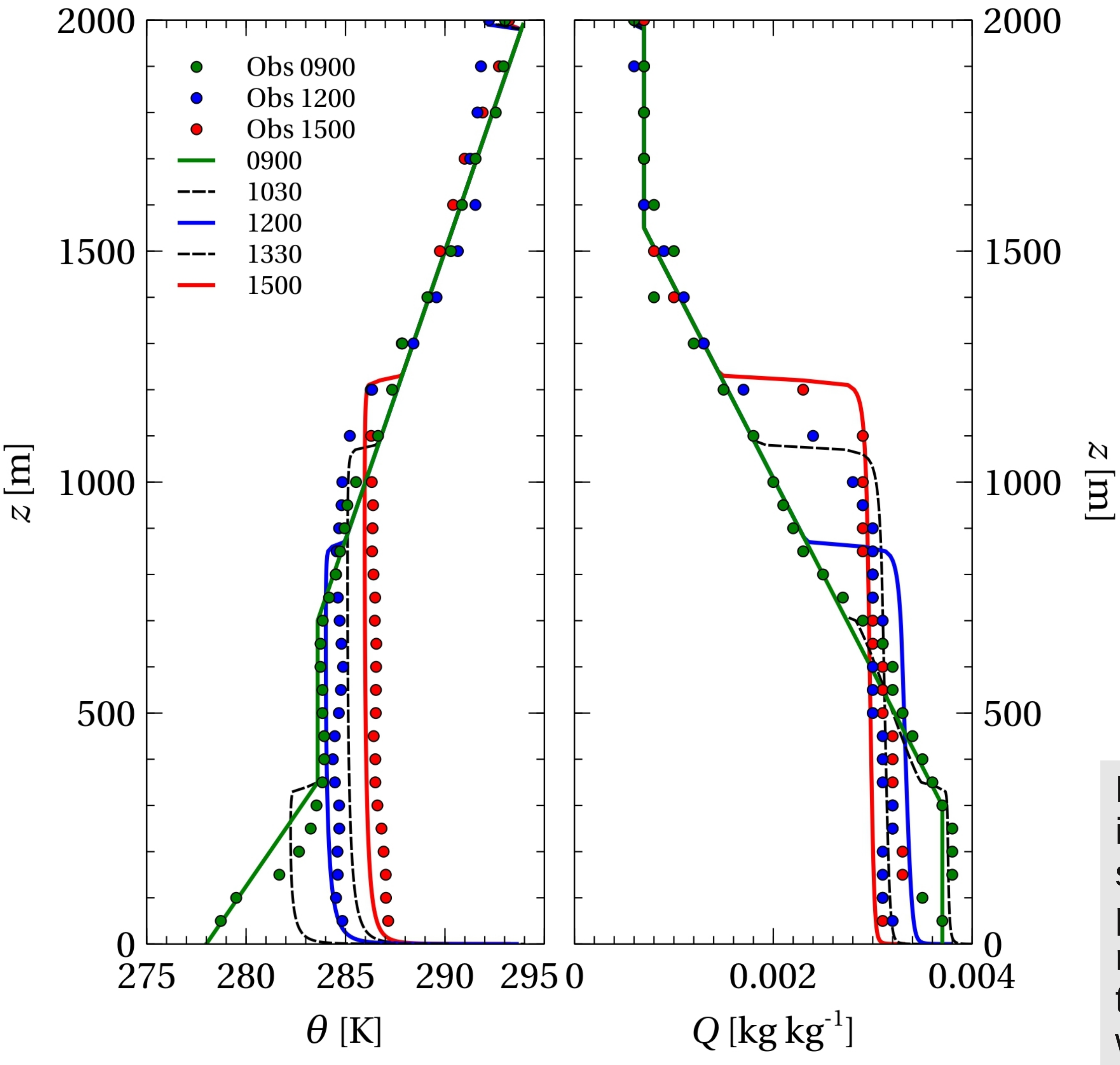
Previously compared with many ABL models

"clear skies, very little horizontal advection of heat or moisture, and lack of any frontal activity within 1000 km" (Deardorff, 1974)

- Late southern hemisphere winter; radiative forcing of modest strength; available energy Q^*-Q_G (net radiation less the soil heat flux density) peaked at a little over 200 W m^{-2} around noon (EST).
- Winds light
- Initialize to profiles observed at 0900



Wangara day 33 – simulated vs. observed profiles of potential temperature and humidity



$C_{BV} = 1$

$\tau_\infty = 10\,\mathrm{min}$

Rate of erosion of capping inversion about right; since sfc heating rate prescribed and depth of mixing about right, model tracks rate of warming without fixing sfc temperature

Conclusion

- This closure had been suggested by Durbin (1991) to treat “wall damping” as $z \rightarrow 0$ (few authors have pursued the idea)

- Can be no less flexible than the $q = k^{1/2}$ type of K-closure

- Therefore has to be *at least* equally useful

- Not necessarily more parsimonious in terms of tuning constants – but if the fundamental idea has the merit of being more *natural* the tuning problem may be easier

- A hint in that regard is that conventional closures are contradictory in regard to dependency of λ on wind shear (through R_i)

- Ginny presently comparing versus a test case (GABLS)