

Atmospheric modelling – decomposition into resolved/unresolved scales of motion

Let $q=q(x,y,z,t)$ be an arbitrary atmospheric variable. We decompose it thus: $q=\langle q\rangle+q'$

where

$$\langle q(x,y,z,t) \rangle = \frac{1}{\Delta_x \Delta_y \Delta_z \Delta t} \int_{-\Delta_x/2}^{\Delta_x/2} \int_{-\Delta_y/2}^{\Delta_y/2} \int_{-\Delta_z/2}^{\Delta_z/2} \int_{-\Delta t/2}^{\Delta t/2} q(x+x', y+y', z+z', t+t') dx' dy' dz' dt'$$

- Δ_x is the zonal gridlength (etc.), Δt is the time step
- apply this averaging operation ("Reynolds averaging") to the equations of motion
- e.g. $\langle \frac{\partial}{\partial x} u w \rangle = \frac{\partial}{\partial x} [\langle u \rangle \langle w \rangle + \langle u' w' \rangle]$
- result is an explicit separation of the resolved and unresolved components
- treatment of transport by the unresolved components can only be statistical, and has to be framed in terms of what is known – that is, in terms of the resolved variables
- will illustrate using the zonal momentum equation

Averaging the zonal momentum equation (in Cartesian coords, under Boussinesq approx.)

$$\frac{\partial u}{\partial t} + \overbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}}^{(\vec{u} \cdot \nabla)u} = \frac{-1}{\rho_0} \frac{\partial p}{\partial x} + f v$$

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} - u \cancel{\nabla \cdot \vec{u}} = \frac{-1}{\rho_0} \frac{\partial p}{\partial x} + f v$$

for simplicity, drop this term (valid to do so under the Boussinesq approx., valid within the ABL)

Apply the Reynolds average:

$$\begin{aligned} \frac{\partial \langle u \rangle}{\partial t} = & - \frac{\partial}{\partial x} [\langle u \rangle \langle u \rangle \langle u'u' \rangle] - \frac{\partial}{\partial y} [\langle u \rangle \langle v \rangle + \langle u'v' \rangle] - \frac{\partial}{\partial z} [\langle u \rangle \langle w \rangle + \langle u'w' \rangle] \\ & - \frac{1}{\rho_0} \frac{\partial \langle p \rangle}{\partial x} + f \langle v \rangle \end{aligned}$$

For notational simplicity, let's switch to uppercase for resolved variables...

Averaging the zonal momentum equation – dynamics/physics decomposition

$$\frac{\partial U}{\partial t} = \left[\frac{\partial U}{\partial t} \right]_{\text{dyn}} + \left[\frac{\partial U}{\partial t} \right]_{\text{phys}}$$

$$\left[\frac{\partial U}{\partial t} \right]_{\text{dyn}} = -U \frac{\partial U}{\partial x} - V \frac{\partial U}{\partial y} - W \frac{\partial U}{\partial z} - \frac{1}{\rho_0} \frac{\partial P}{\partial x} + f V$$

$$\left[\frac{\partial U}{\partial t} \right]_{\text{phys}} = - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{v'u'}}{\partial y} - \frac{\partial \overline{w'u'}}{\partial z}$$

"Friction" is (formally) the divergence of the unresolved momentum flux

$$F_u = - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{v'u'}}{\partial y} - \frac{\partial \overline{w'u'}}{\partial z}$$

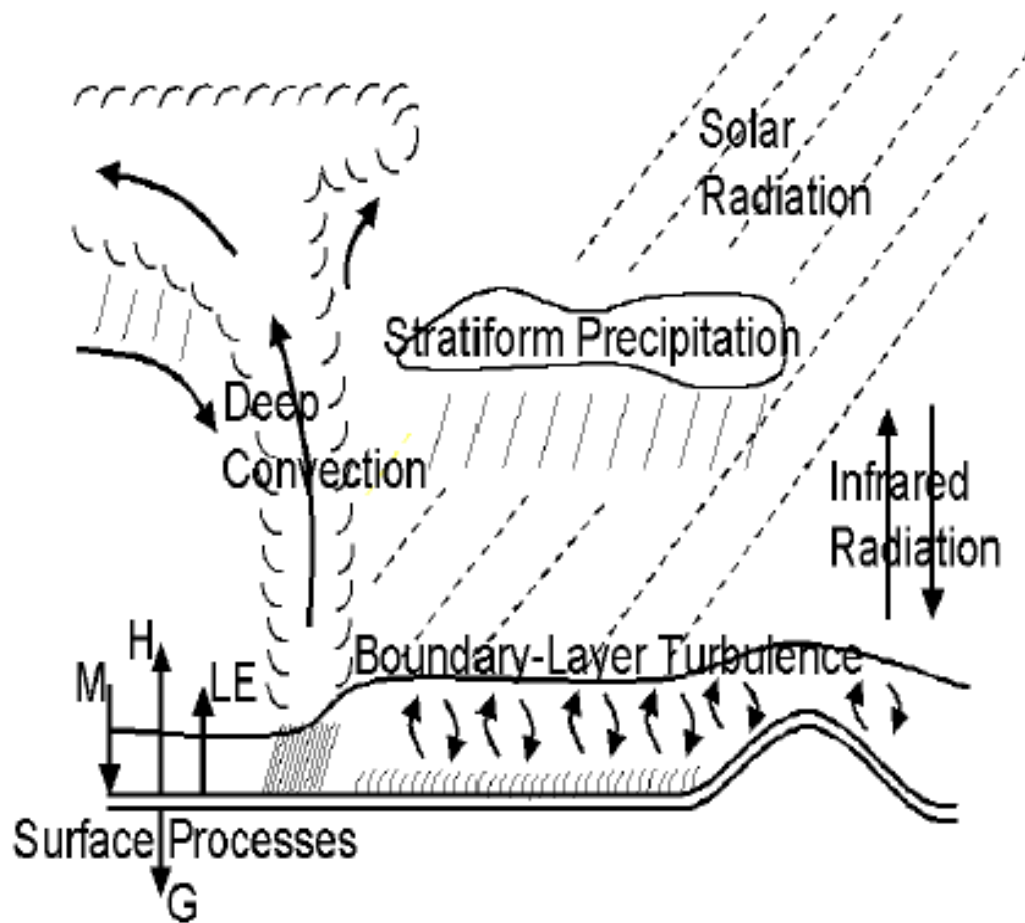
these terms usually neglected in NWP

vertical gradient of the mean vertical flux of u -momentum carried by the unresolved scales of motion

Similarly, the tendency in potential temperature lumped into "physics" is

$$\left[\frac{\partial \bar{\theta}}{\partial t} \right]_{\text{phy}} = - \frac{\partial \overline{w'\theta'}}{\partial z} - \frac{\partial R}{\partial z}$$

Atmospheric modelling – parameterization of cloud & precip



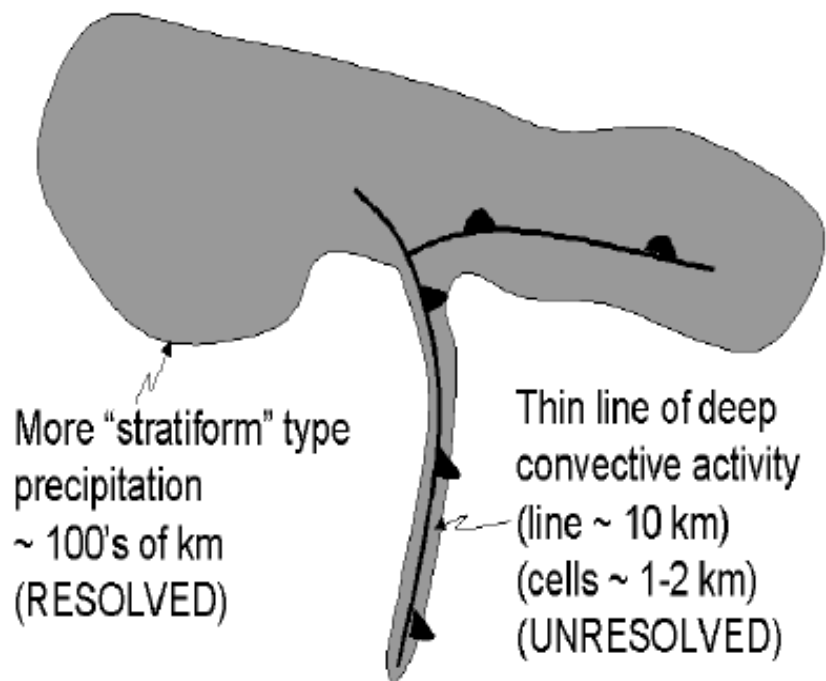
(from Physical Parameterizations in Canadian Operational Models, S. Bélair)

Molinari (1993) defines mesoscale models as hydrostatic models with $10 \leq \Delta x \leq 50$ km

Thus both the Global (25 km) and Regional (10 km) runs of CMC's Global Multiscale Environmental NWP model are mesoscale models

“At a grid spacing of 10 km, the grid scale approaches the preferred scale for instability of convection in nature.”
(Molinari, 1993)

Resolvable versus unresolvable cloud & precip



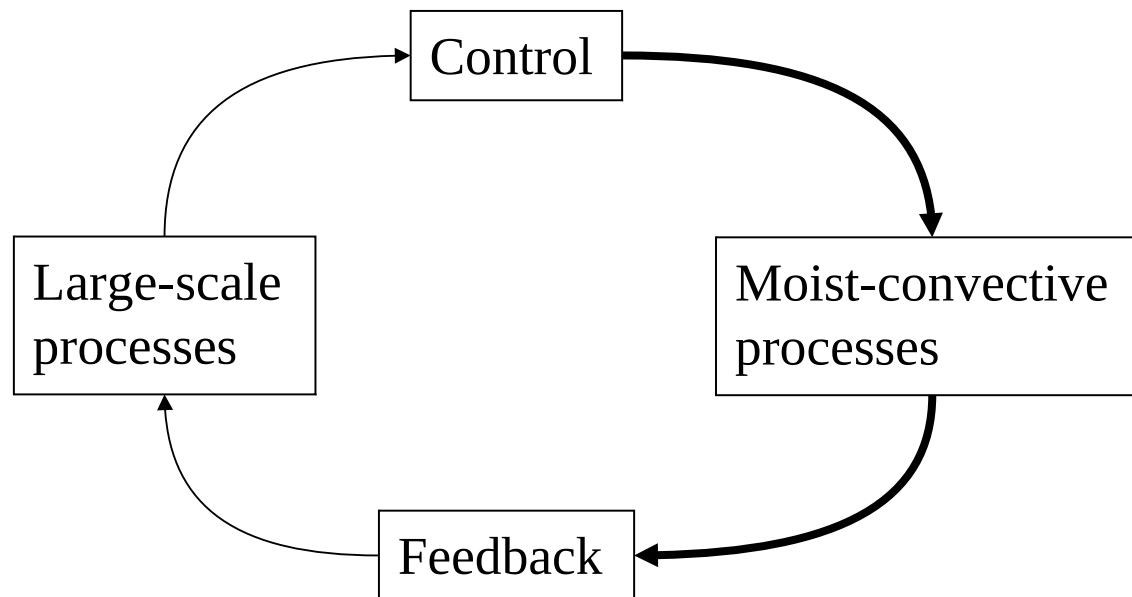
"Use of Primitive Equation models in NWP has made parametrization of the role of convection essential: for otherwise the lapse rate may become unstable during a numerical forecast, and intense (and false) synoptic scale vertical velocities can develop and ruin the large-scale forecast" (Simard and Girard, CMC)...

... but while correcting unstable model lapse rates is a compelling reason in and of itself, obviously sub-grid cloud needs to be accounted for - and is essential if we wish to have quantitative NWP guidance on precip & severe weather

What does “parameterization” mean?

Most individual clouds are sub-grid scale: must formulate the statistical behaviour and collective effects of subgrid-scale clouds in terms of prognostic variables of grid scale (paraphrased from Arakawa, 1993)

Wish for : universal formulation (impossible), valid over some well-defined range of grid lengths Δx . But in practice there are “schemes for large scale models”, and “schemes for mesoscale models” (Arakawa 1993)

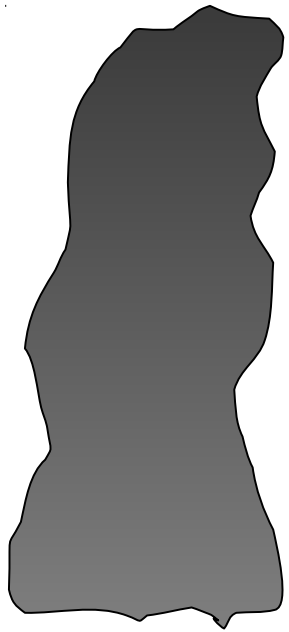


Formulation of the path connected by heavy curves is the purpose of “cumulus parameterization” (Arakawa 1993)

Scale interaction: control and feedback (convective clouds)

Large scale Control

- stratification
- convergent flow
- vertical motion
- humidity



“The premise underlying all physical parameterization is that some aspect of the microscale chaotic process is in statistical equilibrium with the macroscale system” (Emanuel, 1993)

Convective parameterization “requires in principle a spectral gap between scales being parameterized and scales being resolved on the grid” but in practise grid-scale and subgrid scale processes may be inseparable and there is a danger of “double counting the same effect” of parameterized and resolved condensation (Arakawa, 1993).

Feedback onto large scale field

- subsidence (compensates cloud updraft)
- detrainment (mixing) of cloud air with environment
- evaporation of falling precip

A fast developing scientific field – and the proliferation of schemes

“Over the past three decades, significant effort has been devoted to improving our understanding of the interaction between cumulus convection and larger-scale circulations and to modeling such interaction in various approaches of cumulus parameterization. Despite these efforts, a general theory of cumulus parameterization does not exist, and no one single scheme is found to outperform other schemes consistently in a wide range of weather situations.” (Kuo et al. 1996)

“Imperfectly represented cloud processes may interact with the larger-scale model in unrealistic ways that are not permitted with the simpler (convective) adjustment schemes” (Anthes 1977)

“Many efforts are currently under way to unify the representation of low-level convective clouds...There is no a priori correct number of cloud schemes that have to be used to represent clouds in an atmospheric model.” (Belair et al. 2005) . Belair et al. (of Cdn Meteorol. Centre) report “realistic representation of the wide range of clouds that were observed during a large-scale weather event over the Pacific” using GEM in a configuration – now adopted operationally for the four daily “Regional” runs – for “global medium-range weather forecasting with grid sizes on the order of 30–35 km” (GEM REG now 10 km, 5 min)

GEM uses *the combination of four schemes*: “MoistTKE” for boundary layer clouds; Kuo Transient scheme for overshooting shallow cumulus; Kain-Fritsch scheme for deep convection; and a treatment for non-convective clouds (occurring in unconditionally stable layers)

An aside on GEM**

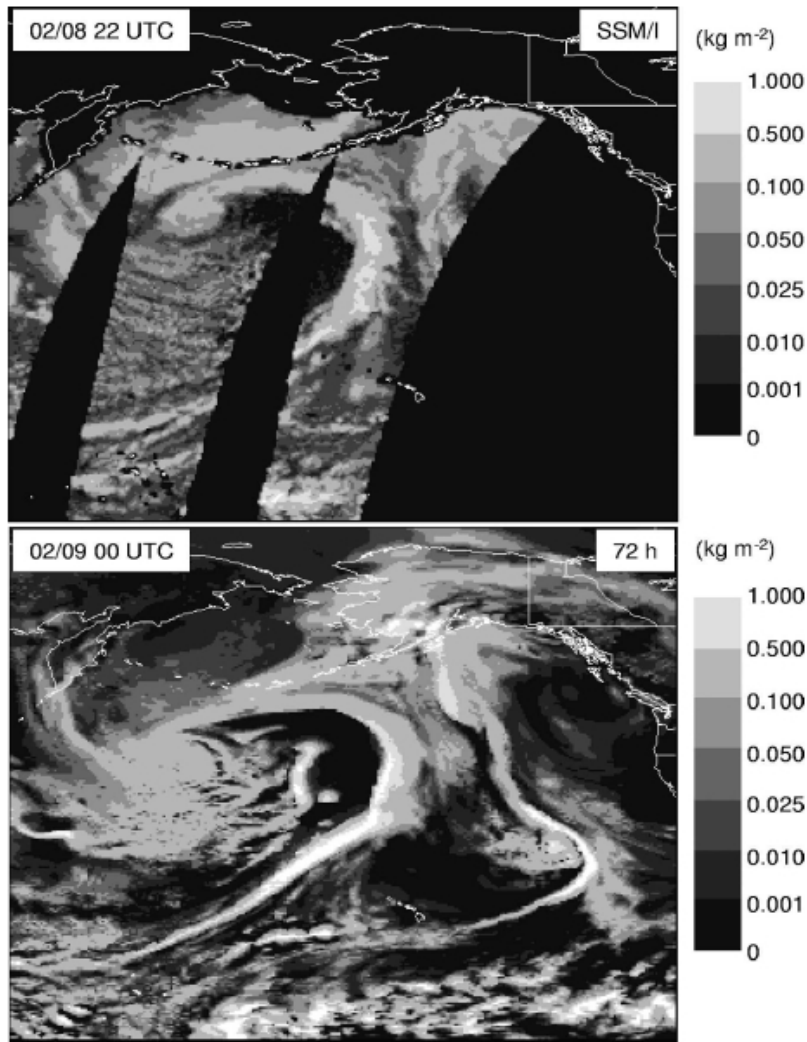


FIG. 5. Vertically integrated liquid cloud water (in kg m^{-2})
 Upper – satellite measurement
 Lower – GEM 72 h forecast: cloud water includes contributions from all four cloud schemes. (**Belair et al. 2005)

TABLE 1. Summary of the GEM forecast system. as of Feb. 2014

| Dynamics/numerics | Global run |
|--|---|
| <ul style="list-style-type: none"> ● Hydrostatic primitive equations; ● Global uniform resolution of 0.45° longitude and 0.30° latitude (1024x800); ● Variable vertical resolution with 80 levels; model top at 10 hPa; ● Time step of 720 s (i.e. 12 min); ● Cell-integrated finite-element discretization on Arakawa C grid; ● Terrain-following hydrostatic pressure vertical coordinate; ● Two-time-level semi-implicit time scheme; ● 3D semi-Lagrangian advection; ● ∇^6 horizontal diffusion on momentum variables; increased horizontal diffusion (sponge) for the four uppermost levels; ● Periodic horizontal boundary conditions; ● No motion across the lower and upper boundaries. | |
| | Physics |
| | <ul style="list-style-type: none"> ● Planetary boundary layer based on TKE with statistical representation of subgrid-scale cloudiness (MoisTKE); ● Fully implicit vertical diffusion; ● Stratified surface layer, distinct roughness lengths for momentum and heat/humidity; ● Four types of surface represented: land, water, sea ice, and glaciers; ● Solar/infrared radiation schemes with cloud-radiation interactions based on predicted cloud radiative properties; ● Kuo Transient scheme for shallow convection; ● Kain–Fritsch scheme for deep convection; ● Sundqvist scheme for nonconvective condensation. |

CMC's RDPS (Regional Deterministic Prediction System) as of 3 Oct. 2012

launched at [00,06,12,18] Z+2:00, with forecast range to 48 hours (or 54 hours)

core is "REG-LAM", run on limited area domain with **uniform $\Delta=10$ km** gridlength

global 4D-Var analysis every 6 h provides initial conditions for REG-LAM

global driving model ("REG-DRIVER" 25 km) provides lateral boundary conditions

80 levels (7 or more below 850 hPa)

timestep **5 min**

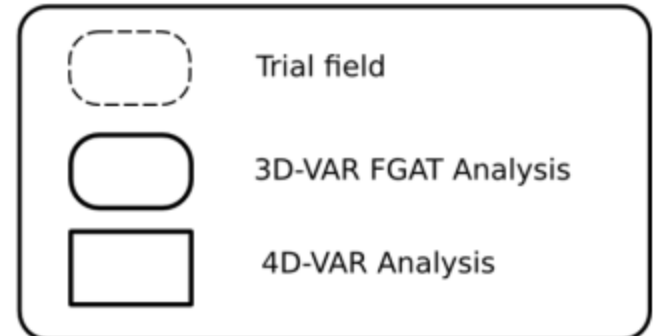
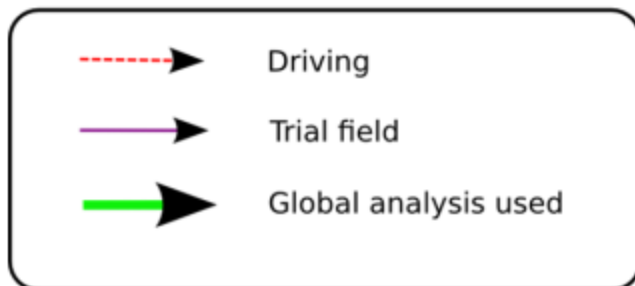
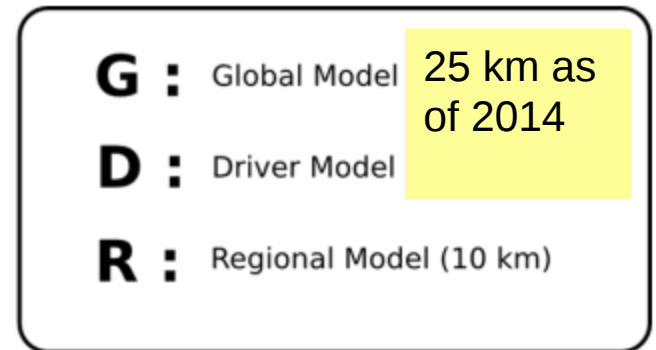
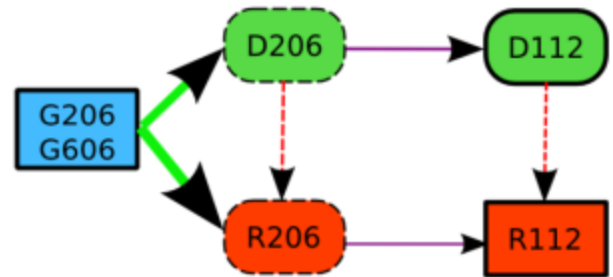


Figure 1 : Flowchart of the RDPS cycle

Domain of the limited area REG-LAM implementation of GEM ($\Delta=10$ km)

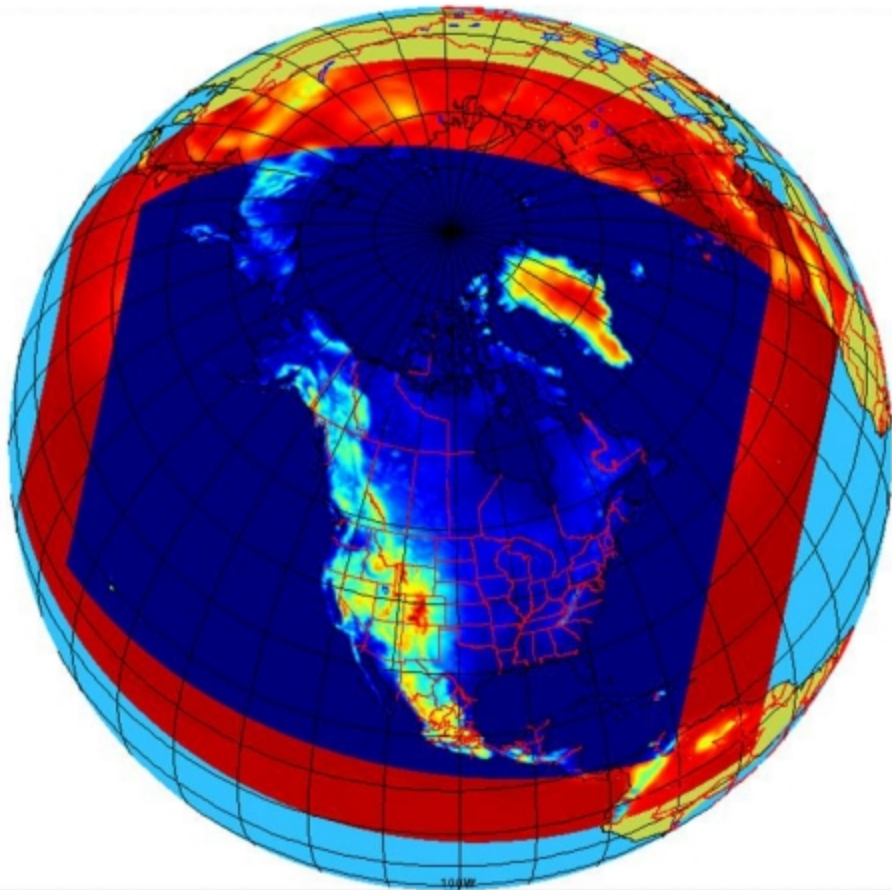


Figure 2: 10 km regional model grid (dark blue) and the tangent-linear/adjoint model grid at 100 km horizontal resolution (red)

- non-convective large scale condensation – condensation assumed to occur when air is supersaturated on the grid (i.e. resolved) scale
- moist-convective adjustment (eg. Manabe et al., 1965) – moist convection assumed to occur where air is conditionally (or absolutely) unstable and supersaturated, at grid scale. Temperature and humidity are adjusted (non-locally) to saturated, moist adiabatic state, subject to energy being conserved in sum across the cloud layer... criticism: *requires* grid scale saturation before invokes subgrid moist convection. Many refinements (eg. Betts and Miller, 1986)
- cloud-model schemes (prototype, Kuo, 1974). Kuo scheme was extensively used in large scale models (ie. *not* mesoscale) and is covered below. Emanuel (1993) states: “one of the earliest and most enduringly popular schemes... convection is assumed to consume water at the rate it is supplied by the macrofluid system... violates causality... convection is *not* caused by the macroscale water supply.”

“What we eventually need... is a unified cloud parameterization, covering deep, shallow, high, low, cumuliform, and stratiform clouds with and without mesoscale organization”
(Arakawa, p15, 1993)

Non-convective large scale condensation

- Prior to the application of the adjustment scheme a model layer $z_B - z_T$ is supersaturated (specific humidity $q > q^*$) but unconditionally stable
- Adjust T, q by amounts $\delta T(z) > 0$, $\delta q(z) < 0$ subject to the constraints:

$$-L \rho \delta q(z) = \rho c_p \delta T(z), \quad (\delta T > 0)$$

$$q(z) + \delta q(z) = q_*(T + \delta T, p)$$

where L is the latent heat of vapourization. There is no vertical energy transport (thus “non-convective” local condensation) so energy is locally exchanged between latent and sensible form.

Dry convection

- Prior to the application of the adjustment scheme a model layer $z_B - z_T$ is unsaturated (specific humidity $q < q^*$) but unstable ($\gamma < \gamma_d = -g/c_p$). Adjust the temperature by $\delta T(z)$ throughout the layer to obtain a neutral lapse rate, subject to the constraint

$$\int_{z_B}^{z_T} \rho c_p \delta T(z) dz = 0$$

Moist-convective adjustment

- The model layer is supersaturated (specific humidity $q > q^*$) and conditionally or unconditionally unstable ($\gamma < \gamma_m$). Readjust by amounts $\delta T(z) > 0$, $\delta q(z) < 0$ to obtain (saturated) neutrality ($\gamma = \gamma_m$), subject to the constraints

$$- \int_{z_B}^{z_T} L \rho \delta q(z) dz = \int_{z_B}^{z_T} \rho c_p \delta T(z) dz$$

$$q(z) + \delta q(z) = q_*(T + \delta T, p)$$

- This is solved numerically by successive approximations. When condensation occurs, the resulting precipitation is

$$P \text{ [kg m}^{-2}\text{]} = - \int_{z_B}^{z_T} \rho \delta q(z) dz$$

and the latent heat is released instantly to the layer.

It has been common for modellers to adjust the humidity threshold for condensation in order to tweak this scheme. The “Manabe scheme” was the first widely used convective adjustment procedure; a more recent convective adjustment scheme is that of Betts and Miller (1986)

Kuo cloud model scheme (deep convection scheme)

The classic scheme, universally employed for models with gridlength order 100 km or more; problematic at modern resolution (is supplemented in GEM by a “Kain-Fritsch scheme”).

- cumulus convection exists only in the presence of deep, conditionally unstable layer in which there is low level convergence and (resulting) net moisture convergence

- moisture supply is sum of large-scale convergent advection of vapour + surface evap'n

scheme instantaneously vertically-redistributes water, and releases latent heat due to condensation; computes cloud fraction (“ μ ”) in the column over the grid square; remixes model layers; produces precipitation

- simplistic computation of cloud location and state: lifts a surface parcel along dry adiabat to LCL; above the LCL, ascent continues along a moist-adiabat slightly modified by entrainment. The top of the cloud layer is the level of non-buoyancy

Kuo cloud model scheme

If μ is the fractional area of sky that is covered by deep cumulus clouds, then the temperature at level z after the dissolution of the cloud (mixing) will be

$$T(z) = \tilde{T}(z) + \mu \left(T_c(z) - \tilde{T}(z) \right)$$

where \tilde{T} is the environmental temperature prior to the mixing of the cloud air, and $T_c(z)$ is the temperature in the cloud.

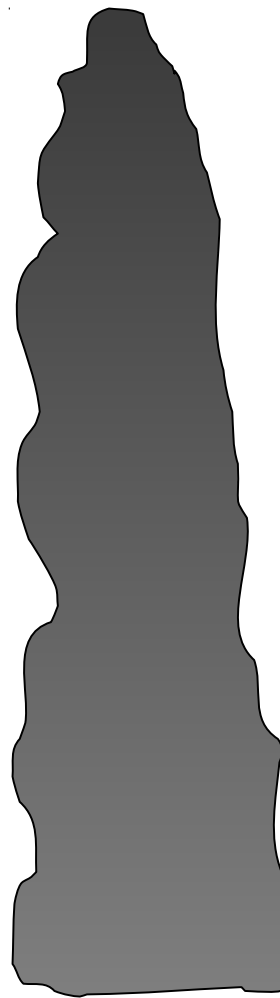
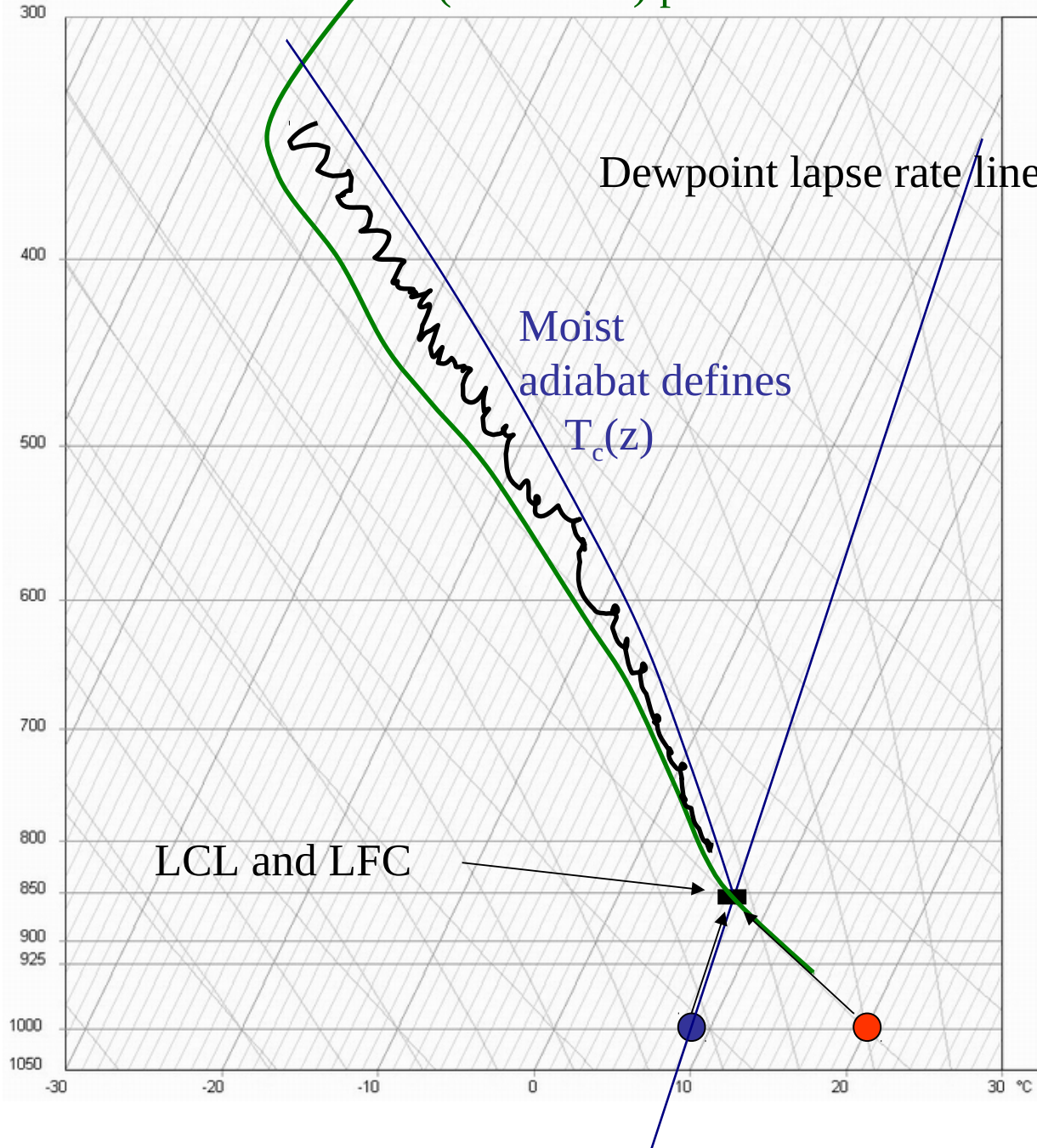
Thus scheme must diagnose μ and $T_c(z)$ for each grid column over which deep convection is inferred to be occurring

Kuo cloud model scheme – cloud properties

- cloud base (z_B) presumed to be the lifting condensation level (LCL) of surface air (in the no longer used CMC Spectral Model, the LCL was computed by assuming a parcel from the surface arrives at the top of boundary layer carrying the height-average properties of the boundary-layer, ie. boundary-layer mean temperature and humidity)
- within the cloud, profiles of temperature $T_c(z)$ and specific humidity $q_c=q^*(T_c)$ are presumed moist adiabatic (profiles of T_c and q_c for the presumed cloud are therefore readily calculated)
- cloud top (z_T) occurs where this moist adiabat from the lifting condensation level crosses the model “sounding” (defined prior to application of the scheme).
- complete and instantaneous (i.e. during the time step) mixing (cloud dissolution) of the cloud, level-by-level, with the environment is assumed

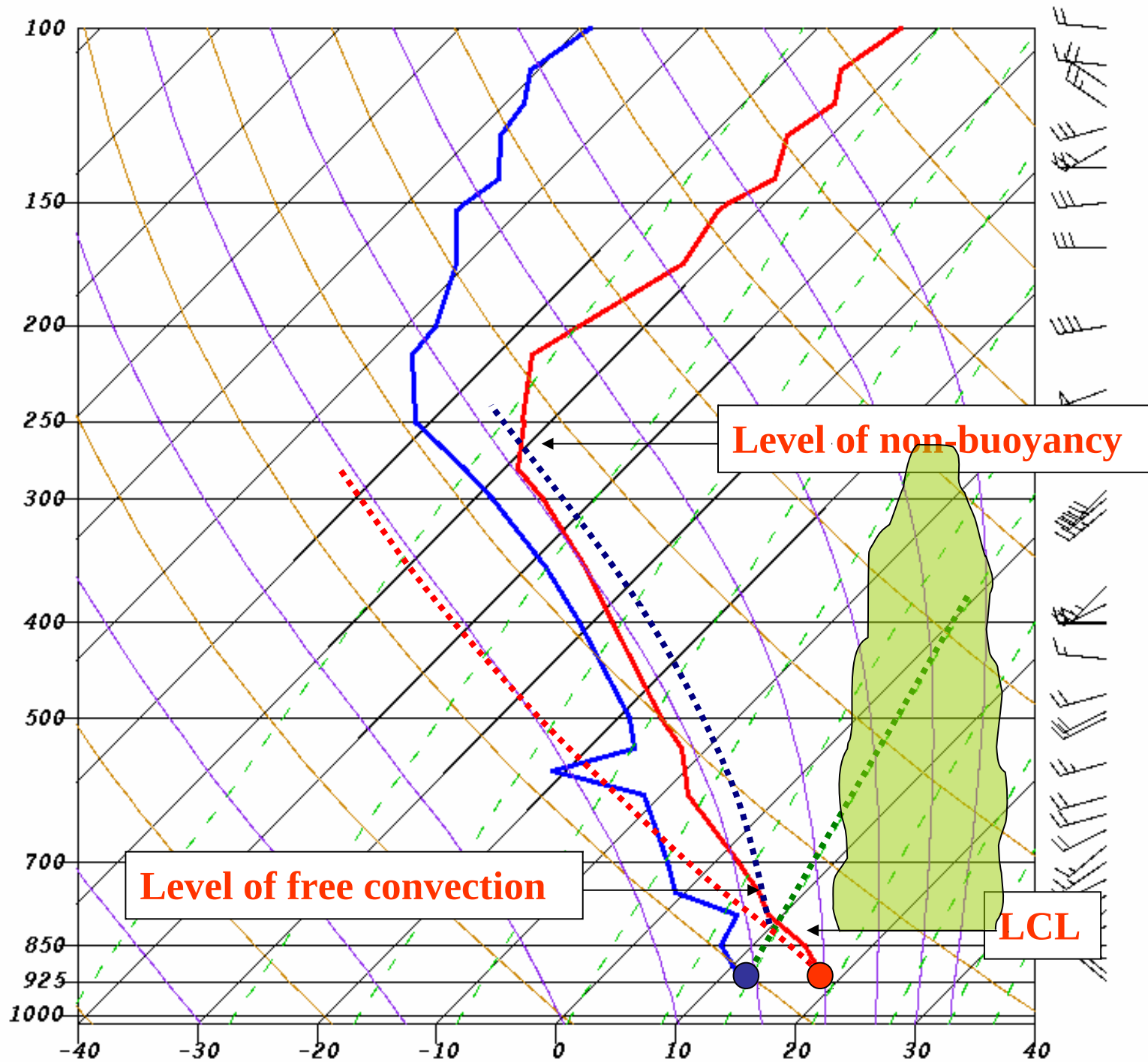
Kuo cloud model scheme – cloud properties

(schematic) pre-correction model sounding



(more typically, LFC is above LCL)

Kuo cloud model scheme – cloud properties



Kuo cloud model scheme – moisture available

Recall that if the low level flow is convergent then the horizontal divergence

$$D = \nabla_H \cdot \vec{V}_H$$

is negative ... area on the constant pressure surface is “shrinking.” The integral

$$M_t = - \int_0^\infty \nabla_H \cdot \left(\rho q \vec{V}_H \right) dz + E_0$$

Circled term is the convective flux density of water vapour carried by the resolved horizontal flow

gives the total rate [$\text{kg m}^{-2} \text{s}^{-1}$] at which water vapour becomes available to the vertical column above unit ground area. Here E_0 is the surface evaporation rate. Kuo speaks of “control of the water vapour supply (for tropical storms) by the low level mean flow field,” presumably meaning that both vapour density and the flow convergence are numerically largest near the ground/ocean (maximum cross-isobar flow).

Multiplying M_t by the model timestep we have the amount $M_t \Delta t$ of vapour (per unit ground area) available to “make” cloud columns from environmental air.

Kuo cloud model scheme – moisture needed

In order to form a cloud column spanning $z_B - z_T$ it is assumed we raise the temperature from environmental temperature T_e (given by the model field \tilde{T} before application of the cloud parameterisation scheme) to T_c (known, see above) by condensing water vapour (it is also assumed that all this condensed water is precipitated out); the amount of water vapour needed per unit ground area is easily calculated as

$$W_1 \text{ [kg m}^{-2}\text{]} = \frac{1}{L} \int_{z_B}^{z_T} \rho c_p \left(T_c(z) - \tilde{T}(z) \right) dz$$

and is to be drawn from the “accession flux” M_t . In addition to this “condensing part,” there is a non-condensing “humidification part,” that raises the humidity of the cloud column to saturation,

$$W_2 \text{ [kg m}^{-2}\text{]} = \int_{z_B}^{z_T} \rho (q_c - \tilde{q}) dz$$

Kuo cloud model scheme – fractional sky cover

The dimensionless ratio

$$\mu = \frac{M_t \Delta t}{W_1 + W_2}$$

is the ratio of the amount of vapour available (the supply) to the amount of vapour needed for cloud formation, over timestep Δt . If the timestep is sufficiently small, we can ensure $\mu < 1$ (moisture supply too small relative to the required moisture for the cloud column); then μ can be considered “the fractional area of the sky that is covered by newly formed cumulus cloud as a result of the accession of moisture by advection and by evaporation from below.” And μ is used in the above-suggested manner to correct the forecast. Kuo suggested μ be interpreted only “somewhat figuratively” as the fractional cloud cover; and he argued that even if Δt is sufficiently large that μ exceeds unity, the correction procedure is still valid.

Thus if \tilde{T} , \tilde{q} are the forecast temperature and specific humidity for the end of the timestep *without* allowing for the effects of cumuli, then the cumulus-corrected temperature and humidity are:

$$T = \tilde{T} + \mu (T_c - \tilde{T})$$
$$q = \tilde{q} + \mu (q_c - \tilde{q})$$

Kuo cloud model scheme – precipitation

The precipitation is that part of the moisture used in warming the air from \tilde{T} to T , so the mass of precipitation falling out of the cloud layer on unit area over the timestep is

$$P \text{ [kg m}^{-2}\text{]} = \frac{\mu}{L} \int_{z_B}^{z_T} \rho c_p \left(T_c - \tilde{T} \right) dz$$

It was soon noted that this formulation underestimated the convective precipitation rate and therefore atmospheric warming in the tropics, apparently because an excessive fraction of the moisture accession is used to humidify the column. Kuo later suggested a more-realistic partitioning of the moisture accession.

What “causes” deep convection?

*precursor to Kain & Fritsch scheme

- moisture accession (as posited by Kuo) ?
- low level frictional convergence?
- existence of Convectively Available Potential Energy?...

Fritsch & Chappell note:

- “studies by (others) indicate that frictional pumping is neither a necessary** nor a sufficient cause for the occurrence of cumulus convection”
- thermals are stronger and larger when low-level convergence is present

**since friction always occurs in the real world, how can one *know* it isn't necessary? Maybe by virtue of modelling studies... in which one could turn off friction? But anyway, its totally obvious that one could have intense buoyancy driven convection with no *mean* motion near the lower boundary

- “moist convection only occurs when air is forced to its LFC by low-level convergence, air mass overrunning, or when low-level heating and mixing remove any stable layers suppressing moist convection (ie. when potential buoyant energy becomes available)”
- each grid column isolated from all others
- assumes deep convection the dominant cloud form
- recognizes convection responds not only to the rate at which the large scale is generating buoyant energy, but also to the buoyant energy generated and stored prior to the onset of deep convection
- accomplishes a vertical rearrangement of mass & eliminates CAPE, through three mechanisms: moist convective updraft, moist convective downdraft, and a dry branch (ascent or descent) all occurring within the grid cell
- precip efficiency is (empirically) related to wind shear across the cloud depth
- “capable of generating convectively driven mesoscale pressure systems”

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