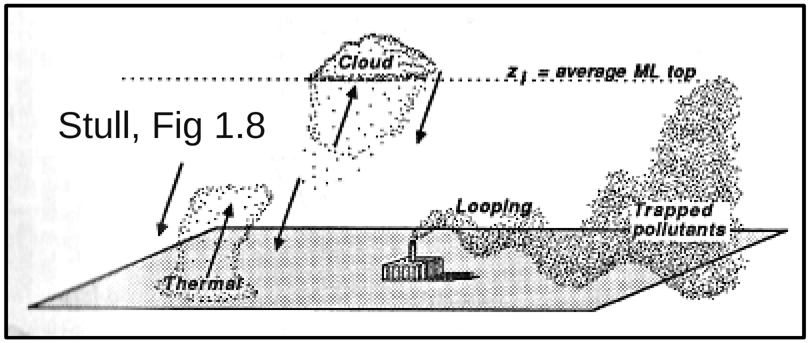
Modelling dispersion in the Convective Boundary Layer – non-Gaussian turbulence

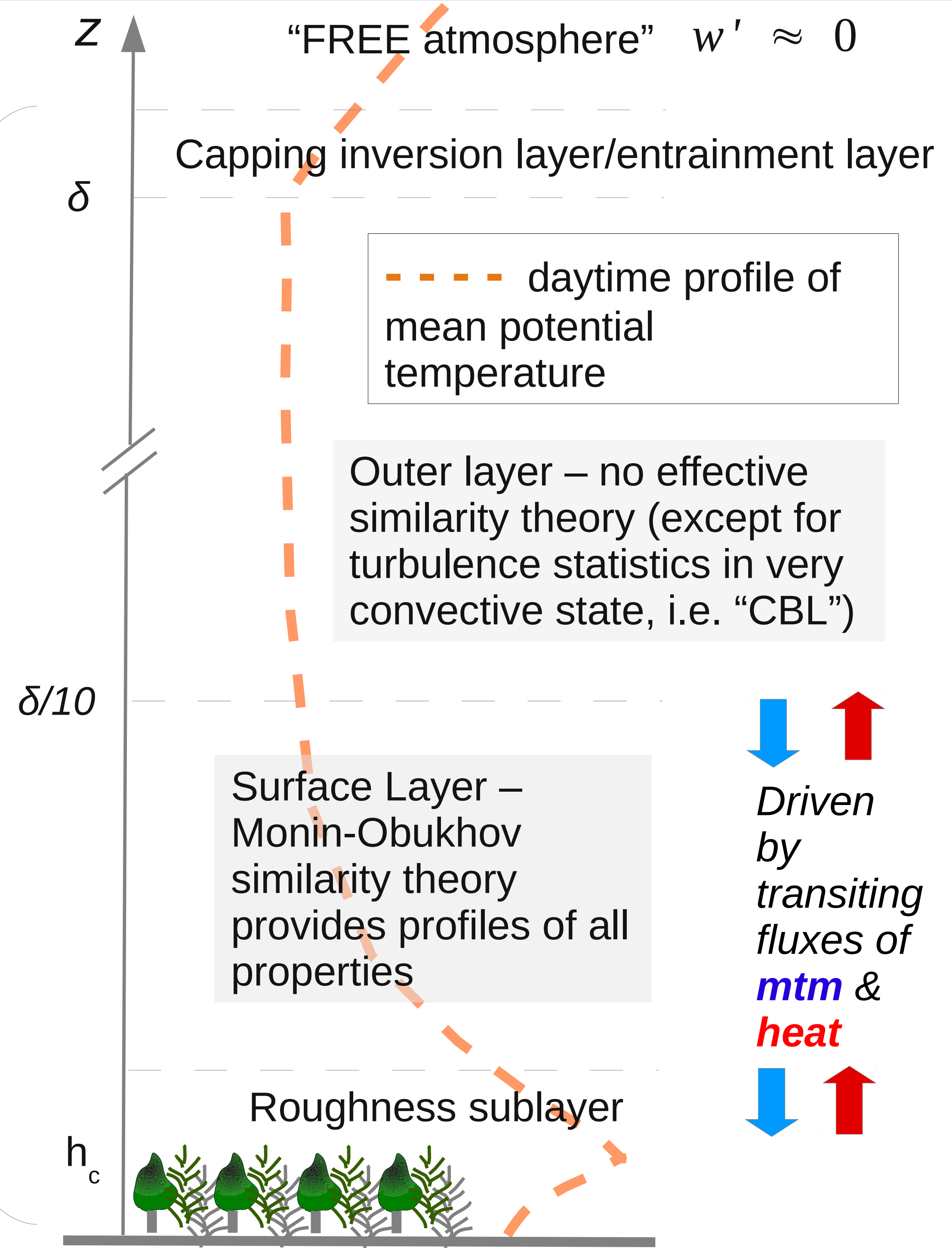


Stull, Fig 1.8

Atmospheric boundary layer
(Friction layer)

Unresolved ("turbulent") velocity fluctuations

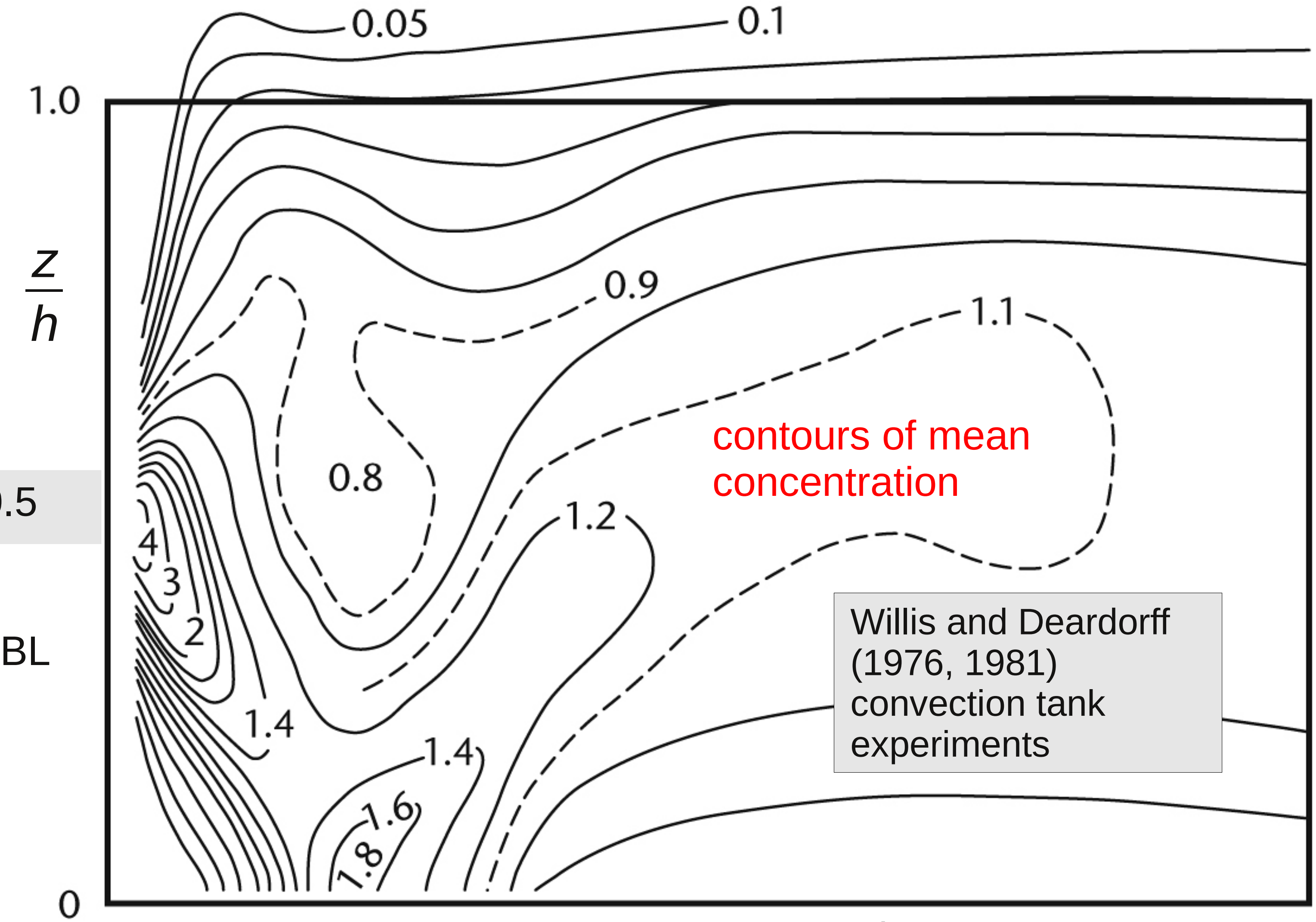
$$u' \approx v' \approx w' = \mathrm{O}[\mathrm{m\ s^{-1}}]$$



eas471_dispersionLS.odp
JDW 13 Mar. 2014

Dispersion in the CBL – non-Gaussian turbulence

• a peculiarity of observed dispersion in the CBL, that Gaussian plume models cannot easily represent, is that the (time-average) plume centreline from an elevated continuous point source initially descends towards ground (with increasing downwind distance), then ascends

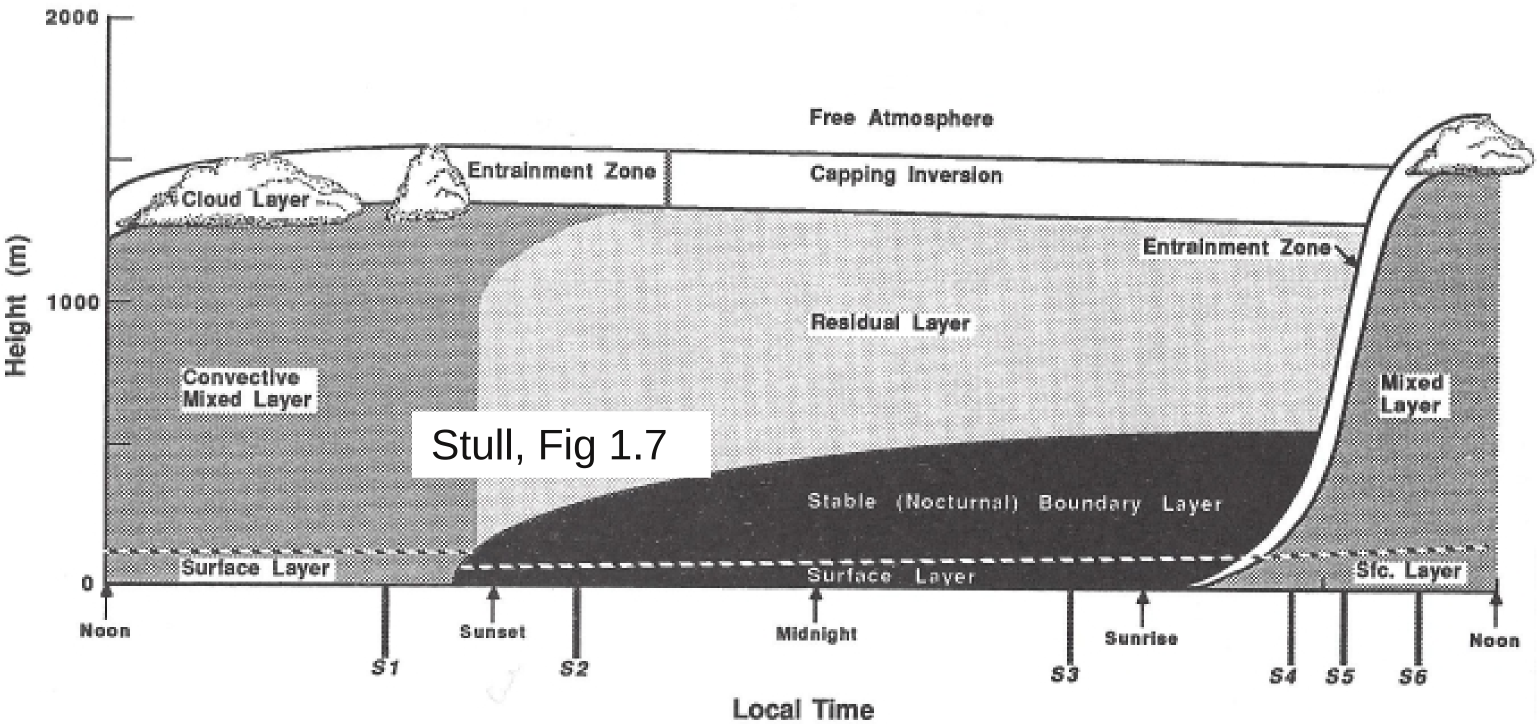


$z_s/h = 0.5$

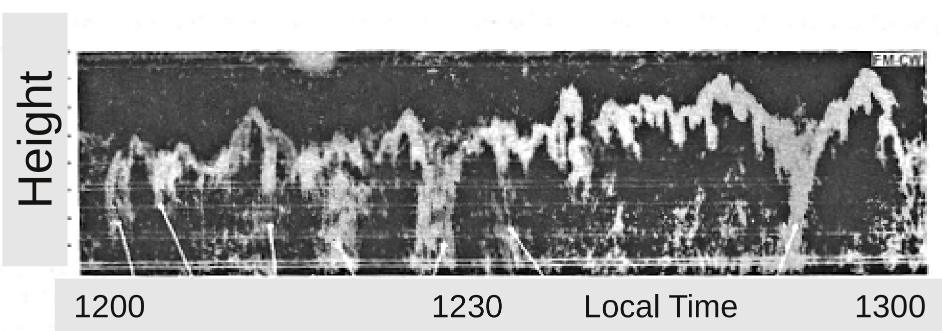
$$X = \frac{x/U}{h/w_*} = \frac{t}{T_L}$$

- *h* (or z_i or δ), ABL depth
- *U*, horiz. wind speed
- *w**, convective velocity scale

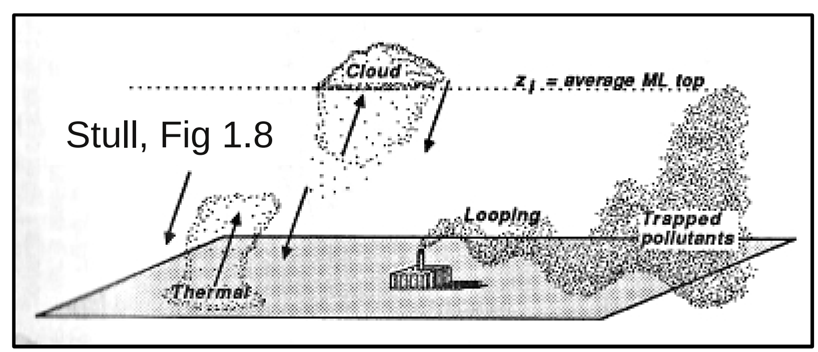
Time evolution of the ABL



Stull, Fig 1.7



Garratt, Fig 6.4



Stull, Fig 1.8

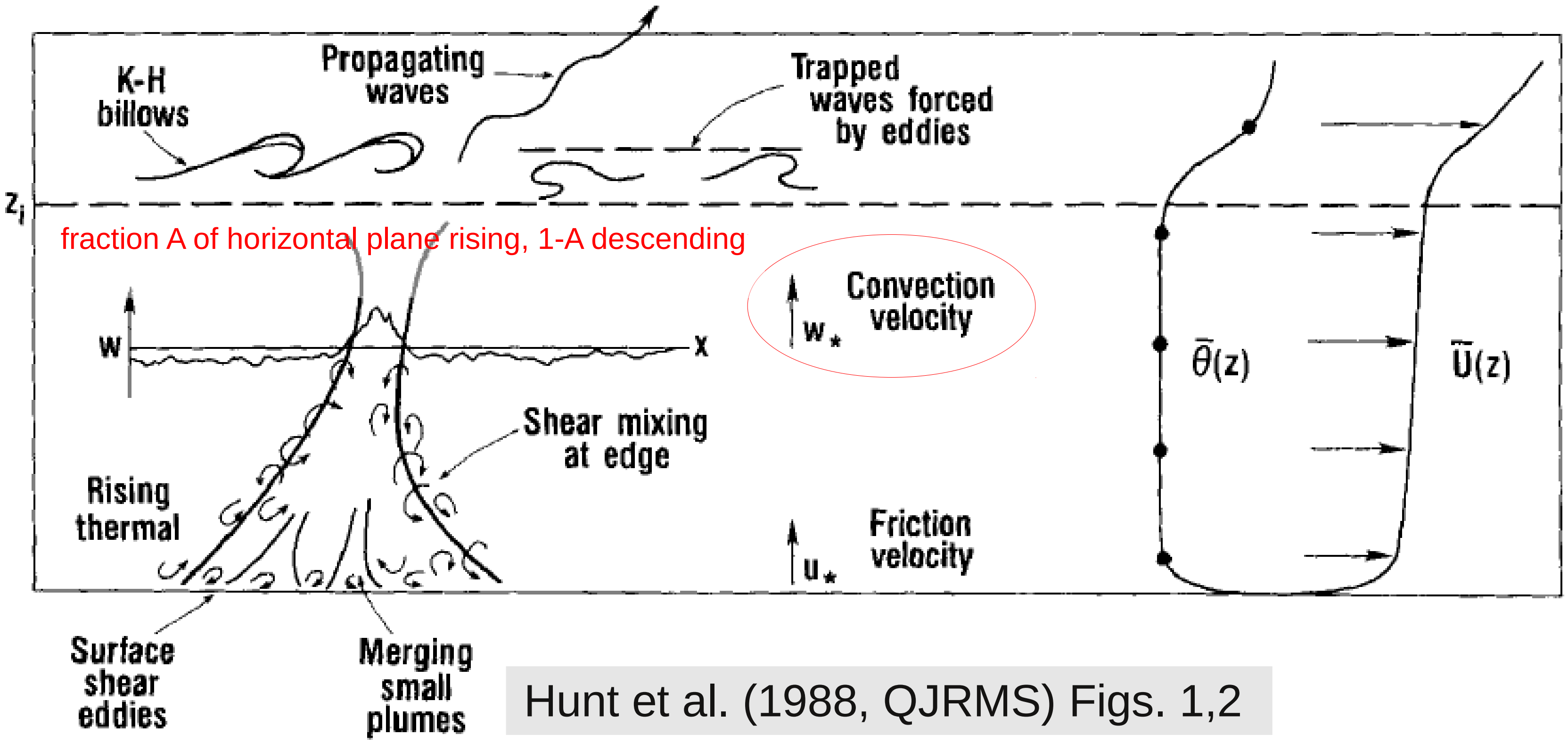
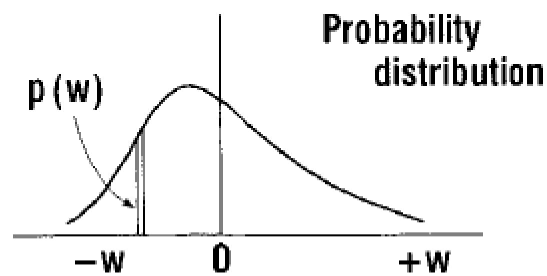
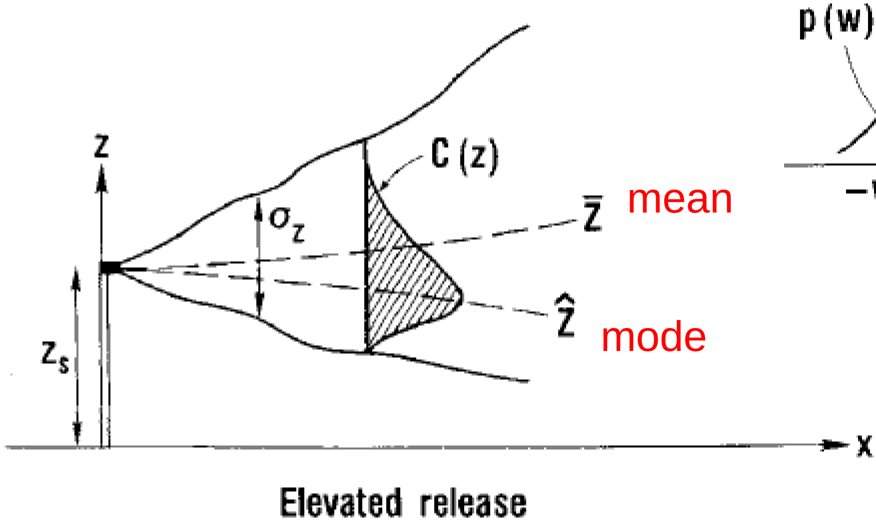
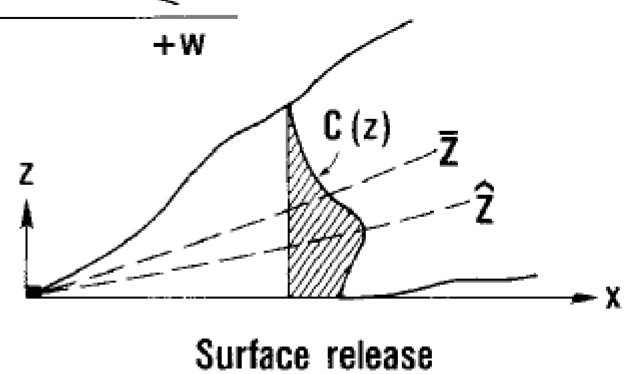


Figure 1. Schematic diagram of the flow, temperature and eddy structure of the convective boundary layer.

- CBL as "windless convection layer"
- skew PDF of *w*

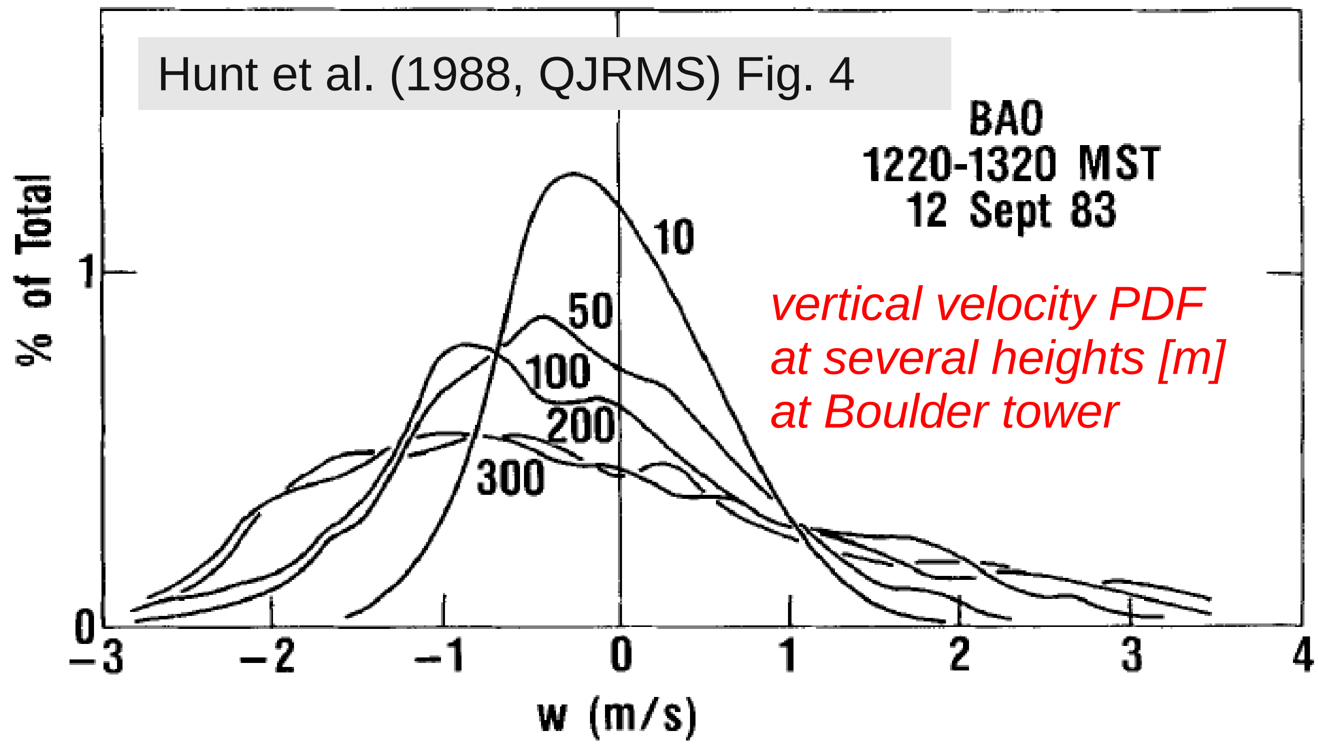


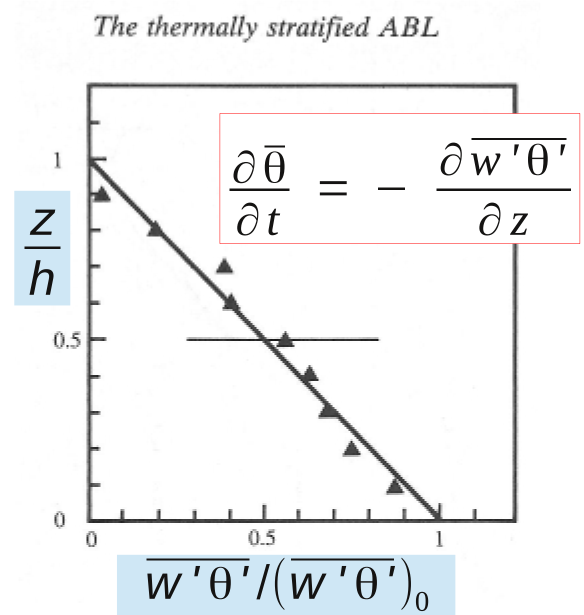




CBL velocity statistics & mixed layer similarity theory

Hunt et al. (1988, QJRMS) Fig. 4





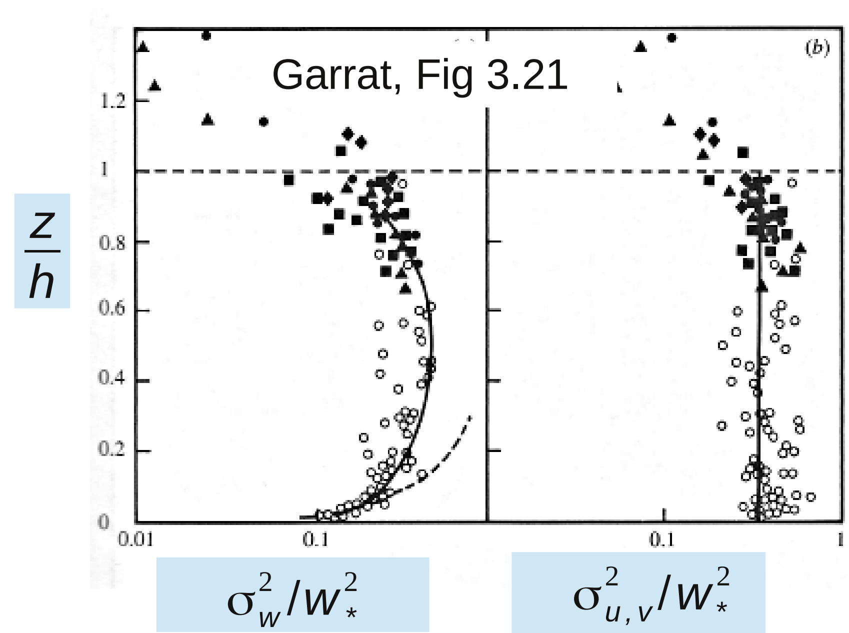
Garratt, Fig 6.13

Mixed-layer turbulence scales: *δ, w*, θ**

$$w_* = \left[\frac{g}{\theta_0}\,\delta\,\left(\overline{w'\theta'}\right)_0\right]^{1/3}$$

$$\theta_* = -\left(\overline{w'\theta'}\right)_0 / w_*$$

Garrat, Fig 3.21



Reynolds-averaged governing equations for ABL

- Decompose every variable into sum of Mean + Fluctuation $w = W + w'$
- Apply the averaging operation to the Navier-Stokes equations → Reynolds' equations

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\left(U^2 + \overline{u'u'}\right) + \frac{\partial}{\partial y}\left(UV + \overline{u'v'}\right) + \frac{\partial}{\partial z}\left(UW + \overline{u'w'}\right) = \frac{-1}{\rho}\frac{\partial P}{\partial x} + fV + \nu\nabla^2 U$$

non-divergence

Viscous friction

Posit horizontal homogeneity of *statistics* – then indicated terms vanish, leaving:

$$\frac{\partial U}{\partial t} = -\frac{\partial \overline{u'w'}}{\partial z} + f\left(V - V_G\right)$$

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial \overline{w'\theta'}}{\partial z}$$

(corresponding heat budget)

“turbulent friction” (divergence of Reynolds stress). Fluctuations *u′* and *w′* are (anti-) correlated – so $\overline{u'w'}$ is a downward flux of *u*-momentum

“Geostrophic deficit” (balances to zero in free atmos) Large scale pressure gradient defines the Geostrophic wind (so $f\,V_G$ is just a substitution)

Preview of the Turbulent Kinetic Energy (TKE) equation

- defined $$k = \frac{1}{2}\left(\overline{u'u'}+\overline{v'v'}+\overline{w'w'}\right) \equiv \frac{1}{2}\left(\sigma_u^2+\sigma_v^2+\sigma_w^2\right)$$

- by manipulating the Navier-Stokes eqns. (under the Boussinesq approx.), the TKE eqn for a <u>horizontally-homogeneous</u> layer is:

$$\frac{\partial k}{\partial t} = \underbrace{-\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z}}_{\text{shear production}} + \underbrace{\frac{g}{\theta_0}\overline{w'\theta'}}_{\text{buoyant prodn}} \underbrace{- \frac{\partial}{\partial z}\overline{w'\left(\frac{p'}{\rho}+\frac{u'u'+v'v'+w'w'}{2}\right)}}_{\text{pressure transport + turbulent transport}} \underbrace{-\ \epsilon}_{\text{viscous dissip'n}}$$

- shear production large in surface layer due to strong near-ground shear
- buoyant production dominates above the surface layer
- at z/L = 1, shear and buoyant production have equal magnitude
- wherever the heat flux is negative (i.e. in inversion layers) buoyancy suppresses turbulence
- the transport term often modeled as "diffusion"
- viscosity damps the motion of the smallest eddies, converting their kinetic energy to heat
- TKE (k) and its dissipation rate (ε) key variables in most turbulence models

Form of the TKE equation for horiz. homog. layer under first-order closure

- invoke

$$\left(\overline{u'w'}, \overline{v'w'}\right) = -K \frac{\partial}{\partial z}(U, V)$$

$$\overline{w'\theta'} = -K_H \frac{\partial \overline{\theta}}{\partial z}$$

$$\epsilon = \frac{(c\, k)^{3/2}}{\lambda}$$

$\lambda=\lambda(z)$ an empirical "length scale", c a tunable constant

$$K = (c\, k)^{1/2} \; \lambda(z)$$ and $$K_H = K / P_r$$

- result:

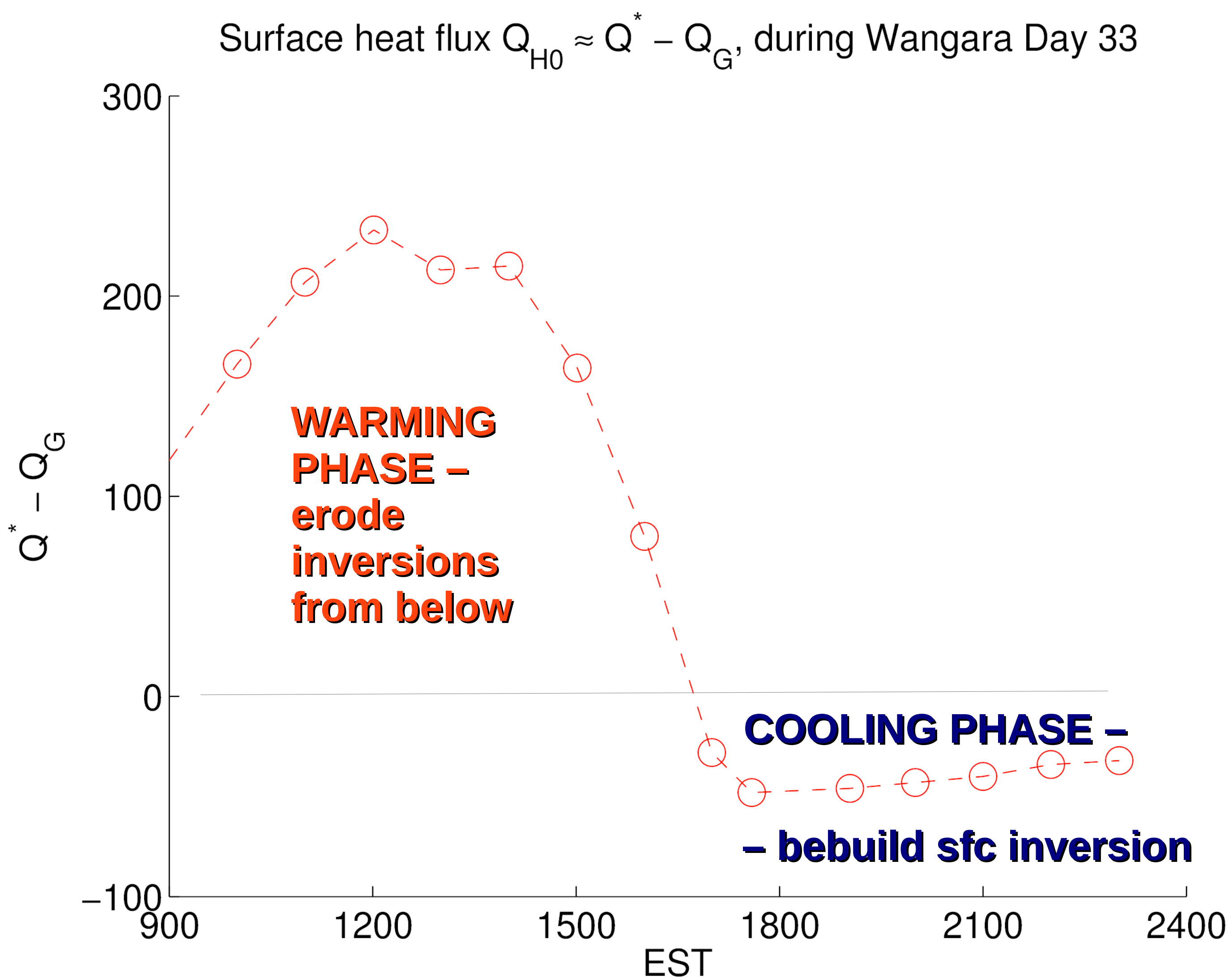
$$\frac{\partial k}{\partial t} = K\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] - \frac{g}{\theta_0} K_H \frac{\partial \overline{\theta}}{\partial z} + \frac{\partial}{\partial z}\left(K_k \frac{\partial k}{\partial z}\right) - \frac{(c\, k)^{3/2}}{\lambda(z)}$$

shear production — buoyant prodn — diffusion — dissipation

Day 33 previously compared with many ABL models

“clear skies, very little horizontal advection of heat or moisture, and lack of any frontal activity within 1000 km” (Deardorff, 1974)

- Late southern hemisphere winter; radiative forcing of modest strength; available energy Q^*-Q_G (net radiation less the soil heat flux density) peaked at a little over 200 W m^{-2} around noon (EST).
- Winds light
- Initialize to profiles observed at 0900

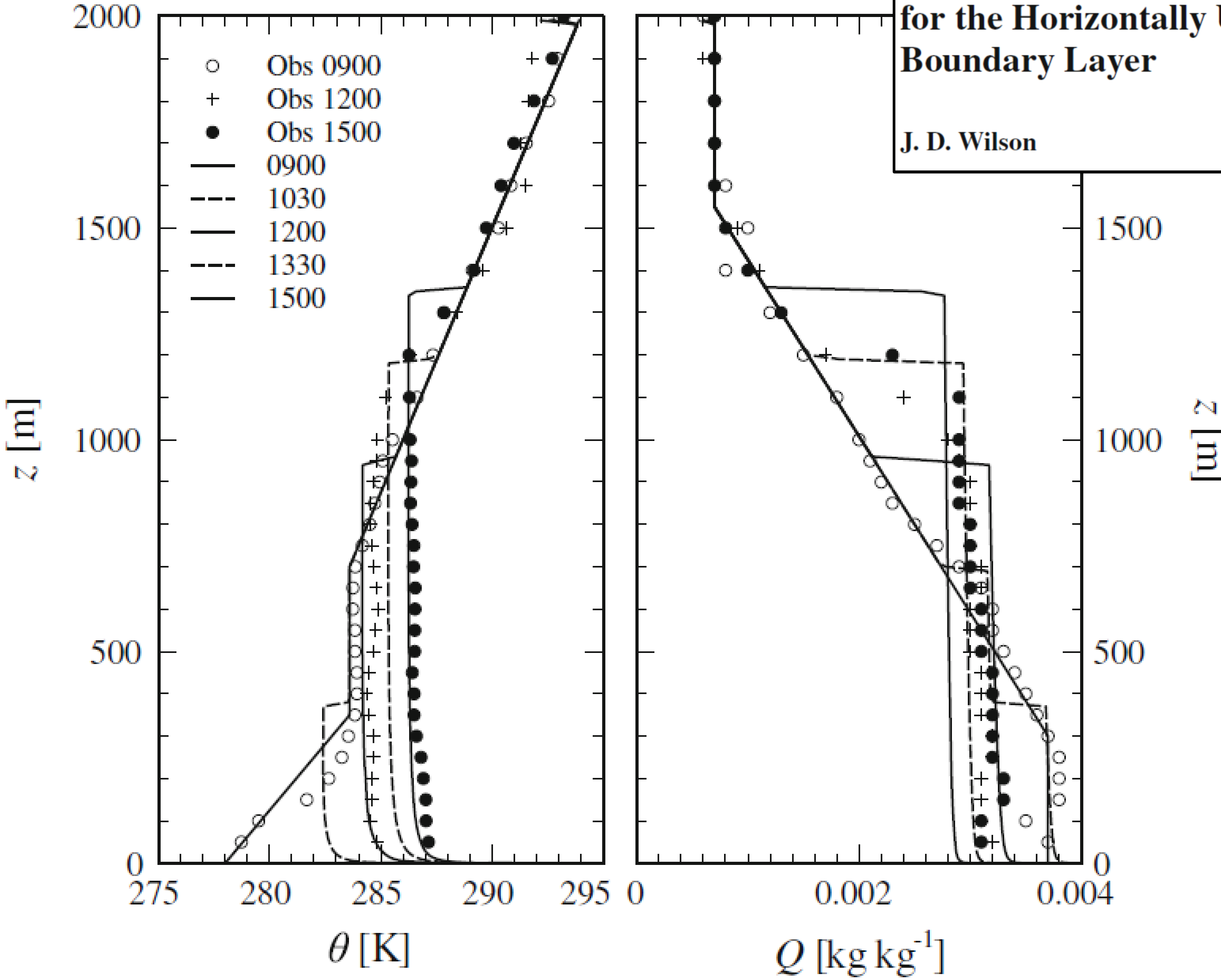


Boundary-Layer Meteorol (2012) 145:165–184
DOI 10.1007/s10546-011-9650-0

ARTICLE

An Alternative Eddy-Viscosity Model for the Horizontally Uniform Atmospheric Boundary Layer

J. D. Wilson



Dispersion in the CBL – well-mixed Lagrangian stochastic model

• ignore specifics of thermals; consider ABL horiz. homog; then, the vertical velocity PDF is skewed (as shown earlier). Ignore the shear in horiz. velocity (constant advection veloc. *U*)

• fractional area (*A*) of "updrafts" (wherein mean velocity is upward, but instantaneous velocity need not be) and complementary fractional area (*B=1-A*), considered the (predominantly) subsiding environmental region. Obsv. give *A* ~ 0.2 – 0.4

• choose a suitable representation of the vertical velocity PDF, e.g.

$$g_a(w;z) = A(z)\, P_A(w;z) + [1 - A(z)]\, P_B(w;z)$$

(Luhar & Britter, 1989, Atmos. Env. Vol. 23) where P_A, P_B are Gaussians (whose moments vary with *z*)... may or may not require *A*=*A*(*z*)

• setting aside the details of "fitting" the parameters of the PDF's, the form of the model is (again)

$$dW = a(Z,W)\, dt + \sqrt{C_0\, \epsilon(z)}\ \ d\xi$$

• applying the well-mixed condition gives $$a = -\frac{\sigma_w^2(Z)}{\tau(Z)}\ \frac{Q(W)}{g_a(W;Z)} + \frac{\varphi(W)}{g_a(Z;W)}$$

Atmospheric Environment Vol. 23, No. 9, pp. 1911–1924, 1989.
Printed in Great Britain.

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A RANDOM WALK MODEL FOR DISPERSION IN INHOMOGENEOUS TURBULENCE IN A CONVECTIVE BOUNDARY LAYER

ASHOK K. LUHAR and REX E. BRITTER

δ = CBL depth
*W** = CBL veloc., scale

$$w_* = \left[\frac{g}{T_0} \left(\overline{w'T'} \right)_0 \delta \right]^{1/3}$$

$$Q(W) = \frac{A}{\sigma_{wA}^2} (W - \overline{w}_A)\, P_A(W) + \frac{1-A}{\sigma_{wB}^2} (W + \overline{w}_B)\, P_B(W)$$

Mean horiz. wind in CBL sometimes treated as const. *U* – a "shearless convection layer"

$$\begin{aligned}
\phi(W) = & -\frac{1}{2} \left(A \frac{\partial \overline{w}_A}{\partial z} + \overline{w}_A \frac{\partial A}{\partial z} \right) erf \left(\frac{W - \overline{w}_A}{\sqrt{2}\, \sigma_{wA}} \right) \\
& + \frac{1}{2} \left(B \frac{\partial \overline{w}_B}{\partial z} + \overline{w}_B \frac{\partial B}{\partial z} \right) erf \left(\frac{W + \overline{w}_B}{\sqrt{2}\, \sigma_{wB}} \right) \\
& + \left[\frac{A}{2} \frac{\partial \sigma_{wA}^2}{\partial z} \left(\frac{W^2}{\sigma_{wA}^2} + 1 \right) + \sigma_{wA}^2 \frac{\partial A}{\partial z} \right] P_A(W) \\
& + \left[\frac{B}{2} \frac{\partial \sigma_{wB}^2}{\partial z} \left(\frac{W^2}{\sigma_{wB}^2} + 1 \right) + \sigma_{wB}^2 \frac{\partial B}{\partial z} \right] P_B(W)
\end{aligned}$$

The moments $\overline{w}_A(z),\ \sigma_{wA}(z) \ldots$ of the component Gaussians vary with height and are related to suitable empirical profiles of $\overline{w'^2},\ \overline{w'^3},\ \overline{w'^4}$ for the CBL

Dispersion in the CBL – well-mixed Lagrangian stochastic model

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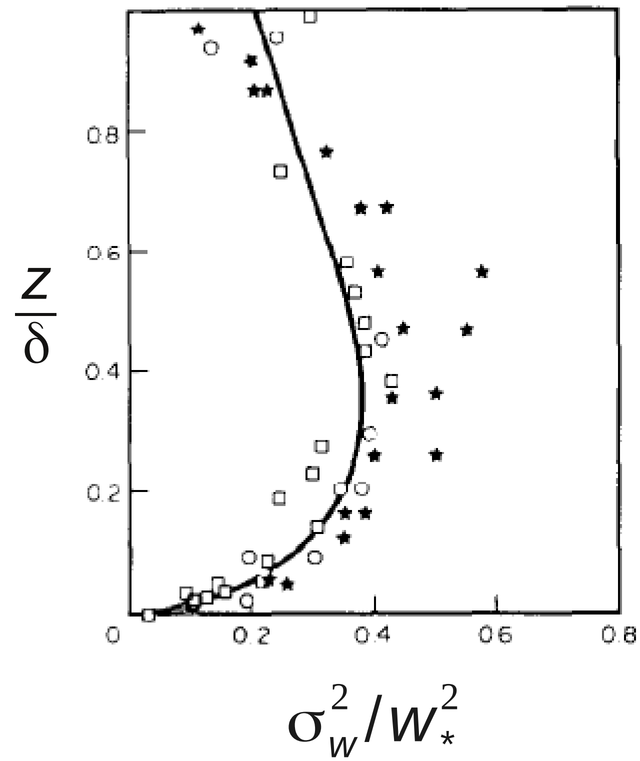
A RANDOM WALK MODEL FOR DISPERSION IN INHOMOGENEOUS TURBULENCE IN A CONVECTIVE BOUNDARY LAYER

ASHOK K. LUHAR and REX E. BRITTER



- L&B fitted empirical curves to observations:

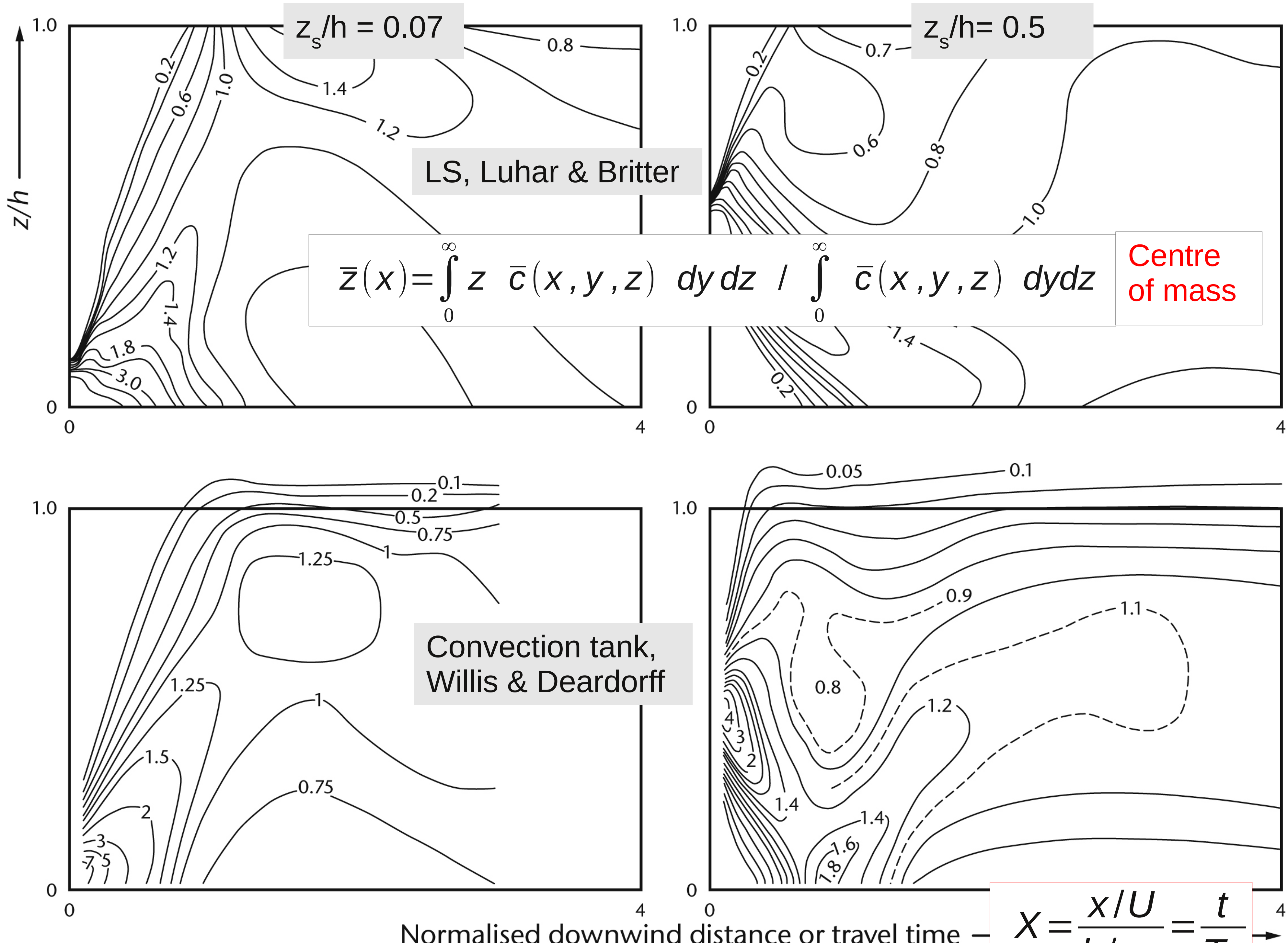
$$\frac{\sigma_w^2(Z)}{w_*^2} = 1.1\ \left(\frac{Z}{\delta}\right)^{2/3}\ \left(1-\frac{Z}{\delta}\right)^{2/3}\left[1-4\frac{(Z/\delta-0.3)}{(2+|Z/\delta-0.3|)^2}\right]$$

$$\frac{\overline{w'^3}}{w_*^3} = 0.8\ \left(\frac{\sigma_w^2}{w_*^2}\right)^{3/2}$$

$$\frac{w_*\tau(Z)}{\delta} = \left[1.5-1.2\left(\frac{Z}{\delta}\right)^{1/3}\right]^{-1}\ \frac{\sigma_w^2(Z)}{w_*^2}$$

$$\frac{\delta\,\epsilon(Z)}{w_*^3} = \left[1.5-1.2\left(\frac{Z}{\delta}\right)^{1/3}\right]$$

Note: these profiles do not resolve an ASL



$$\bar{z}(x) = \int_0^\infty z \ \bar{c}(x,y,z) \ dy\,dz \ / \int_0^\infty \bar{c}(x,y,z) \ dydz$$

Centre of mass

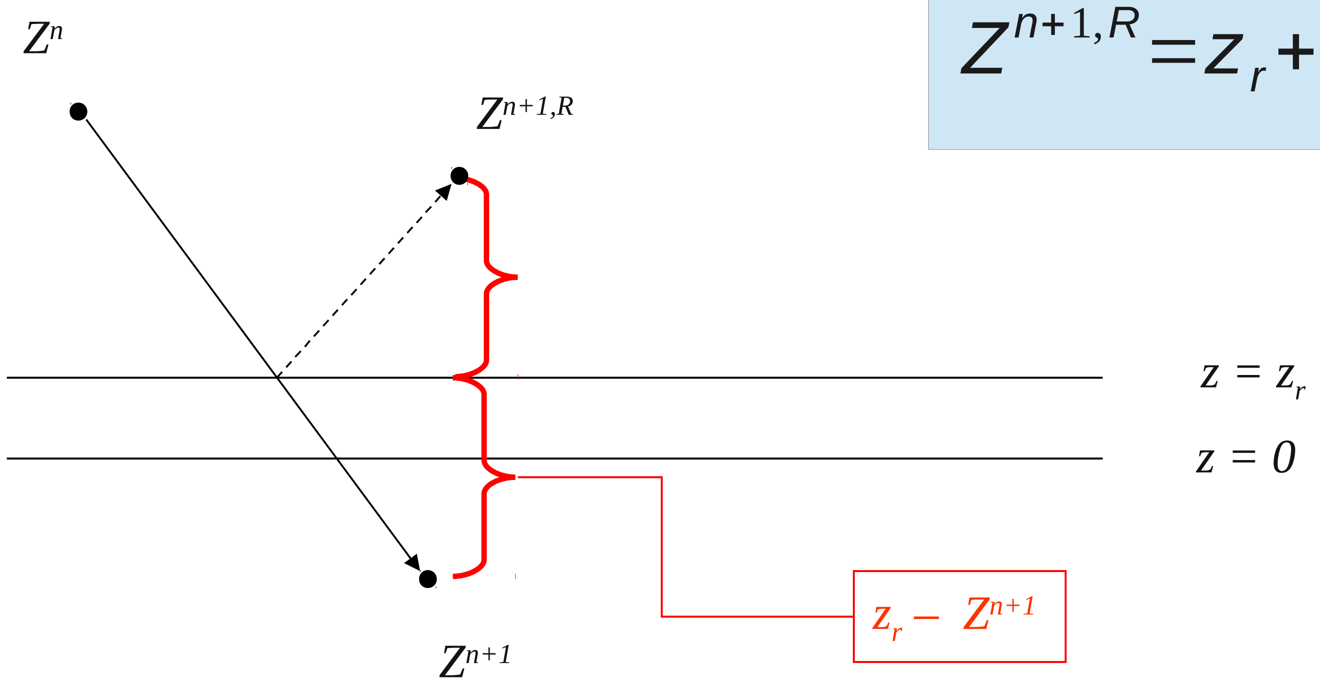
$$X = \frac{x/U}{h/w_*} = \frac{t}{T_L}$$

Reflection of Trajectories in LS models

• a reflection algorithm tacked to a well-mixed LS model can cause violation of the w.m.c. (e.g. Wilson & Flesch, 1993, "Flow boundaries in random flight dispersion models: enforcing the well-mixed condition," J. Appl. Meteorol. 32, 1695-1707)

• however "perfect reflection" ("smooth wall reflection") at an artificial boundary (reflection height z_r) is acceptable in Gaussian turbulence provided $\partial \sigma_w / \partial z \rightarrow 0$ as $z \rightarrow z_r$

$$Z^{n+1,R} = z_r + \left[z_r - Z^{n+1} \right]$$



• in practise acceptable to set z_r much larger than z_0 to reduce computation time
• if simulating whole ABL may need reflection at $z = \delta$ as well

How to judge if reflection is problematic?

Compute the evolution of an initially well mixed particle distribution… e.g. release N_P particles, with the initial height chosen $Z^0 \in U\left[z_r,\, \delta\right]$ which implies $p(z,0) = \dfrac{1}{\delta - z_r}$

MONTE CARLO SIMULATION OF PLUME DISPERSION IN THE CONVECTIVE BOUNDARY LAYER

J. HØGNI BÆRENTSEN* and RUWIM BERKOWICZ

Atmospheric Environment Vol. 18, No. 4, pp. 701–712, 1984
Printed in Great Britain.

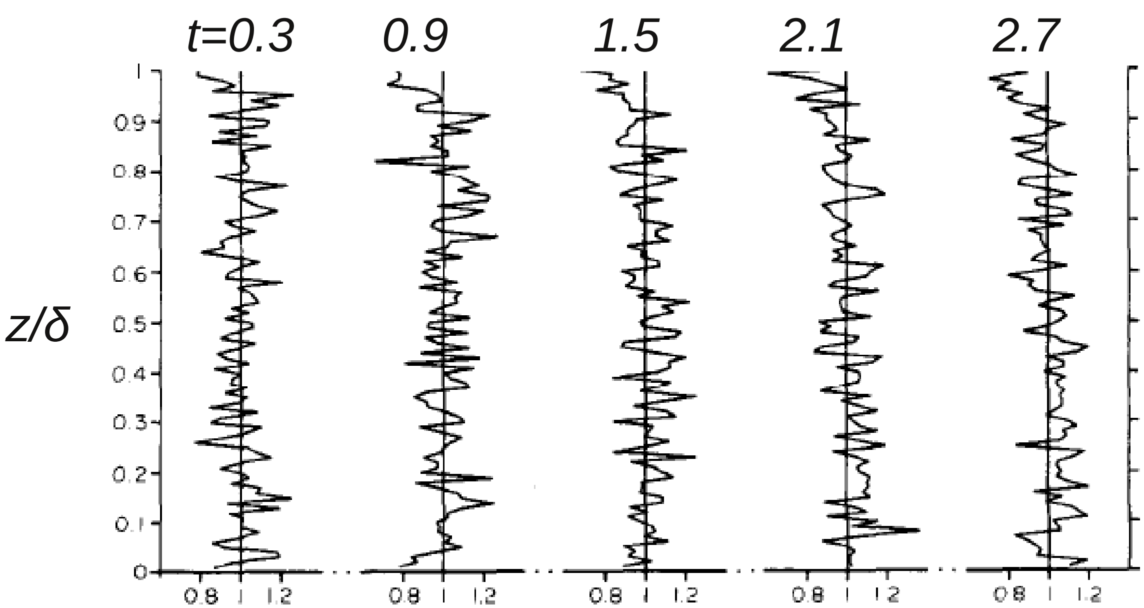
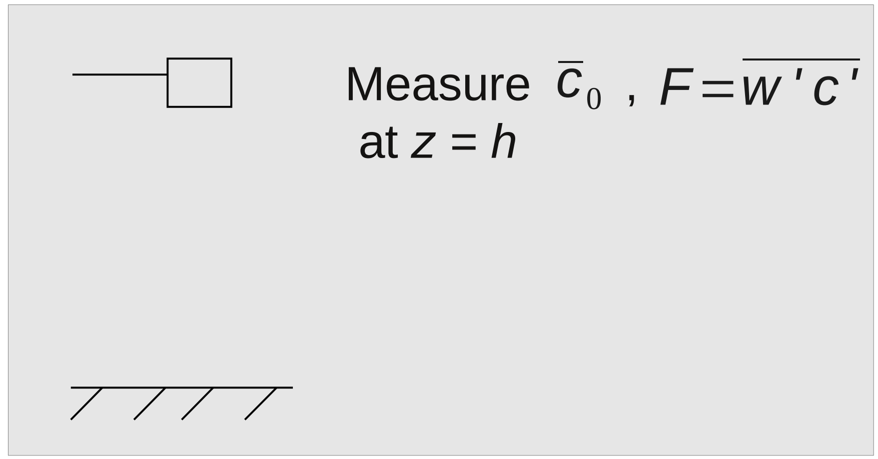


Fig. 4. Distribution of particles after different travel times as simulated by the Monte Carlo model. The source is distributed uniformly through the whole boundary layer. 10,000 particles were used in the simulation.

This work done prior to Thomson's (1987) well-mixed condition

Deposition to ground or canopy

Uptake at ground is often parameterised in terms of a "deposition velocity," defined as the ratio $w_d = F / \bar{c}_0$ of the magnitude of the flux density to the surface to a mean concentration $\bar{c}_0$ measured at an arbitrary reference location above the surface (this only makes sense if the flux and concentration are measured within the constant flux layer)



If deposition velocity known, can incorporate in LS model by performing *partial* reflection: a fraction *A* of particles contacting the surface is absorbed, and the complementary fraction *R=1-A* is reflected in the usual way. Wilson et al. (1989, Agric. Forest Meteo. Vol. 47) show *R* relates to w_d as:

$$\frac{1-R}{1+R} = \sqrt{\frac{\pi}{2}} \frac{w_d}{\sigma_w}$$

Thomson's LS model for 2-D Gaussian, horiz. homogeneous turbulence

$$dU = -\frac{b^2}{2\sigma^2}\left[U\,{\sigma_w}^2 - W\,\overline{u'w'}\right]dt + \frac{\phi_u}{g_a}dt + b\,d\xi_u$$

$$dW = -\frac{b^2}{2\sigma^2}\left[W\,{\sigma_u}^2 - U\,\overline{u'w'}\right]dt + \frac{\phi_w}{g_a}dt + b\,d\xi_w$$

$$dX = [\,\overline{u}(Z) + U\,]\,dt$$

$$dZ = W\,dt$$

where $b^2 = C_0 \epsilon$,

$$\sigma^2 = \sigma_u^2\,\sigma_w^2 - u_*^4 \qquad (\ u_*^2 \equiv \overline{u'w'}\ )$$

and $g_a = g_a(u', w'; z)$ is the joint PDF of the Eulerian velocity fluctuations, specifically, the joint Gaussian:

$$g_a(u', w'; z) = \frac{1}{2\pi\sigma}\ \exp\left[-\ \frac{\sigma_w^2\,(u')^2 + \sigma_u^2\,(w')^2 - 2u_*^2\,u'\,w'}{2\,\sigma^2}\right]$$

Thomson's LS model for 2-D Gaussian, horiz. homogeneous turbulence

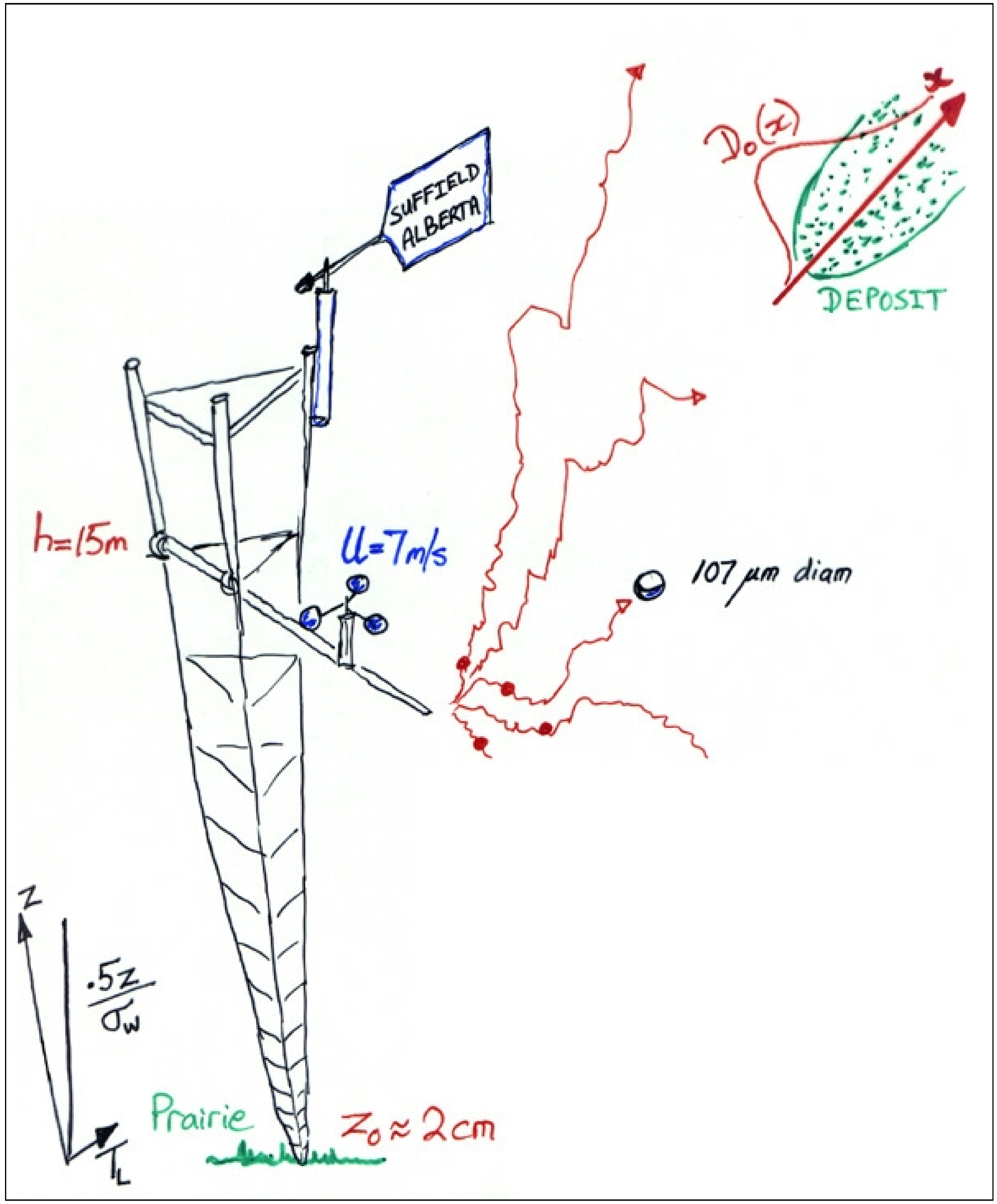
$$\frac{\phi_u}{g_a} = \frac{1}{2}\frac{\partial \overline{u'w'}}{\partial z} + W\,\frac{\partial \overline{u}}{\partial z}$$
$$+ \frac{1}{2\sigma^2}\left[\frac{\partial {\sigma_u}^2}{\partial z}\left({\sigma_w}^2\,U\,W - \overline{u'w'}\,W^2\right) + \frac{\partial \overline{u'w'}}{\partial z}\left({\sigma_u}^2\,W^2 - \overline{u'w'}\,U\,W\right)\right]$$

$$\frac{\phi_w}{g_a} = \frac{1}{2}\frac{\partial {\sigma_w}^2}{\partial z}$$
$$+ \frac{1}{2\sigma^2}\left[\frac{\partial {\sigma_w}^2}{\partial z}\left({\sigma_u}^2\,W^2 - \overline{u'w'}\,U\,W\right) + \frac{\partial \overline{u'w'}}{\partial z}\left({\sigma_w}^2\,U\,W - \overline{u'w'}\,W^2\right)\right]$$

- necessary to include the *u*' fluctuation if, for instance, the turbulence intensity $\sigma_u / \overline{u}$ is large, as (for instance) within a plant (or urban) canopy
- however often there is little (if any) penalty to neglecting the correlation $\overline{u'w'}$ in which case the model above simplifies radically
- 3D generalization is straightforward

Heavy particle dispersion

- inertia
- gravitational settling
- deposition on ground
- what is the “well-mixed state”?
- lack rigorous criteria for LS models
- path of a heavy particle is not a fluid trajectory



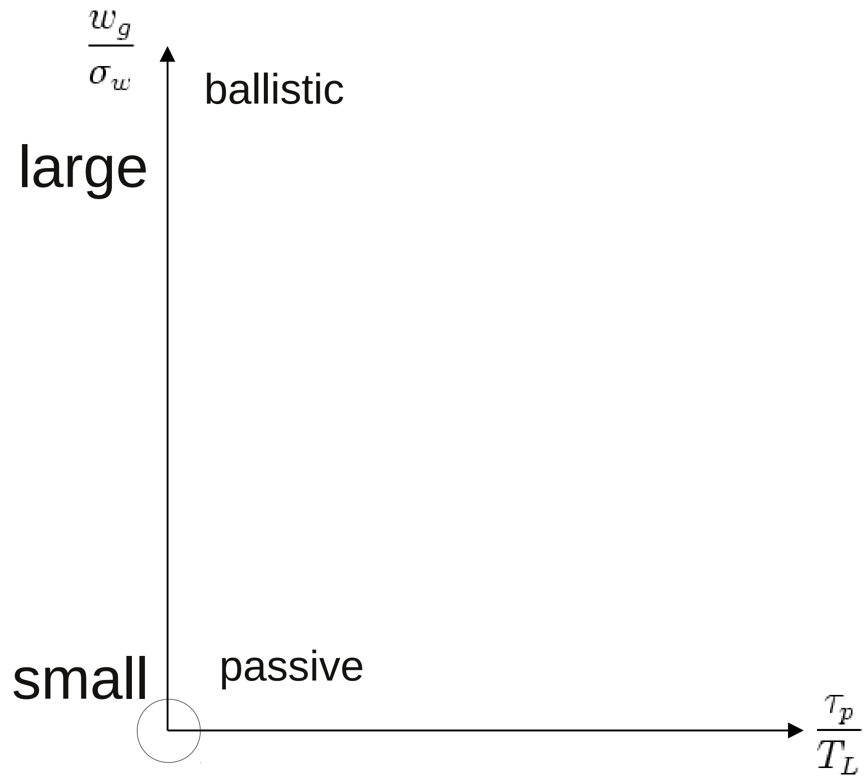
For spherical particles (diam. d) at low slip Reynolds number

$$R = \frac{\mid \vec{U}_p - \vec{u} \mid \ d}{\nu}$$

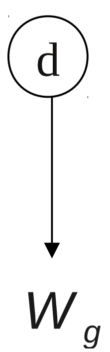
the eqn of motion is

$$\frac{d\,W_p}{dt} = \frac{w(t) - W_p}{\tau_p} - g$$

(drag depends linearly on relative velocity)



At steady state with Eulerian velocity w = const., the terminal velocity is $W_g = \tau_p \ g$



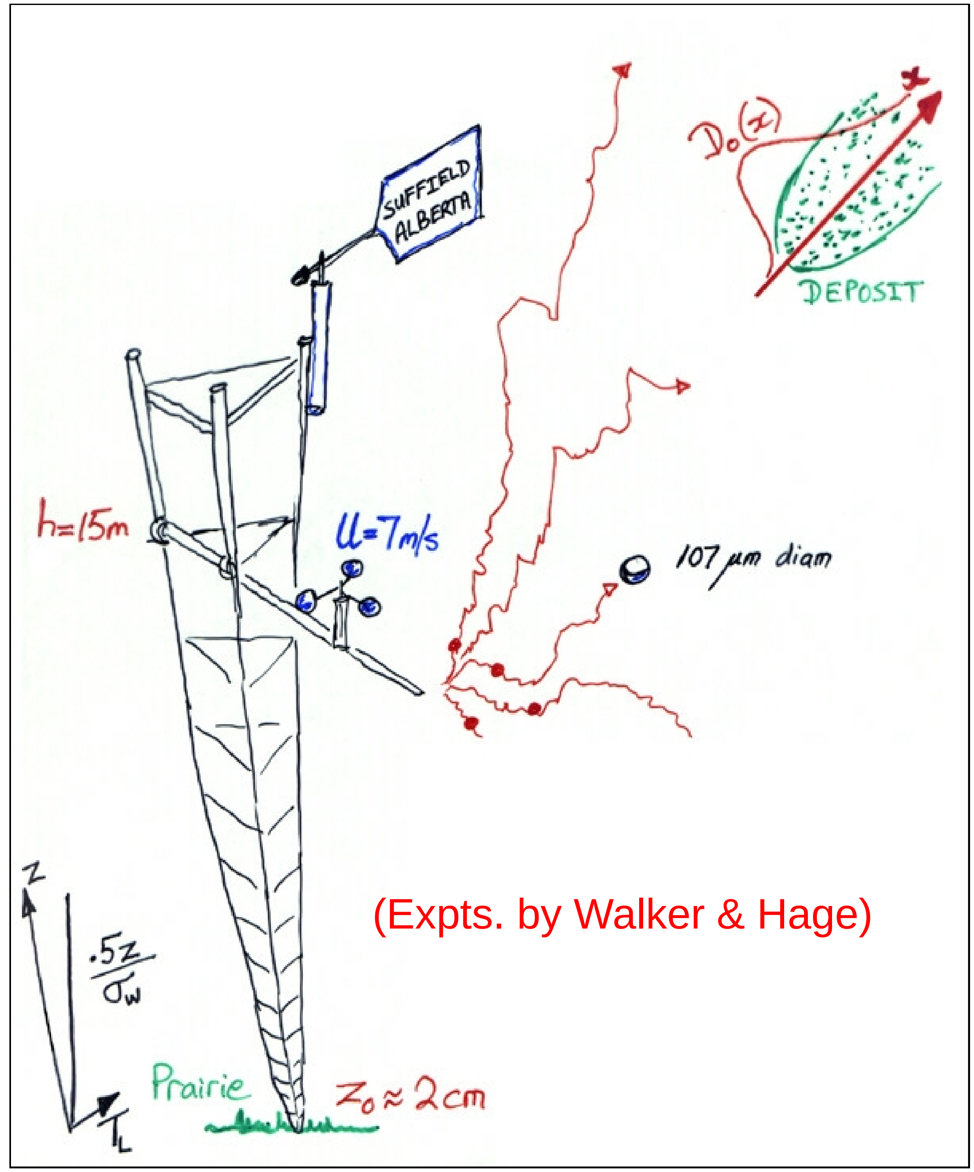
$\mu, \ \rho, \ \rho_p$

What does a dimensional analysis suggest for W_g ?

Stokes' analysis (linearized treatment) $\frac{\tau_p}{d^2/\nu} = \frac{1}{18} \ \frac{\rho_p}{\rho}$

Heavy particle dispersion – Deposition of Glass Beads

$$
\begin{aligned}
d &= 107\ \mu\mathrm{m} \\
\tau_p &= 0.06\ \mathrm{s} \\
w_g &= 0.6\ \mathrm{m\ s^{-1}}
\end{aligned}
$$

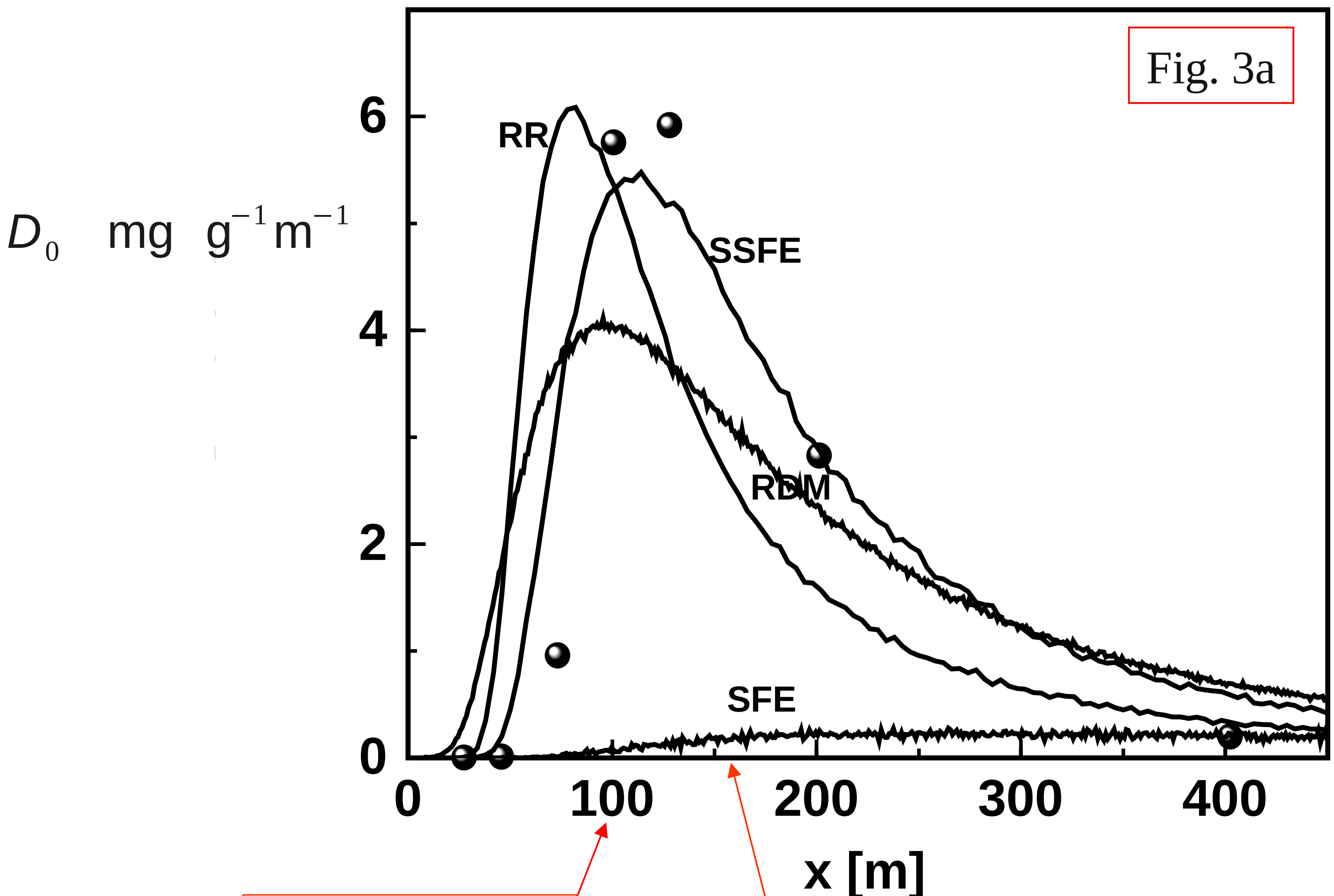


Trajectory Models for Heavy Particles in Atmospheric Turbulence: Comparison with Observations

JOHN D. WILSON

JOURNAL OF APPLIED METEOROLOGY (2000) VOLUME 39

Run C: neutral stratification, L = 341 m



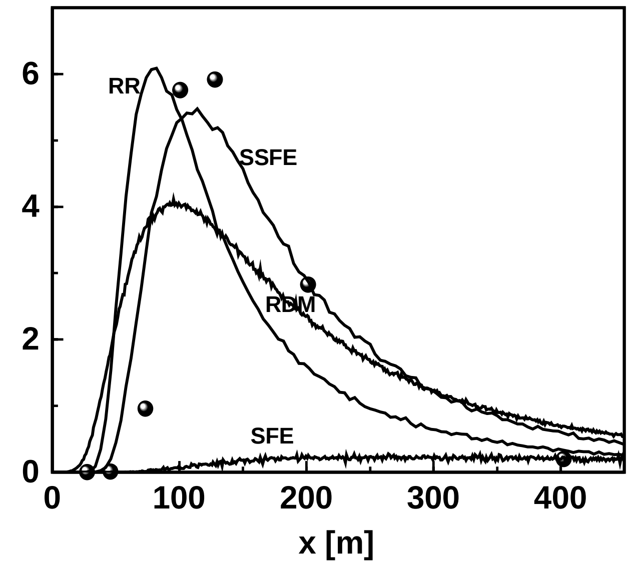
$$\frac{x}{\overline{u}(h)\, T_L(h)} \approx 1$$

Ballistic impact point x=159 m

$$\mathrm{RR}: \;\; W = \mathrm{const.} = \sigma_w(h)\, r \; - w_g, \;\; r \in \mathrm{N}(0,1)$$

$$\mathrm{RDM}: \;\; dZ = \frac{\partial(\sigma_w^2\, T_L)}{\partial z}\, dt + \sqrt{\frac{2\, \sigma_w^2\, dt}{T_L}}\; r \; - w_g\, dt, \;\; r \in \mathrm{N}(0,1)$$

SSFE ("Settling sticky fluid element" model):



$$
\begin{aligned}
dW &= a\,dt + \sqrt{C_0\,\epsilon}\,d\xi \\
W &= W + dW \\
W_p &= W - w_g \\
dZ_p &= W_p\,dt
\end{aligned}
$$

where $a(W) = -\frac{W}{T_L} + \frac{1}{2}\frac{\partial {\sigma_w}^2}{\partial z}\left(\frac{W^2}{{\sigma_w}^2} + 1\right)$ and $T_L = \frac{2{\sigma_w}^2(z)}{C_0\,\epsilon(z)}$

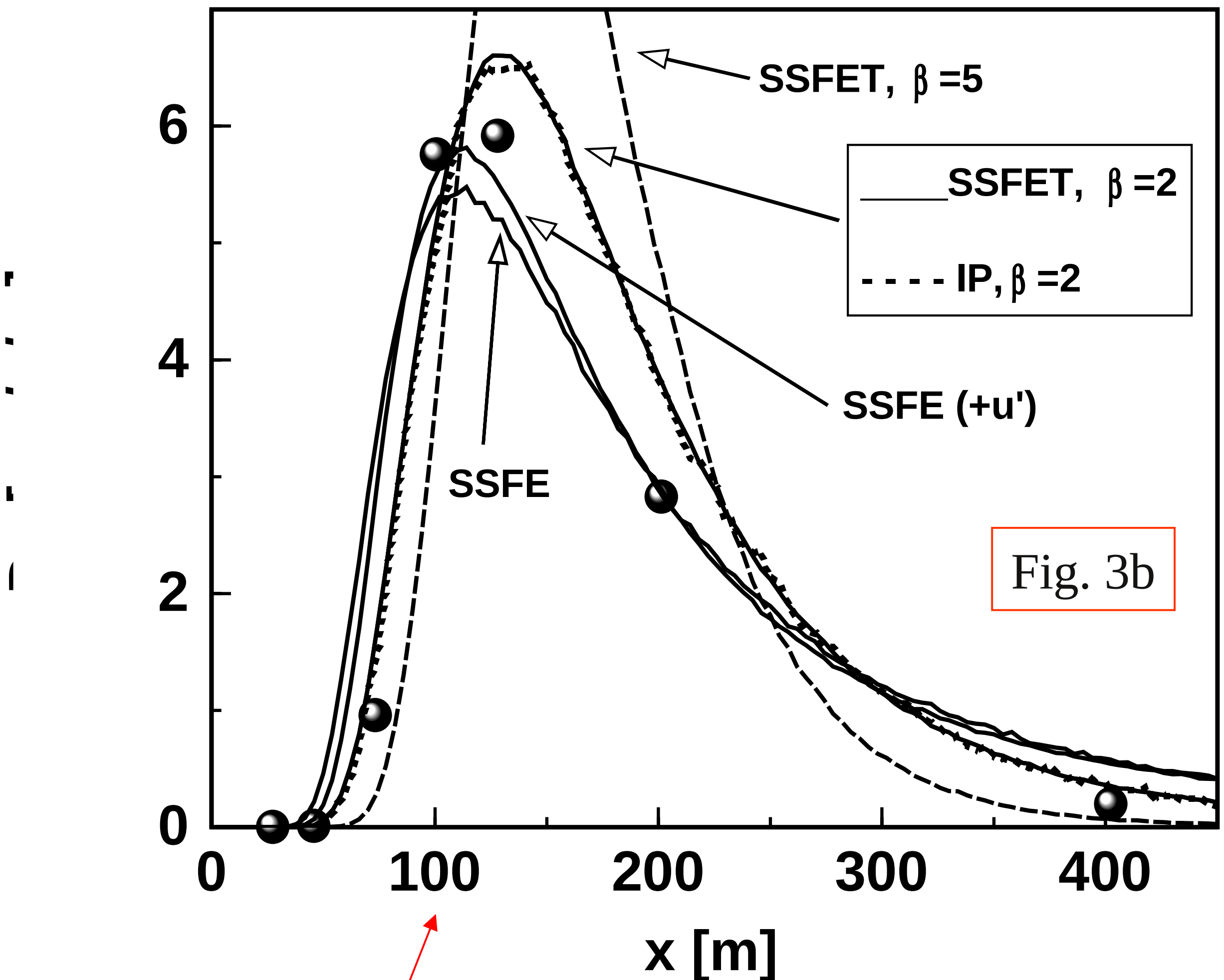
(*a* as for unique 1-D model for Gaussian inhomogeneous turbulence).

SSFET ("Settling sticky fluid element, reduced T" model):

Replace T_L in the above by T_p, a reduced timescale for the fluid velocity along the particle's path – this accounts empirically for the "crossing trajectories effect"

$$T_p = \frac{T_L}{\sqrt{1 + (\beta\,w_g/\sigma_w)^2}}$$

Run C: neutral stratification, L = 341 m



$$\frac{x}{\overline{u}(h)\, T_L(h)} \approx 1$$

Neither an Eulerian nor a Lagrangian sequence. Computed using Thomson's well-mixed 1st – order model, with timescale reduced relative to T_L

contrast with "IP" model

IP:

$$\frac{dW_p}{dt} = \frac{w(\mathbf{X}(t)) - W_p}{\tau_p} - g$$

$$dZ = W_p \, dt$$