EAS 471 **MODERN LS MODEL(S)**

ⓐ Zeroth order 1D LSM is the RDM — Z is Markovian

$$dZ = (\partial K/\partial z)\, dt + \sqrt{2K\, dt}\; r \quad , \quad r \in N(0,1)$$

Equivalent to eddy diffusion approach, therefore not rigorous in near field of source(s).

ⓑ First order LSM — (Z, W) is a (compound) Markovian variable — is the generalized Langevin model, $dW = a\, dt + b\, d\xi$

ⓒ Second order LSM — (A, W, Z) jointly Markovian — is a random walk in accel^n, $dA = a\, dt + b\, d\xi$

We'll focus on ⓑ, option for assignment 3.

Let $dU_i = (dU, dV, dW)$ be the increments in Lagr. velocity (particle velocity) over time step dt, where dt is "small" w.r.t. all signif. time scales

1D case $$\boxed{dZ = W\,dt}$$ $$\boxed{dW = a\,dt + b\,d\xi}\,,\qquad d\xi \in N(0, dt)$$

most general form of Markovian model for (W, Z)

(i) To determine the coeff't "b", invoke Kolmogorov's postulate that

$$\langle dU_i\, dU_j \rangle = C_0\, \varepsilon\, dt\, \delta_{ij}\,,$$

C_0 dimensionless const.

ε turb. kinetic energy dissip^n rate

Implies $\langle dW^2 \rangle = C_0\, \varepsilon\, dt$

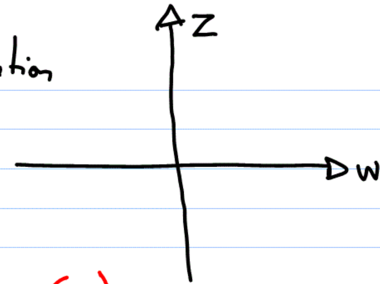
$$= \langle a^2 dt^2 + b^2 d\xi^2 + 2ab\, dt\, d\xi \rangle$$

$$= \langle a^2 \rangle \underbrace{\cancel{dt^2}}_{\text{neglect}} + b^2 \underbrace{\langle d\xi^2 \rangle}_{dt} + 2b\, dt \underbrace{\langle a\, d\xi \rangle}_{\text{zero as } d\xi \text{ is chosen randomly}}$$

$\therefore\ C_0\, \varepsilon\, dt = b^2 dt$ or $b = \sqrt{C_0\, \varepsilon}$

(ii) How to get the other coefficient? Thomson's (1987) "well mixed condition" units are $m^{-1}\,(m\,s^{-1})^{-1}$

Let $p(z, w, t)$ be the joint PDF for particle vert. veloc. + position



Imagine that at $t = 0$ we had particles "well mixed" in the flow. (I'll restrict to constant density flow). Let the Eulerian velocity pdf be $g_a(w)$.

$p(z, w, 0) \propto \rho\, g_a(w)$, this is the maximally disordered (max entropy) state

Therefore $p(z, w, dt)$ must also be well mixed. But p is known to evolve according to the Fokker-Plank eqn., which for this case reads

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial z}(w p) - \frac{\partial}{\partial w}(a p) + \frac{b^2}{2}\frac{\partial^2 p}{\partial w^2}$$

$$= -\frac{\partial}{\partial z}(w p) - \frac{\partial}{\partial w}\left(a p - \frac{b^2}{2}\frac{\partial p}{\partial w}\right)$$

Thomson's w.m.c. implies that $g_a(w)$ must be a steady solution of this F.P. equation. This gives a constraint on the coefficient a.

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Vertically inhomogeneous Gaussian turbulence

$$g_a(w) = \frac{1}{\sqrt{2\pi}\,\sigma_w(z)} \exp\left[-\frac{w^2}{2\sigma_w^2}\right]$$

Eulerian velocity PDF

(we'll assume horiz. homog. flow with $\nabla\cdot\vec{u} = 0$

i.e. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \underbrace{\cancel{\frac{\partial \bar{u}}{\partial x}}}_{hh} + \underbrace{\cancel{\frac{\partial \bar{v}}{\partial y}}}_{hh} + \frac{\partial \bar{w}}{\partial z} = 0$

$\bar{w}(z) = \bar{w}(0) = 0$)

$$a = -\frac{W}{\tau} + \frac{1}{2}\frac{\partial \sigma_w^2}{\partial z}\left(\frac{W^2}{\sigma_w^2} + 1\right), \quad \tau = \frac{2\sigma_w^2}{C_0\,\varepsilon}$$

hh ASL

$$\boxed{\begin{matrix} \bar{u}(z) \\ \sigma_w(z) \\ \varepsilon(z) \end{matrix}}$$

$$dX = \bar{u}(Z)\,dt$$

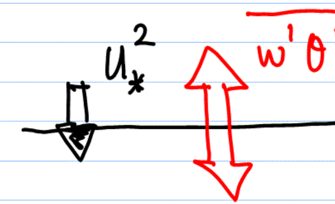
$$dt = \cancel{\mu}\,\tau = \frac{2}{C_0}\,\frac{\sigma_w^2(Z)}{\varepsilon(Z)}$$

$$dZ = W\,dt$$

$$dW = a(Z, W)\,dt + \sqrt{C_0\,\varepsilon(Z)}\,d\xi$$

ABOUT THE IDEAL HORIZ. HOMOG. ASL *

Do a dimensional analysis for the property $\partial\bar{u}/\partial z$ ($n = 5$, $m = 3$)



z , g/T_0

Monin-Obukhov scaling

* shallow layer $z_0 \ll z \ll \delta$

sfc roughness

ABL depth

across which height variation of the (kinematic) fluxes of sensible heat and horiz. momentum can be neglected.

Monin & Obukhov conjectured that properties in the ASL "scale with"

z, g/T_0, u_*, $\overline{w'\theta'}$ where T_0 is a bulk Kelvin temp.

$$\frac{\partial \bar{u}}{\partial z} = \sum_i c_i \; z^{\alpha_i} \, u_*^{\beta_i} \left(g/T_0\right)^{\gamma_i} \left(\overline{w'\theta'}\right)^{\varepsilon_i}$$

method of indices

$$\Downarrow$$

$$\frac{\partial \bar{u}}{\partial z} = \sum_i c_i \; z^{\gamma_i - 1} \, u_*^{-3\gamma_i + 1} \left(g/T_0\right)^{\gamma_i} \, \overline{w'\theta'}^{\,\gamma_i}$$

$$\Rightarrow \quad \frac{z}{u_*}\frac{\partial \bar{u}}{\partial z} = \sum_i c_i \left(\frac{z}{\hat{L}}\right)^{\gamma_i} = f\left(\frac{z}{\hat{L}}\right), \quad \hat{L} = \frac{u_*^3}{(g/T_0)\,\overline{w'\theta'}}$$