EAS471, "Atmospheric Modelling"   Exam   15 April, 2016

Professor: J.D. Wilson   Time available: 150 mins   Value: 30%

Symbols have their usual meteorological interpretation.

Multi-choice   (32 x $\frac{1}{2}$% = 16%)

1. Which option expresses the quantity $\nabla \cdot \nabla \phi$ in Cartesian coordinates (x, y, z)?

   (a) 0
   (b) $\frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial z}$
   (c) $\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z}$
   (d) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$ ✓✓
   (e) 1

2. Suppose ϕ represents the quantity per unit volume of a scalar property of the air, and $\mathbf{u}$ the velocity field. Which choice correctly gives the convective flux density $\mathbf{F}$ of ϕ?

   (a) $\mathbf{F} = \mathbf{u} \cdot \nabla \phi$
   (b) $\mathbf{F} = \mathbf{u} \nabla \phi$
   (c) $\mathbf{F} = \phi \nabla \cdot \mathbf{u}$
   (d) $\mathbf{F} = \mathbf{u} \times \nabla \phi$
   (e) $\mathbf{F} = \mathbf{u}\, \phi$ ✓✓

3. What constraint would permit the function

$$f(x) = \begin{cases} 2 - |x|/b & , \ |x| \le b \\ 0 & , \ |x| > b \end{cases}$$

   to qualify as being a "probability density function?

   (a) $b = 4$
   (b) $b = 2$
   (c) $b = 1$
   (d) $b = 1/2$ ✓✓
   (e) $b = 1/4$

4. Assuming b is chosen correctly, what is the probability that a random sample of x is greater than $-1/4$?

   (a) 1/8
   (b) 1/4
   (c) 1/2
   (d) 3/4
   (e) 7/8 ✓✓

5. With T_I^n representing the temperature at position $x = I\,\Delta x$ and time $t = n\,\Delta t$, the Crank-Nicolson scheme for the one-dimensional heat equation is

$$\frac{T_I^{n+1} - T_I^n}{\Delta t} = \frac{\kappa}{2}\,\frac{T_{I+1}^{n+1} + T_{I-1}^{n+1} - 2T_I^{n+1}}{\Delta x^2} + \frac{\kappa}{2}\,\frac{T_{I+1}^n + T_{I-1}^n - 2\,T_I^n}{\Delta x^2}\,,$$

where κ is the thermal diffusivity. Which statement is **false**?

(a) κ has unit $[\mathrm{m^2\,s^{-1}}]$
(b) this is an explicit method ✗✗
(c) this is not a "leapfrog scheme"
(d) this method can be used to obtain T_I^1 from the initial state T_I^0
(e) this discretization is "consistent" with the heat equation (i.e. its truncation error $\epsilon^{\mathrm{Trunc}} \to 0$ as $\Delta t, \Delta x \to 0$)

6. $f(x)$ is the probability density function for a random variable x that can take on any real value, whose mean is μ and whose variance is σ^2. Which of the statements below is **false**?

(a) $\int\limits_{-\infty}^{\infty} f(x)\,dx = 1$

(b) $\int\limits_{-\infty}^{\infty} x\,f(x)\,dx = \mu$

(c) $\int\limits_{-\infty}^{\infty} (x-\mu)\,f(x)\,dx = 0$

(d) $\int\limits_{-\infty}^{\infty} x^2\,f(x)\,dx = \mu^2$ ✗✗

(e) $\int\limits_{-\infty}^{\infty} (x-\mu)^2\,f(x)\,dx = \sigma^2$

7. Which of the properties given **is not** an attribute of the idealized mathematical "delta function"? (Note: x_1 is a constant.)

(a) $\int\limits_{-\infty}^{\infty} \delta(x-x_1)\,dx = 1$

(b) $\int\limits_{-\infty}^{\infty} \delta(x-0)\,dx = 1$

(c) $\int\limits_{-\infty}^{\infty} x_1\,\delta(x-0)\,dx = 1$ ✗✗

(d) $\int\limits_{-\infty}^{\infty} g(x)\,\delta(x-x_1)\,dx = g(x_1)$

8. Suppose that a layer $z_B \leq z \leq z_T$ of an NWP model is supersaturated at the end of a model time step (but prior to application of the gridpoint comptations) such that the resolved humidity $q > q_*(T)$, where $T = T(z)$ is the resolved temperature and $q_*(\cdot)$ the saturation specific humidity function. Which unresolved process is parameterized the following adjustment?

$$
\begin{aligned}
-L\,\rho\,\delta q(z) &= \rho\,c_p\,\delta T(z)\ ,\ \delta T \geq 0\,, \\
q(z) + \delta q(z) &= q_*(T + \delta T)
\end{aligned}
$$

(a) non-convective large scale condensation ✓✓
(b) dry convection
(c) moist convection
(d) unresolved evaporation
(e) vertical mixing by the unresolved scales of motion

9. Which option best (verbally) defines the mathematical quantity $\nabla_H \cdot (\rho\, q\, \mathbf{V}_H)$, where $\mathbf{V}_H$ is the horizontal velocity vector, ∇_H the horizontal gradient operator, ρ the density and q the specific humidity?

(a) the moisture accession flux M_t of Kuo's convective parameterization
(b) divergence of the horizontal moisture flux ✓✓
(c) surface evaporation rate
(d) vertical component of the relative vorticity
(e) resolved precipitation rate

10. Kuo's parameterization for unresolved moist convention entails a dimensionless variable μ, loosely interpreted as cloud fraction, and given by

$$
\mu = \frac{M_t\,\Delta t}{W_1 + W_2}
$$

where Δt is the model time step. The model cloud spans the layer defined by the LCL and LNB, as given by the model's resolved temperature and humidity $T(z), q(z)$ prior to application of the parameterization, and the cloud properties are $T_c(z) \geq T(z)$, $q_c(z) = q_*(T_c(z))$. Which statement is **false**?

(a) M_t is the rate ($[\mathrm{kg\,m^{-2}\,s^{-1}}]$) at which moisture is *supplied* to the local column of gridpoints
(b) the sum $W_1 + W_2$ is the amount of moisture required to warm the cloud layer to $T_c(z)$ and humidify it to $q_*(T_c(z))$
(c) the lapse rate of $T_c(z)$ follows a moist adiabat
(d) the physics embodied in Kuo's scheme guarantees that $\mu \leq 1$ ✗✗

11. Let U be the resolved zonal velocity component in an NWP model whose timestep is Δt and whose zonal gridlength is Δx. If the numerics of the model are subject to the Courant condition, which of the following is a necessity? (D/Dt denotes the Lagrangian derivative following the resolved motion, and $(\partial/\partial t)_{\mathrm{dyn}}$ denotes the tendency due to the "dynamical" processes.)

    (a) $|U\Delta t/\Delta x| \leq 1$ ✓✓
    (b) $|U\Delta t/\Delta x| \geq 1$
    (c) $(\partial U/\partial t)_{\mathrm{dyn}} \geq 0$
    (d) $(DU/\partial t)_{\mathrm{dyn}} = 0$
    (e) $\Delta x/(g\,\Delta t^2) \leq 1$

12. In an NWP model, which process is parameterized by revising each gridpoint column of resolved temperatures T according to

$$\left(\frac{\partial T}{\partial t}\right)_{\mathrm{phys}} = \frac{\partial}{\partial z}\left(K_{\mathrm{H}}\,\frac{\partial T}{\partial z}\right) ,$$

    where K_{H} is an eddy diffusivity?

    (a) advection by the resolved velocity field
    (b) radiative convergence
    (c) evaporation of liquid water/condensation of vapour
    (d) vertical heat transport by unresolved scales of motion ✓✓
    (e) molecular conduction

13. In NWP parlance (terminology), which momentum-related process or term is **not** considered an aspect of the model dynamics, i.e. does not contribute to the dynamical tendency $(\partial/\partial t)_{\mathrm{dyn}}$?

    (a) advection of momentum by the resolved velocity field
    (b) Coriolis terms
    (c) resolved pressure gradient force
    (d) buoyancy
    (e) Reynolds stress divergence ✗✗

14. Let $X(t)$ denote the displacement (along direction x) of a particle from its point of release at time $t = 0$. Assuming the motion is "diffusive" (with an unvarying diffusivity K), which choice correctly gives the statistic $\sigma_x^2(t) \equiv \overline{\left(X(t) - \overline{X}(t)\right)^2}$ defined over an ensemble of trials? (The overbar denotes an ensemble average.)

    (a) 0
    (b) $\sqrt{2K\,t}$
    (c) $2K\,t$ ✓✓
    (d) $(2K\,t)^2$
    (e) ∞

15. Consider the vertical dispersion of a passive tracer gas from a point source in the horizontally-homogeneous atmospheric surface layer, and the suitability of its description by the Random Displacement Model (RDM, or Drunkard's Walk)

$$dZ = \frac{\partial K}{\partial z}\, dt + \sqrt{2\,K}\, d\zeta\ ,$$

where dt is the time step; $K = (k_v/S_c)u_* z$ is the eddy diffusivity (k_v the von Karman constant, S_c the Schmidt number, u_* the friction velocity); and $d\zeta$ is a Gaussian random variable with variance dt and vanishing mean. Which statement is **true**?

(a) this RDM is exactly consistent with an eddy diffusion treatment using the same eddy diffusivity $K = K(z)$ ✓✓
(b) implementation of the RDM requires that one introduce a "gridded field" for the diffusivity (say, K_{J} , $\mathrm{J} = 1...M$), whereas Eulerian numerical methods are grid-free
(c) the RDM is valid in both the near field and the far field of the source
(d) the first term on the right hand side ("drift term") tends to "push" computational particles towards lower heights
(e) the Gaussian random variable $d\zeta$ has standard deviation dt^2

16. The momentum budget controlling the profile $\overline{u}(z)$ of mean horizontal velocity in a (horizontally-uniform) forest canopy (of height H) can be approximated as

$$\frac{\partial \overline{u}}{\partial t} = 0 = -\frac{\partial\, \overline{u'w'}}{\partial z} - c_d\, a\, \overline{u}^2$$

where c_d is the foliage drag coefficient and a is the foliage area density. Which option is correct, if it is assumed that the downward momentum flux $\overline{u'w'}(H)$ impinging on the canopy is entirely absorbed by the plants?

(a) $\overline{u'w'}(0) = \int_0^H c_d\, a\, \overline{u}^2\, dz$

(b) $\overline{u'w'}(H) = -\int_0^H c_d\, a\, \overline{u}^2\, dz$ ✓✓

(c) $\overline{u'w'}(H) = -\,\overline{u}^2(H)$

(d) $\overline{u'w'}(H) = \overline{u}^2(H)$

(e) $\int_0^H c_d\, a\, \overline{u}^2\, dz = 0$

17. Reynolds' averaging rules are invoked to formally separate atmospheric fields (say f, g) into their resolved ($\overline{f}$, $\overline{g}$) and unresolved (f', g') components. Which is **false**?

(a) $\overline{f + g} = \overline{f} + \overline{g}$
(b) $\overline{\alpha\, f} = \alpha\, \overline{f}$ (α any constant)
(c) $\overline{f\, g} = \overline{f}\,\overline{g} + \overline{f'g'}$
(d) $\overline{f'} = \overline{g'} = 0$
(e) $\overline{f - f'} = 0$ ✗

18. Accounting for velocity skewness is an essential ingredient of models for dispersion from sources in the CBL, whereas horizontal velocity fluctuations (u', v') may be neglected. On the other hand in modelling dispersion in a plant canopy the effect of velocity skewness is often neglected, while fluctuations (u', v') are retained. Which of the following justifications is **false**?

    (a) above the surface layer – which for some purposes need not be resolved or represented – velocity statistics in the CBL vary weakly with height
    (b) vertical inhomogeneity is extreme within plant canopies
    (c) turbulence intensities (e.g. $\sigma_u/\sqrt{\overline{u}^2 + \overline{v}^2}$) are small in the CBL
    (d) turbulence intensities may be large in a canopy
    (e) it is not possible to model all three fluctuating velocities while also incorporating their skewness ✗✗

19. The temperature $T(x,t)$ along an insulated copper wire satisfies the 1-D heat equation $\partial T/\partial t = \kappa\, \partial^2 T/\partial x^2$, and may be represented

$$T(x,t) \;=\; \int_{-\infty}^{\infty} \widehat{T}(k,0)\, e^{j\,k\,x}$$

    as a superposition of waves: $\widehat{T}(k,t)$ is the Fourier transform of $T(x,t)$, (loosely) the amplitude of the complex wave $e^{jkx} \equiv \cos(kx) + j\sin(kx)$ with wavenumber $k\,[\mathrm{m}^{-1}]$ (wavenumber and wavelength λ are related by $k = 2\pi/\lambda$). Taking the Fourier Transform of the heat equation with respect to x leads to a solution $\widehat{T}(k,t) = \widehat{T_0}(k)\,\exp(-k^2\,\kappa\,t)$. Which interpretive statement is **false**? (Note: κ is the thermal diffusivity of copper.)

    (a) each wave component of the initial state $T(x,0)$ decays exponentially in time
    (b) a wave with wavenumber k decays on time scale $\tau = 1/(k^2\,\kappa)$
    (c) short waves (large k) are damped more slowly than long waves (small k) ✗✗
    (d) the action of the diffusion operator $\partial^2/\partial x^2$ (1-D Laplacian) is wavelength selective
    (e) the diffusion operator tends to diminish the curvature $\partial^2 T/\partial x^2$ of the T-profile

20. Suppose $T(x,t)$ defined on $-1 \le x \le 1$, $0 \le t$ is governed by the heat equation $\partial T/\partial t = \kappa\,\partial^2 T/\partial x^2$ with boundary conditions $T(-1,t) = T(1,t) = 0$, and initial condition $T(x,0) = \cos k\pi x/2$, where k is an arbitrary positive integer. Initially T has a maximum magnitude of 1, i.e. $|T|_{\mathrm{mx}} = 1$. Which option best describes the T profile along x at later times?

    (a) $|T|_{\mathrm{mx}} \le 1$ ✓✓
    (b) $|T|_{\mathrm{mx}} \ge 1$
    (c) $|T|_{\mathrm{mx}} = k$
    (d) $|T|_{\mathrm{mx}} = \kappa\,\Delta t/\Delta x^2$
    (e) $|T|_{\mathrm{mx}} = 0$

21. In a horizontally-homogeneous layer the steady state conservation equation for turbulent kinetic energy k (the KE residing in unresolved scales of motion) is

$$\frac{\partial k}{\partial t} = 0 = -\underbrace{\overline{u'w'}\,\frac{\partial U}{\partial z}}_{\mathrm{I}} - \underbrace{\overline{v'w'}\,\frac{\partial V}{\partial z}}_{\mathrm{II}} + \underbrace{\frac{g}{\theta_0}\,\overline{w'\theta'}}_{\mathrm{III}} - \underbrace{\epsilon}_{\mathrm{IV}} - \underbrace{\frac{\partial}{\partial z}\,\overline{w'\left(\frac{p'}{\rho_0} + \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right)\right)}}_{\mathrm{V}}$$

where ϵ denotes the TKE dissipation rate, and other terms have their usual meaning. As an idealization or for the purposes of modelling, sometimes the TKE budget is assumed to be in "local equilibrium," meaning that local sources and sinks of TKE balance. Under that assumption, which term would be dropped?

(a) I
(b) II
(c) III
(d) IV
(e) V ✓✓

22. In the TKE equation given above, the quantity $\partial/\partial z\,(K\,\partial k/\partial z)$ might be used to model (i.e. parameterize) which term?

(a) I
(b) II
(c) III
(d) IV
(e) V ✓✓

23. The flux Richardson number R_f is (minus) the ratio of buoyant to shear production terms for turbulent kinetic energy, negative in unstable stratification. Thus referring to the terms in the TKE budget above, which ratio correctly defines R_f?

(a) IV/III
(b) III/(I+II) ✓✓
(c) IV/V
(d) V/IV
(e) III/IV

24. Non-linear computational instability (NLCI) of a finite difference scheme for the Navier-Stokes equations is a consequence of which factor(s)?

(a) inconsistency of the difference scheme with the underlying differential equation
(b) discretization of the advection – rather than the flux – form of the equation
(c) wave-wave interaction originating in non-linear terms, and aliasing ✓✓
(d) truncation error and neglect of small terms in the differential equations
(e) staggering of the grid for different variables

25. Let $\phi(x,t)$ be the exact solution to a partial differential equation (PDE), ϕ^* be the exact solution to a discretization (i.e. difference equation, DE) representing the PDE with gridlengths $(\Delta x, \Delta t)$, and let ϕ^{Num} be the numerical solution to the difference equation. The "Truncation error" *in the difference equation* is defined $\epsilon^{\mathrm{Trunc}} = \mathrm{PDE} - \mathrm{DE}$, and usually abbreviated to an expression of form $\mathrm{O}(\Delta x^{\mathrm{M}}) + \mathrm{O}(\Delta t^{\mathrm{N}})$, where O() means "of the order of". Errors in the *solution* are defined:

- Discretization error $\epsilon^{\mathrm{Disc}} = \phi - \phi^*$,
- Stability error $\epsilon^{\mathrm{Stab}} = \phi^* - \phi^{\mathrm{Num}}$,
- Total error $\epsilon^{\mathrm{Tot}} = \phi - \phi^{\mathrm{Num}}$.

The Lax Equivalence Theorem states: "If a difference equation is *consistent* with the differential equation it represents, then (numerical) stability[1] is the necessary and sufficient condition for *convergence*." Which option correctly states the technical meaning of "convergence"?

(a) $\epsilon^{\mathrm{Trunc}} \to 0$ as $(\Delta x, \Delta t) \to 0$

(b) $\epsilon^{\mathrm{Disc}} \to 0$ as $(\Delta x, \Delta t) \to 0$

(c) $\epsilon^{\mathrm{Stab}} \to 0$ as $(\Delta x, \Delta t) \to 0$

(d) $\epsilon^{\mathrm{Tot}} \to 0$ as $(\Delta x, \Delta t) \to 0$ ✓✓

26. Noting the generic Taylor series expansion

$$\phi(s \pm \Delta s) = \phi(s) \pm \left(\frac{\partial \phi}{\partial s}\right)_s \Delta s + \frac{1}{2!}\left(\frac{\partial^2 \phi}{\partial s^2}\right)_s \Delta s^2 \pm \frac{1}{3!}\left(\frac{\partial^3 \phi}{\partial s^3}\right)_s \Delta s^3 + ...,$$

suppose the 1D heat equation were discretized as

$$\frac{T_I^{n+1} - T_I^{n-1}}{2\,\Delta t} = \kappa\,\frac{T_{I+1}^n + T_{I-1}^n - 2T_I^n}{\Delta x^2}\,, \tag{1}$$

where I labels gridpoints on the x-axis separated by distance Δx, etc. Which option correctly states the order of the truncation error?

(a) $\mathrm{O}[\Delta t]$

(b) $\mathrm{O}[\Delta x^2]$

(c) $\mathrm{O}[\Delta t^2] + \mathrm{O}[\Delta x^2]$ ✓✓

(d) $\mathrm{O}[\Delta t^2] + \mathrm{O}[\Delta x]$

(e) $\mathrm{O}[\Delta t] + \mathrm{O}[\Delta x^2]$

[1] Numerical stability, loosely speaking, means that the solution does not "blow up." More formally, it is said that "a difference scheme is stable if its solutions remain uniformly bounded functions of the initial state for sufficiently small values of the timestep."

27. Suppose $\phi(x,t)$ is defined on $-\infty \le x \le \infty$, and is governed by the 1D linear advection equation

$$\frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial x} = 0$$

where U = const. If $\phi(x,0) \equiv \phi_0(x) = \sin(kx)$ where k is the wavenumber, which option gives the true solution $\phi(x,T)$?

(a) $\sin[k(x - U\,T)]$ ✓✓
(b) $\sin(kx) - U\,T$
(c) $\phi_0(x) + \sin(kx)$
(d) $\phi_0(x) + U\,T$
(e) $\phi_0(x)\ \exp(-Ut/x)$

28. Suppose that in a certain disturbed micro-meteorological flow the mean spatial field of a property θ were modelled by

$$\overline{u}(z)\ \frac{\partial \overline{\theta}(x,z)}{\partial x} = -\frac{\partial \overline{u'\theta'}}{\partial x} - \frac{\partial \overline{w'\theta'}}{\partial z}$$

with $\overline{u}(z) \ge 0$. Which of the following **is not** a valid inference?

(a) the field of $\overline{\theta}$ is steady state and (spatially) two-dimensional
(b) the mean velocity vector has but one component, $\overline{u}$, directed along the x axis
(c) the mean velocity field is horizontally homogeneous
(d) the $\overline{\theta}$ field is horizontally homogeneous ✗✗
(e) there are no sources or sinks of θ

29. Continuing from question (28), suppose the modeller invokes eddy diffusion closure, viz.

$$\overline{u}(x,z)\ \frac{\partial \overline{\theta}}{\partial x} = \frac{\partial}{\partial x}\left(K_x \frac{\partial \overline{\theta}}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial \overline{\theta}}{\partial z}\right) .$$

Which option correctly states the circumstance that would permit his or her treatment of the x-axis as "one-way"?

(a) $\left|\overline{u}(x,z)\ \frac{\partial \overline{\theta}}{\partial x}\right| \ll \left|\frac{\partial}{\partial x}\left(K_x \frac{\partial \overline{\theta}}{\partial x}\right)\right|$
(b) $\left|\overline{u}(x,z)\ \frac{\partial \overline{\theta}}{\partial x}\right| \gg \left|\frac{\partial}{\partial x}\left(K_x \frac{\partial \overline{\theta}}{\partial x}\right)\right|$ ✓✓
(c) $\left|\frac{\partial}{\partial z}\left(K_z \frac{\partial \overline{\theta}}{\partial z}\right)\right| \gg \left|\frac{\partial}{\partial x}\left(K_x \frac{\partial \overline{\theta}}{\partial x}\right)\right|$
(d) $\left|\frac{\partial}{\partial z}\left(K_z \frac{\partial \overline{\theta}}{\partial z}\right)\right| = 0$
(e) $\overline{u} = 0$

30. Still with the scenario of question (28), let the inflow and outflow boundaries of the solution domain lie at $x = (X_{\text{in}}, X_{\text{out}})$. Assuming that the x axis *is* one-way, which of the listed conditions is most suitable as a condition on $\overline{\theta}$ at the downwind boundary?

   (a) $\overline{\theta}(X_{\text{out}}, z) = 0$
   (b) $\overline{\theta}(X_{\text{out}}, z) = \overline{\theta}(X_{\text{in}}, z)$
   (c) $\left(\partial\overline{\theta}/\partial z\right)_{X_{\text{out}}} = 0$
   (d) $\left(\partial\overline{\theta}/\partial x\right)_{X_{\text{out}}} = 0$ ✓✓
   (e) $\left(K_z\, \partial\overline{\theta}/\partial z\right)_{X_{\text{out}}} = \left(K_z\, \partial\overline{\theta}/\partial z\right)_{X_{\text{in}}}$

31. Suppose advection by the resolved wind field $\mathbf{U} = (U, V, W)$ were the sole mechanism causing evolution of the temperature T at a point in the atmosphere (or ocean). Which option correctly gives the sign and magnitude of the local tendency in (resolved) temperature $\partial T/\partial t$?

   (a) $-\,\mathbf{U} \cdot \nabla T$ ✓✓
   (b) $\mathbf{U} \cdot \nabla T$
   (c) $-\,\mathbf{U} \times \nabla T$
   (d) $\mathbf{U} \times \nabla T$
   (e) $-\,\mathbf{U}\, T$

32. Gridpoint computations for the influence of unresolved scales of motion in the ABL on the resolved absolute humidity ρ_v will involve the equation

$$\left(\frac{\partial \rho_v}{\partial t}\right)_{\text{physics}} = -\ [.]\ \overline{w'\rho_v'} \tag{2}$$

   Which option correctly defines the missing operator "[.]"?

   (a) $\partial/\partial z$ ✓✓
   (b) $\partial/\partial x$
   (c) $U\,\partial/\partial x + V\,\partial/\partial y$ (U, V the resolved horizontal velocity components)
   (d) $W\,\partial/\partial z$ (W the resolved vertical velocity)
   (e) $K\,\partial/\partial z$ (K the eddy diffusivity)

Long answer: **2 x 7% = 14%**

Answer any ***two*** *questions from this section. Use point or essay format, as you wish. You are invited to think and respond rather broadly (but coherently, and relevantly) on these somewhat open-ended questions.*

1. Explain the significance of Figure (1) in relation to the eddy diffusion closure ("K-theory") for eddy fluxes like $\overline{w'\theta'}$ (etc). Interpret the diagram, and deduce any implication regarding the legitimacy of K-theory (also known as "first-order closure") in canopy flows. You might relate your argument to the utility of K-theory in another regime of turbulent flow, e.g. the neutrally-stratified and horizontally-homogeneous ASL in which the vertical eddy momentum flux is $-u_*^2$ (which defines the friction velocity u_*) and the mean wind shear is $\partial \overline{u}/\partial z = u_*/(k_v z)$ (where k_v is the von Karman constant).

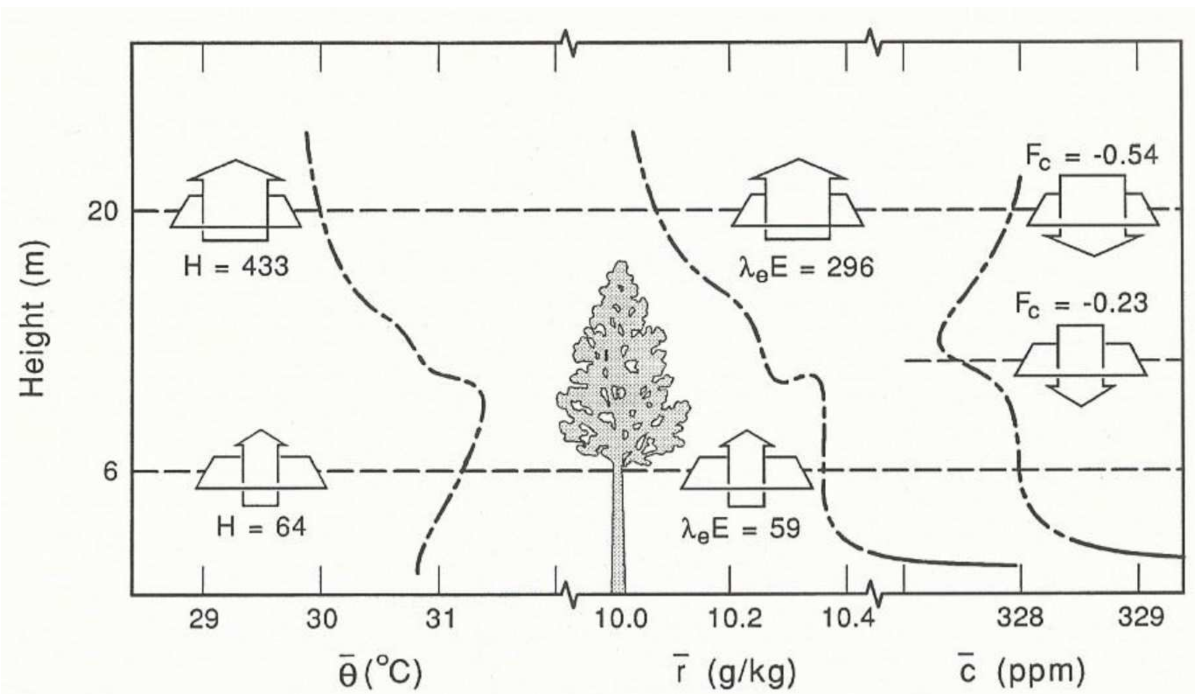

Figure 1: Measured 60-min mean vertical eddy fluxes of sensible heat H [W m^{-2}], latent heat λE [W m^{-2}] and carbon dioxide F$_c$ [mg m^{-2} s^{-1}] near noon in Uriarra Forest (Australia), with corresponding mean profiles of potential temperature $\overline{\theta}$, water vapour mixing ratio $\overline{r}$ and carbon dioxide mixing ratio $\overline{c}$. (Source: Kaimal & Finnigan, Fig. 3.9).

Answer: In the context of the *legitimacy* of K-theory, the key thing to notice about the diagram is the inconsistent behaviour of the various eddy diffusivities: the eddy diffusivity for heat is *negative* at the 6 m level (the mean heat flux is travelling *against* the mean temperature gradient); the eddy diffusivity for water vapour is infinite at the 6 m level (there's a vapour flux, but no gradient to "drive" it); and the eddy diffusivity for carbon dioxide is negative in the canopy crown (at about 12 m). Only above the canopy are the fluxes directed down-gradient. In contrast, the momentum flux ($-u_*^2$) and the mean wind shear $\partial \overline{u}/\partial z = u_*/(k_v z)$ in the neutral ASL together imply $K = -\text{flux/gradient} = k_v u_* z$, and this is a trusted, useful relationship we'd seen several times, in different contexts but always applying near ground, over the lectures of this year's course.

Those were the key points. It was also worth noting that the fluxes have greater magnitudes above the canopy than within the canopy, and that there is an inversion in the trunk space below the canopy crown. Not expected, but a bonus if given, was an explanation that active photosynthesis in the leafy canopy crown (where much solar radiation was absorbed, causing the peak in $\overline{\theta}$) had resulted in a "drawdown" in CO_2 concentration.

2. Consider an event like the Fukushima-Daiichi nuclear disaster, when predicting (or retrospectively diagnosing) the spread of an injection of gases and/or fine particles into the atmosphere is of concern. One has access to forecast NWP fields (analyses, or if needbe, forecasts), for instance the mean velocity vector $(\overline{u}, \overline{v}, \overline{w})$ representing the *resolved* motion field and the mean potential temperature $\overline{\theta}$ (etc). Bearing in mind however that the NWP wind field does not capture all scales of motion, and given that those unresolved scales must contribute to the dilution of any emitted plume of material, **explain in general terms what options one might have and what approaches one might take in order to simulate an ensemble of trajectories** representing the plume's advection and growth over the large (say, inter-continental) scale, taking account not only of the resolved winds, but also the impact of the finer scales of motion. In considering this, recall that NWP models produce some information regarding the statistics of unresolved motion, in the context of their parameterizations for boundary layer friction and mixing, e.g. in many cases the TKE equation is solved to provide the field of k and which in turn provides a basis to compute an eddy diffusivity $K \propto \lambda \sqrt{k}$ (where λ is a length scale) or to approximate the variances $(\sigma_u^2, \sigma_v^2, \sigma_w^2)$ of the unresolved velocity components (one might assume the TKE is partitioned evenly, such that $\sigma_u^2 = \sigma_v^2 = \sigma_w^2 = 2k/3$).

   Specify and justify an algorithm you might use to compute trajectories on the stated scale, and how the needed inputs would be provided. Be as specific as you can, e.g. do you or do you not include horizontal velocity fluctuations, and why or why not? Is the diffusion paradigm legitimate on the scale of interest, and why or why not?

   Some key aspects of an answer:

(Notice the instruction to **specify and justify an algorithm**, and that a response to two particular questions is invited.)

- the concern being long range transport, one's interest is the far field of the source: hence a Random Displacement Model would suffice for computing the desired trajectories
- horizontal motion can be based on the NWP fields for the resolved (mean) horizontal velocities $(\overline{u}, \overline{v})$, because mean horizontal wind speed is much larger than the fluctuations, except very close to ground
- the RDM requires an eddy diffusivity, which can be procured from the model's gridpoint computations
- will want to have an option to allow gravitational settling of particulates, and perhaps precipitation scavenging
- model form something like

$$\begin{aligned} dX &= \overline{u}(X,Y,Z,t)\,\Delta t\ , \\ dY &= \overline{v}(X,Y,Z,t)\,\Delta t\ , \\ dZ &= \frac{\partial K}{\partial z}\,\Delta t\,+\,\sqrt{2\,K)\,\Delta t}\,+\,[\overline{w}(X,Y,Z,t)\,-\,w_g]\,\Delta t\ , \end{aligned}$$

  where (X, Y, Z) is the position of a particle, $K = K(X, Y, Z, t)$ is the eddy diffusivity (interpolated from the NWP model) at the particle's position, $\overline{w}$ is the resolved (NWP). vertical motion and w_g a gravitational settling velocity.
- how is the source to be treated? Is it a time-continuous release or an instantaneous release? If explosive, perhaps represent the "plume" as having an initial volume — extending to the top of the troposphere? (Or not).

Other, more minor (or obvious) elements:

- No need to retain molecular diffusion

3. Suppose the following (highly simplistic) model were adopted for the nocturnal evolution of mean potential temperature and turbulent kinetic energy (TKE, denoted k) in a horizontally homogeneous atmospheric surface layer, under fairweather (cloud free) conditions, viz.

$$\begin{aligned}\frac{\partial \overline{\theta}}{\partial t} &= \frac{\partial}{\partial z}\left(K\,\frac{\partial \overline{\theta}}{\partial z}\right) ,\\ \frac{\partial k}{\partial t} &= K\left(\frac{\partial U}{\partial z}\right)^2 - \frac{g}{\theta_0}\,K\,\frac{\partial \overline{\theta}}{\partial z} - \frac{(a\,k)^{3/2}}{k_v\,z} ,\end{aligned}$$

with the eddy diffusivity parameterized as[2] $K = k_v z\,\sqrt{a\,k}$. Continuing with the theme of simplicity, the (kinematic) sensible heat flux at the surface, $\overline{w'\theta'}(0) \equiv \left(-\,K\,\partial\overline{\theta}/\partial z\right)_0$, is constant (and negative, cooling the layer from below); and the wind shear $\partial U/\partial z$ is treated as *fixed*, i.e. it does not respond to changes in stratification and TKE), and is of the order of $|\mathbf{U}_g|/h$, where h is the depth of the surface-based turbulent layer and $\mathbf{U}_g$ is the geostrophic wind (assumed constant).

Within this framework, explain why it is that on some occasions wind speeds near ground remain appreciable overnight, whereas on other occasions near ground winds become calm. Critique this model, i.e. list the (many) ways in which it is a gross oversimplification.

Elements of response:

In hindsight, this was an awkward question: on the one hand you're told the wind shear is held constant, while on the other hand you're being asked to explain why winds can die down (if $|\mathbf{U}_g|/h$ is small). Nevertheless useful points could be made:

- the sensible heat budget equation, given the downward (negative) surface heat flux, will "grow" an inversion
- this means the buoyant production term in the TKE equation will be negative, a *sink* for TKE. It therefore acts *in opposition* to the shear production term which (clearly) is positive, such that the net TKE source term is reduced (less positive) or might even become negative – either way, tending to weaken vertical mixing
- on a windy night, i.e. with large $|\mathbf{U}_g|/h$ and so large mean shear $\partial U/\partial z$, the shear production term in the TKE budget can overcome the turbulence-suppressing effect of buoyancy, so that mixing is sustained and a continuous downward supply of momentum is assured

[2] As an aside that is not necessarily essential to understanding the qualitative implications of the model, one would choose (calibrate) the constant a to ensure that $\sqrt{a\,k}$ equates to the friction velocity u_*, in the case that $\partial\overline{\theta}/\partial z = 0$, i.e. in the neutral limit. The final term of the TKE eqn parameterizes TKE dissipation to heat, and in the neutral limit would reduce to $u_*^3/k_v z$, thereby balancing shear production.

- but conversely if $\partial U/\partial z$ is small, the turbulence-suppressing effect of buoyancy can result in calm conditions – there is no longer any turbulent mixing to carry down momentum from aloft

Critiquing the model:

- unrealistic that wind speed does not respond to changes in stability – should include the momentum equations in the model (e.g. as in Délage's model). This would allow the mean wind shear to respond to changes in the state of the ABL, an essential feedback.
- related to the above point, this model cannot account for (i.e. simulate) the possible developement of a low level jet
- the model does not include *radiative* energy transport, which can be rather important
- surface heat flux dos not in fact stay constant overnight; a more realistic surface boundry condition would entail solving the surface energy balance
- effects of moisture entirely neglected; the excludes the possibility of condensation, haze and fog, with potentially major effects on radiation

Other (less obvious) criticisms of the model:

- the TKE budget equation imposes a *local* balance of source and sink terms, i.e. does not allow for vertical transport of TKE from one level to another (which is an element of a fuller TKE equation, such as that given in multichoice question 21).
- the length scale $k_v z$ appearing in the TKE dissipation term $\epsilon = (a\,k)^{3/2}/(k_v z)$ ought (also) to respond to stratification
- the eddy diffusivity for heat and the eddy viscosity have not been distinguished: their ratio is the turbulent Prandtl number, which though often treated as being equal to 1, remains uncertain