EAS 471 Discretization (ctd). 21 JAN 2016

$$\frac{T_I^{n+1} - T_I^n}{\Delta t} = K \frac{T_{I+1}^n + T_{I-1}^n - 2T_I^n}{\Delta x^2}$$

This is "explicit" ~ suppose we have the solution at time n, then

$$T_I^{n+1} = T_I^n + \frac{K\Delta t}{\Delta x^2}\left[T_{I+1}^n + T_{I-1}^n - 2T_I^n\right]$$

Compare
~~Contrast~~ this with, eg. Richardson scheme

$$\frac{T_I^{n+1} - T_I^{n-1}}{2\Delta t} = K\left(\frac{T_{I+1}^n + T_{I-1}^n - 2T_I^n}{\Delta x^2}\right)$$

3 time levels

explicit

unconditionally unstable
numerically

centred on

I, n I, n

Crank-Nicolson $$\frac{T_I^{n+1} - T_I^n}{\Delta t} = \frac{K}{2}\left[\frac{T_{I+1}^{n+1} + T_{I-1}^{n+1} - 2T_I^{n+1}}{\Delta x^2}\right] + \frac{K}{2}\left[\frac{T_{I+1}^n + T_{I-1}^n - 2T_I^n}{\Delta x^2}\right]$$

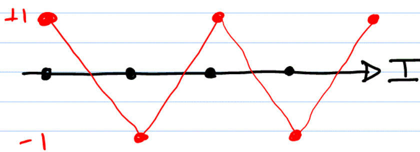
The Crank-Nicolson scheme is "implicit" → simultaneous eqns for T_I^{n+1} (matrix inversion problem). In general, implicit schemes less likely to be unstable.

There is a formal method (Von Neumann) to determine the stability of schemes for linear eqns.

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2}$$

Is our simple explicit scheme* for heat eqn stable?

Suppose that at time n, $T_I^n = (-1)^I$



If $\left|T_I^{n+1}\right| > 1$, scheme unstable

$$T_I^{n+1} = T_I^n + \frac{K\Delta t}{\Delta x^2}\left[T_{I+1}^n + T_{I-1}^n - 2T_I^n\right]$$

Let I be odd $\quad T_I^{n+1} = -1 + \frac{K\Delta t}{\Delta x^2}\left[1 + 1 - 2(-1)\right] = -1 + 4\frac{K\Delta t}{\Delta x^2}$

Stability requires $\left| -1 + 4 \frac{K \Delta t}{\Delta x^2} \right| < 1$

$$4 \frac{K \Delta t}{\Delta x^2} < 2$$

$$\boxed{\frac{K \Delta t}{\Delta x^2} < \frac{1}{2}}$$

criterion on the diffusion number.