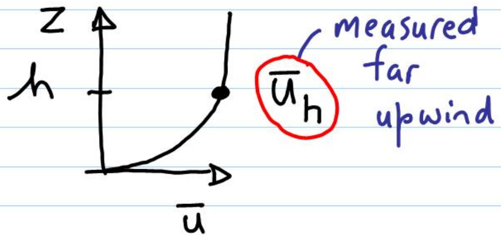
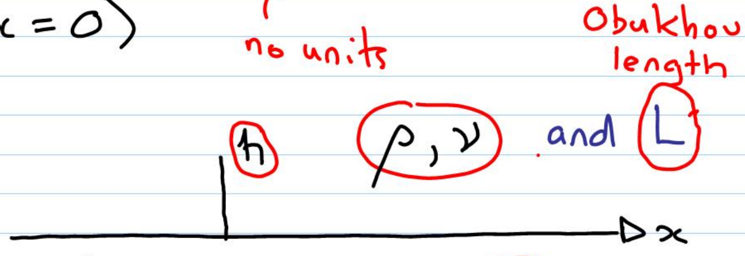
EAS 471 23 Feb 2016

① Another dimensional analysis. Surface layer flow, mean wind runs down the x-axis. Windbreak (height h, porosity ϕ) runs to $\pm\infty$ along the y-axis (and stands at $x = 0$)

no units





Surface roughness length z_0

Please do a dimensional analysis to find an expression for the drag (per unit length in y direction) D_x $\left[\frac{N}{m}\right]$

Method $n = 8$, $m = 3$ (length, time, mass) ∴ seek relationship between $n - m = 5$ dimensionless variables.

D_x has unit of pressure × length, ∴ choose $\dfrac{D_x}{\rho\, \overline{u}_h^2\, h}$ as l.h.s.

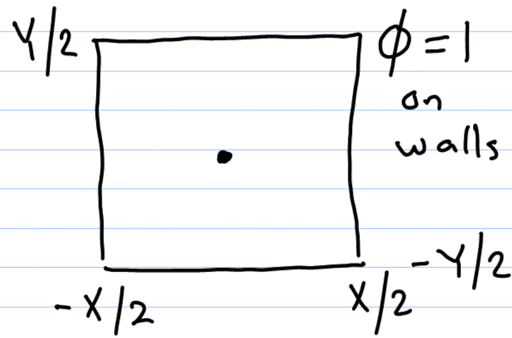
$$\therefore \quad \frac{D_x}{\rho\, \bar{u}_h^2\, h} = f\left(\frac{h}{z_0}, \frac{L}{z_0}, \phi, \underbrace{\frac{\nu}{\bar{u}_h h}}_{1/R_e}\right)$$

$$\xrightarrow[\infty]{|L|} \hat{f}\left(\frac{h}{z_0}, \phi, \cancel{\frac{\nu}{\bar{u}_h h}}\right) \xrightarrow[\infty]{R_e} \hat{\hat{f}}\left(\frac{h}{z_0}, \phi\right)$$

Note: what about ABL depth δ (order kms)? Gives us another Reynolds no. $R_e' = \delta\, \bar{u}_h / \nu$ (probably better to add R_e' to the argument list rather than, say, δ / z_0)

② Background for assignment 2: elliptic pde's

Laplace's eqn $\quad \nabla^2 \phi = 0$



- prototypical elliptic problem ("jury problem")
- both axes are "2-way"

- steady state, linear
- homogeneous (no term that doesn't involve ϕ or one of its derivatives)
- looks like $0 = -\nabla\cdot(\underbrace{-D\nabla\phi}_{\vec{F}_\phi})$ with $D = 1$

  $\vec{F}_\phi$ a purely diffusive flux
- "want" no curvature in the interior

Poisson's eqn $\nabla^2\phi = \underbrace{F(x,y)}$ forcing inhomogeneity

③ "Relaxation method" for solving elliptic pde's.

Suppose $\dfrac{\partial T}{\partial t} = 0 = K\nabla^2 T + Q$ in 2 space dimensions.

Let $T^n_{i,j}$ be n^{th} guess of solution at gridpoint i,j

$$K\frac{T^n_{i+1,j}+T^n_{i-1,j}-2T^n_{i,j}}{\Delta x^2}+K\frac{T^n_{i,j+1}+T^n_{i,j-1}-2T^n_{i,j}}{\Delta y^2}+Q_{ij}=R^n_{ij}$$

Correction: $T^{n+1}_{i,j} \longleftarrow T^n_{i,j}+\alpha R^n_{i,j}$

$$K\frac{T^n_{i+1,j}+T^n_{i-1,j}-2\left(T^n_{i,j}+\alpha R^n_{i,j}\right)}{\Delta x^2}+K\frac{T^n_{i,j+1}+T^n_{i,j-1}-2\left(T^n_{i,j}+\alpha R^n_{i,j}\right)}{\Delta y^2}$$

$$+Q_{ij}=R^{n+1}_{i,j}$$

but we can choose α to make $R^{n+1}_{i,j}\equiv 0$

$$=0=\cancel{R^n_{i,j}}-2\alpha \cancel{R^n_{i,j}}\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)K$$

common factor

Choose $\alpha = \frac{1}{2\kappa}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)^{-1} = \frac{1}{2\kappa} \frac{\Delta x^2 \Delta y^2}{\Delta x^2 + \Delta y^2}$

__When to stop?__ Aim to get the residual "small" in magnitude everywhere.

Define

$$\sigma_R = \left(\frac{1}{I_{max} J_{max}} \sum_{\text{all } I,J}\sum R_{I,J}^2 \right)^{1/2}$$

root mean square residual

__Two variants__

Sequential relaxation $\left\{ \begin{array}{l} \text{compute residual} \\ R_{IJ}^n \text{ and immediately} \\ \text{update } T_{IJ} \end{array} \right.$ "under relaxation"

reduces α to ensure numeric stability.

Simultaneous relaxation ⇒ compute all residuals R_{IJ}^n before updating any T_{IJ}