EAS 471

25 FEB 2016

(A) Linking Eulerian + Lagr. approaches to turbulent dispersion (statistical description)

order $10^{-5}\ m^2 s^{-1}$ in atmos.

Mass consv. eqn $$\frac{\partial c}{\partial t} = -\nabla\cdot\left(\vec{u}c - \mathcal{D}\nabla c\right) + Q$$

Eulerian approach: Reynolds average $c = \bar{c} + c'$ ($\bar{c}$ rather than C as wish to keep upper case for Lagr. variables)

$$\frac{\partial}{\partial t}\left(\bar{c} + c'\right) = \dots$$

Then average

$$\frac{\partial \bar{c}}{\partial t} = -\frac{\partial}{\partial x}\left[\bar{u}\,\bar{c} + \overline{u'c'} - \mathcal{D}\frac{\partial \bar{c}}{\partial x}\right] - \frac{\partial}{\partial y}\left[\quad\right] - \frac{\partial}{\partial z}\left[\bar{w}\,\bar{c} + \overline{w'c'} - \mathcal{D}\,\partial\bar{c}/\partial z\right] + \bar{Q}$$

(flux form; will convert to advection form & transport form)

assuming $\nabla\cdot\vec{u} = 0$

align axes to make $\bar{v} = 0$

$$\cancel{\frac{\partial \bar{c}}{\partial t}} + \bar{u}\frac{\partial \bar{c}}{\partial x} + \cancel{\bar{v}}\frac{\partial \bar{c}}{\partial y} + \bar{w}\frac{\partial \bar{c}}{\partial z} = \cancel{\mathcal{D}\nabla^2\bar{c}} + \cancel{\bar{Q}} - \frac{\partial\overline{u'c'}}{\partial x} - \frac{\partial\overline{v'c'}}{\partial y} - \frac{\partial\overline{w'c'}}{\partial z}$$

stationarity

0

zero by continuity eqn in homog flow

neglect if $P_e = \frac{UL}{\mathcal{D}} \gg 1$, U and L are veloc. + length scales

neglect

neglect**

zero if problem has symmetry along y

In the ABL $\bar{u} \gg u'$ except very close to ground. Common to assume

$$\left| \frac{\partial}{\partial x} \bar{u}\, \bar{c} \right| \gg \left| \partial \overline{u'c'} / \partial x \right|$$

vertical eddy flux of mass (unknown)

unknown

Now we have $\bar{u} \dfrac{\partial \bar{c}}{\partial x} = - \dfrac{\partial}{\partial z} \overline{w'c'}$ and let $\bar{u} = \bar{u}(z)$

given

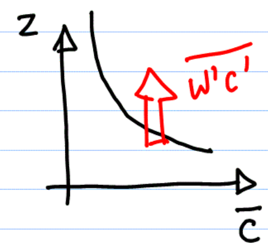
"The turbulence closure problem"

(i) higher order closure, introduce (derive) eqn for $\dfrac{\partial \overline{w'c'}}{\partial t} = - \bar{u} \dfrac{\partial \overline{w'c'}}{\partial x} - \dots$

but this brings in more unknowns, like $\overline{w'w'c'}$

or (ii) invoke eddy diffusion closure (K-theory, flux-gradient theory)

e.g. $\overline{w'c'} = - K_{zm} \dfrac{\partial \bar{c}}{\partial z}$, K_{zm} eddy diffusivity (for mass) $[m^2 s^{-1}]$, huge w.r.t. $\mathcal{D}$



$$\bar{u}(z) \frac{\partial \bar{c}}{\partial x} = \frac{\partial}{\partial z}\left[K_{zm}(z) \frac{\partial \bar{c}}{\partial z} \right] + \frac{\partial}{\partial y}\left[K_{ym}(z) \frac{\partial \bar{c}}{\partial y} \right]$$

Gaussian Plume Model

if we wanted to retract the assumption of symmetry along y

Treat the K's as constants, and $\bar{u}(z) \rightarrow U$

$$U \frac{\partial \bar{c}}{\partial x} = K_z \frac{\partial^2 \bar{c}}{\partial z^2} + K_y \frac{\partial^2 \bar{c}}{\partial y^2} \; * ,$$

solutions are easy, analytical, Gaussians

For a continuous point source of strength Q [$kg\,s^{-1}$] at height h above a non-absorbing ground surface, solution is

$$\bar{c}(x,y,z) = \frac{Q}{2\pi U \sigma_y \sigma_z} e^{-y^2/2\sigma_y^2} \left[e^{-(z-h)^2/2\sigma_z^2} + e^{-(z+h)^2/2\sigma_z^2} \right]$$

where $\sigma_z = \sqrt{2 K_z x / U}$, $\sigma_y = \sqrt{2 K_y x / U}$

in practice these are evaluated from empirical curves, eg. the "Pasquill-Gifford" curves

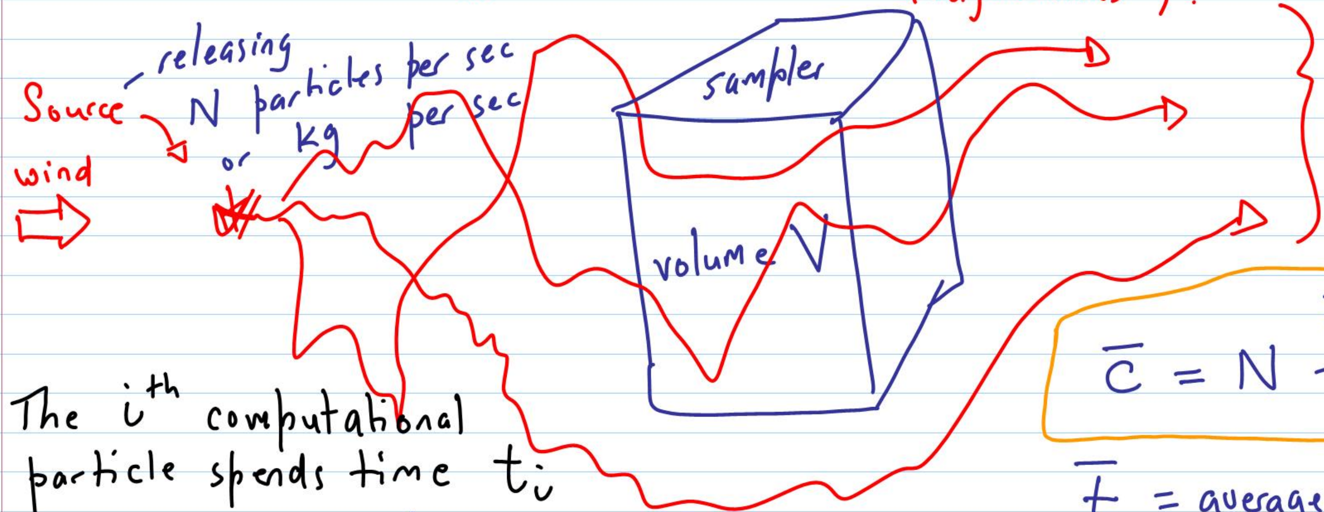
(since x/U = travel time t, can write as $\sigma_z = \sqrt{2 K_z t}$ etc.

* a "Fickian" diffusion eq^n

B) Lagrangian approach. Our mass consv. eqn can be written

$$\frac{dc}{dt} = -\nabla\cdot\big(\underbrace{\cancel{-\mathcal{D}}}_{\text{assume } P_e \gg 1}\nabla c\big) + \underbrace{\cancel{Q}}_{0} = 0$$

, i.e. to a good approx. conc. is conserved (const along fluid element trajectories).



$$\boxed{\bar{c} = N\,\frac{\bar{t}}{V}}$$

link between position statistics and mean conc

The i^{th} computational particle spends time t_i in the sampler (and we compute N_p paths).

$$\bar{t} = \text{average residence time} = \frac{1}{N_p}\sum_{i=1}^{N_p} t_i$$