EAS 471 **Discretizations of the 1D linear advection eqn** 2 FEB 2016

$\phi = \phi(x,t)$ is governed by $\frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial x} = 0$, U = given const.

Will need init cond & one b/cond on x axis. We know solution is

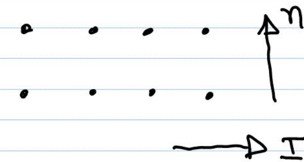
$$\phi(x,t) = \phi(x - Ut, 0) = \phi_0(x - Ut) \; , \quad \phi_0(x) \text{ is initial field.}$$

(this true because $\frac{D\phi}{Dt} = 0$, i.e. ϕ const. following the motion.)

Discretizing

$$\phi_I^n = \phi(I\Delta x, n\Delta t)$$

$$= \phi_0(\underbrace{I\Delta x - nU\Delta t}_{= I'\Delta x})$$



$= I'\Delta x$, is I' an integer?

Named algorithms

Euler

$$\frac{\phi_I^{n+1} - \phi_I^n}{\Delta t} = -u \frac{\phi_{I+1}^n - \phi_{I-1}^n}{2\Delta x}$$

$O[\Delta t]$ $O[\Delta x^2]$

centred on $n + 1/2$ centred on n

explicit, unconditionally unstable

Upstream differencing

explicit, conditionally stable

$$\frac{\phi_I^{n+1} - \phi_I^n}{\Delta x} = -u \begin{cases} \dfrac{\phi_I^n - \phi_{I-1}^n}{\Delta x} & u > 0 \\[2ex] \dfrac{\phi_{I+1}^n - \phi_I^n}{\Delta x} & u < 0 \end{cases}$$

Damped or neutral solution provided

$$0 \leq \frac{u \Delta t}{\Delta x} \leq 1$$

Courant number ($\frac{u \Delta t}{\Delta x}$)

Leapfrog scheme (3 time level)

$$\frac{\phi_I^{n+1} - \phi_I^{n-1}}{2\Delta t} = -u \frac{\phi_{I+1}^{n} - \phi_{I-1}^{n}}{2\Delta x}$$

$O[\Delta t^2]$ centred on n, I $\qquad$ $O[\Delta x^2]$ centred on n, I

We showed diagrammatically that for any "nearest neighbour" scheme there is liable to be a problem unless

$$\frac{u \Delta t}{\Delta x} \leq 1$$

The CFL condition

Courant - Freidrich - Levy

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aside: 1:2:1 spatial filter

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$I-1 \quad I \quad I+1$

$$T_I^{corr} = \frac{T_{I-1} + 2T_I + T_{I+1}}{4}$$

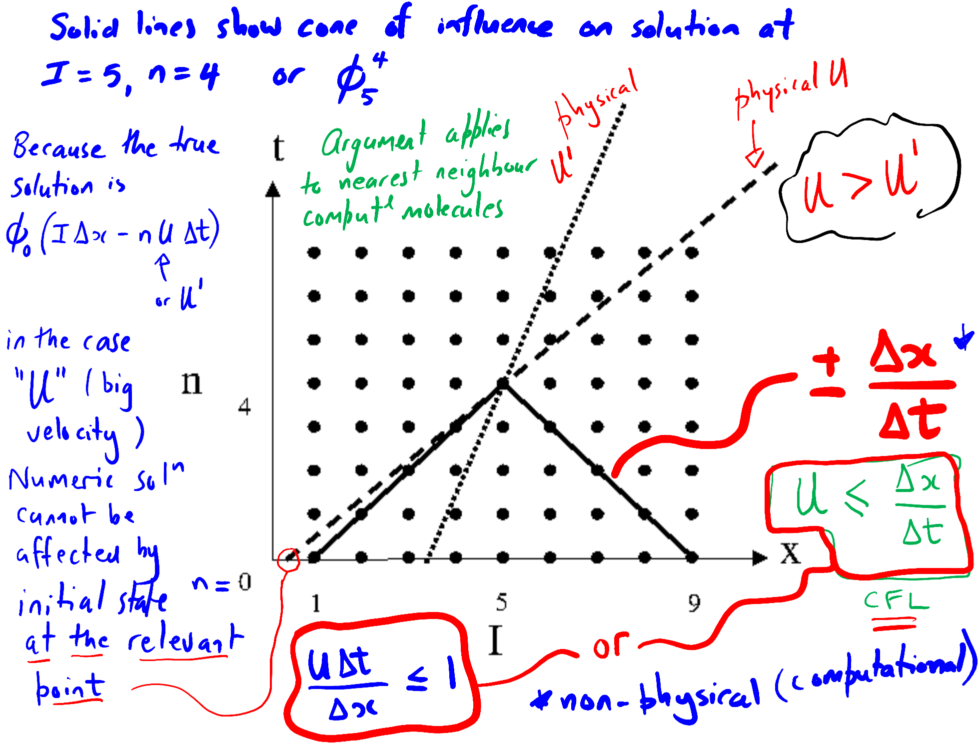

Figure 2.1: Cone of influence; from Haltiner and Williams p121