Thought expt. [How to arrive at a quantitative relationship between rate of emission of some gas and the resulting gas concentration at a downwind point]

EAS 471, 5 Jan 2016

What can affect c? Wind, distance from source, ~~topography~~ big complication, stratification

- Reduce to 2D ≡ wind blows along $x$

Philosophy / approach ≡ statistical

What tools do we have?

1. empiricism
2. dimensional analysis
3. principles of physics
4. symmetries?
5. Idealization

- Define $\bar{u}(x,y,z,t)$ as 30 min mean horiz wind speed

- assume "horizontally homog." and "stationary" 30 min mean wind

$Q\,[\mathrm{kg\,s^{-1}}]$

cts source at $(0,0,h)$

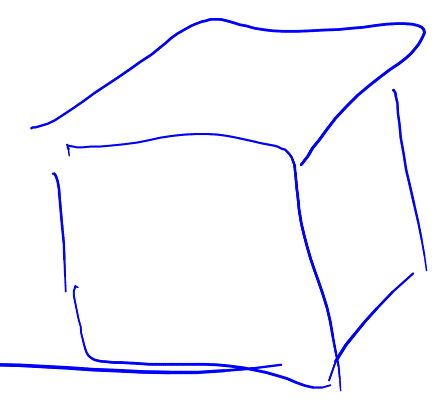
plume

passive gas

- assume problem dominated by convection

detector

$\bar{c}(X,Y,0)$



We can let $\bar{u}(x, y, z, t) \longrightarrow \bar{u}(z)$ [horiz. homog. and stationary flow]

So far we have:

| $Q$ | $X$ | $\bar{u}$ | $\bar{c}$ |
|---|---|---|---|
| forcing | where we are | the atmos. | what we want to know |

$n = 4$ vrbls

$m = 3 \begin{cases} kg \\ m \\ s \end{cases}$

Suppose we believed these four variables delineate the problem & nothing else relevant. Then

the Pi theorem says $\exists$ law connecting $n-m$ unitless ratios formed from these $n=4$

$$\frac{\bar{u}\;\bar{c}\;X^2}{Q} \quad \left[\frac{kg}{m^3}\;\frac{s}{kg}\;\frac{m}{s}\;m^2\right] = \text{const.} = c$$

$$\frac{\bar{u}\;\bar{c}}{Q} = \frac{c}{X^2}$$

If our conjecture (that only $Q, X, \bar{u}, \bar{c}$ are needed) were correct then this is the correct solution. But were we realistic?

Oops! $\bar{u}$ varies with $z$! Add $z$, $n=5$, $m=3$

$$\frac{\bar{u}_{10}\,\bar{c}(z)}{Q}x^2 = f\left(\frac{z}{x}\right)$$

$\bar{u}_{10}$ mean wind speed at $z = 10\,m$

(One could have refined the dimensional analysis much further)

Physics approach?

Mass conservation. Let $c$ be instantaneous local conc $c = c(x,y,z,t)$

$$\frac{\partial c}{\partial t} = ?$$

Vector convective flux of $c$ is

$$\vec{F} = \vec{u}\,c$$

$$= -\nabla \cdot \vec{F} + Q_*$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$= -\frac{\partial}{\partial x}uc - \frac{\partial}{\partial y}vc - \frac{\partial}{\partial z}wc + Q_*$$

$\mathrm{kg\,m^{-3}\,s^{-1}}$

Reynolds averaging gives

$$\frac{\partial \bar{c}}{\partial t} = -\frac{\partial}{\partial x}\overline{uc} - \frac{\partial}{\partial y}\overline{vc} - \frac{\partial}{\partial z}\overline{wc} + \overline{Q_*}$$

where (e.g.) $\overline{wc} = \bar{w}\,\bar{c} + \overline{w'c'}$

eddy flux

$w = \bar{w} + w'$ etc....

$$\frac{\partial \bar{c}}{\partial t} + \underbrace{\bar{u}\frac{\partial \bar{c}}{\partial x} + \bar{v}\frac{\partial \bar{c}}{\partial y} + \bar{w}\frac{\partial \bar{c}}{\partial z}}_{\text{advection by the resolved velocity field}} = -\frac{\partial \overline{u'c'}}{\partial x} - \frac{\partial \overline{v'c'}}{\partial y} - \frac{\partial \overline{w'c'}}{\partial z} + \overline{Q_*}$$

$\vec{\bar{u}} \cdot \nabla \bar{c}$ or $\underset{\sim}{u} \cdot \nabla \bar{c}$

our problem

$Q\,\delta(x-0)\,\delta(y-0)\,\delta(z-h)$