MO Wind Profile

We found $$\frac{z}{u_*}\frac{\partial \bar{u}}{\partial z} = f\left(z/\hat{L} \right)$$

Define the Obukhov length as $$L = \frac{-u_*^3}{k_v \left(g/T_0\right) \overline{w'\theta'}}$$

$$\frac{k_v z}{u_*}\frac{\partial \bar{u}}{\partial z} = \phi_m\left(\frac{z}{L}\right)$$

Aside: properties M&O did **not** include in their dimensional analysis.
latitude? f? U_g, V_g? δ?

Only experimentation can fix the function

Stable stratification $\quad \phi_m = 1 + \beta \frac{z}{L}, \quad \beta \simeq 5$

$$\frac{k_v}{u_*}\frac{\partial \bar{u}}{\partial z} = \frac{1}{z} + \beta / L \longrightarrow \frac{k_v}{u_*}\bar{u} = \ln z + \beta\frac{z}{L} + c_3$$

Define $z = z_0$ as height where $\bar{u} = 0$, implying $c_3 = -\ln z_0 - \beta \frac{z_0}{L}$

$$\bar{u} = \frac{u_*}{k_v}\left[\ln\frac{z}{z_0} + \beta\frac{z - z_0}{L} \right] \xrightarrow[\infty]{|L|} \frac{u_*}{k_v}\ln\frac{z}{z_0}$$

MO Profile for temperature

15 March 2016

Define $\theta_* = -\overline{w'\theta'}/u_*$ "turbulent temperature scale"

u_*, $\overline{w'\theta'}$, z, g/T_o

want $\overline{\theta}(z)$

$$\frac{k_v z}{\theta_*}\frac{\partial\overline{\theta}}{\partial z} = \dots = \theta_h\left(\frac{z}{L}\right)$$

Stable temp profile $\overline{\theta}(z) = \overline{\theta}(z_{0T}) + \frac{\theta_*}{k_v}\left[\ln\frac{z}{z_{0T}} + \beta\frac{z - z_{0T}}{L}\right]$

Humidity?

Define $q_* = -\overline{w'q'}/u_*$ (or $\rho_{v*} = -\overline{w'\rho_v'}/u_*$)

"turbulent humidity scale"

$$\boxed{L = \frac{u_*^2}{k_v \, g/T_o \, \theta_*}}$$

hh-ASL fully "characterized" by (u_*, L) or (u_*, θ_*) plus z_o, wind dir, q_* … δ

Refresher on the ABL and the ASL – idealized as horizontally-homogeneous

Variables of interest:

$\delta(t), u_*, L, w_*, \theta_*$ scaling variables

$\bar{u}(z), \bar{v}(z), \bar{\theta}(z), \bar{q}(z)$

$\overline{u'^2}(z), \overline{v'^2}(z), \overline{w'^2}(z), \overline{\theta'^2}(z), \overline{q'^2}(z)$

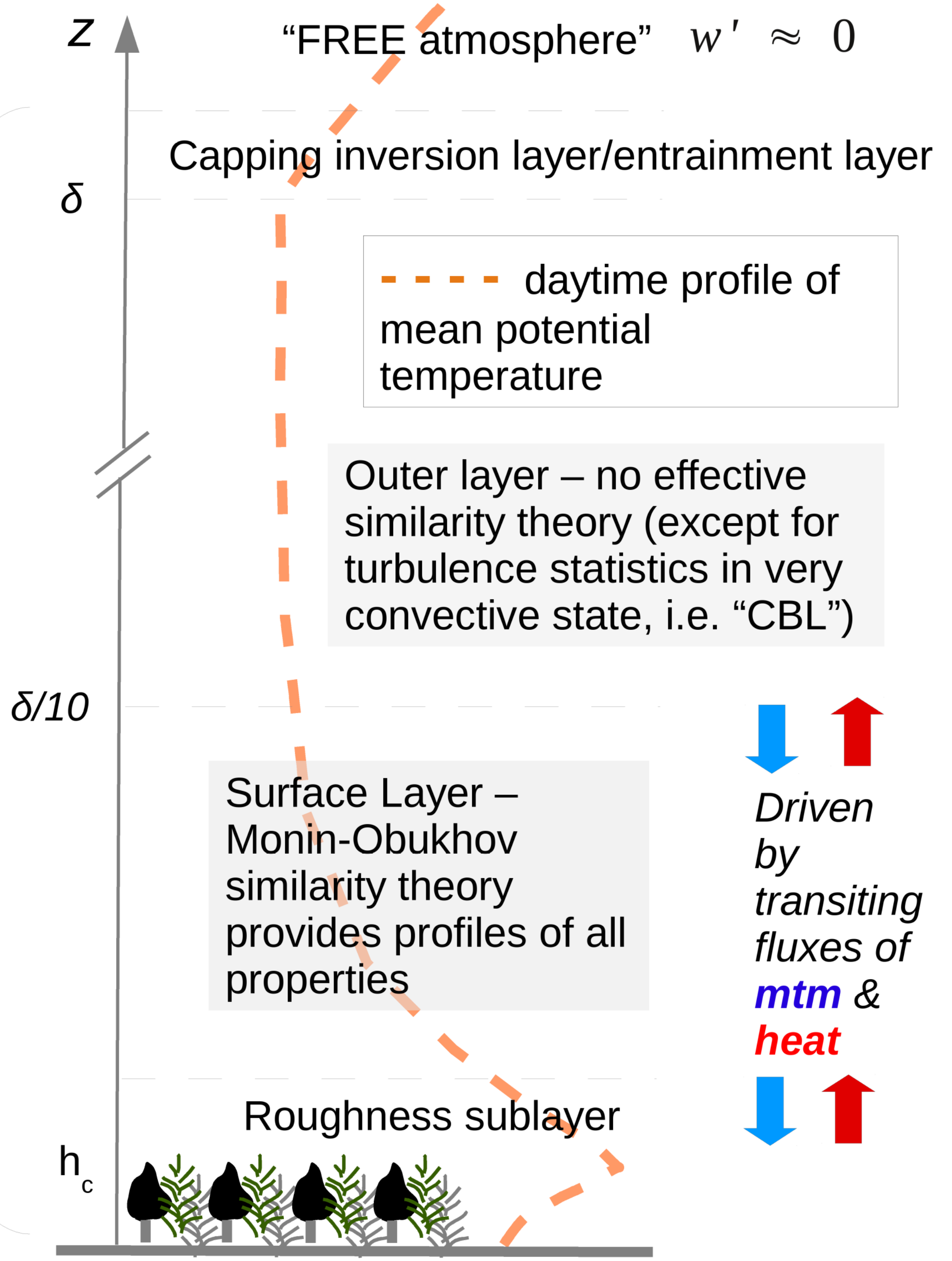
$\overline{u'w'}(z), \overline{v'w'}(z), \overline{w'\theta'}(z), \overline{w'q'}(z)$

$k(z), \epsilon(z)$

Atmospheric boundary layer
(Friction layer)

Unresolved ("turbulent") velocity fluctuations

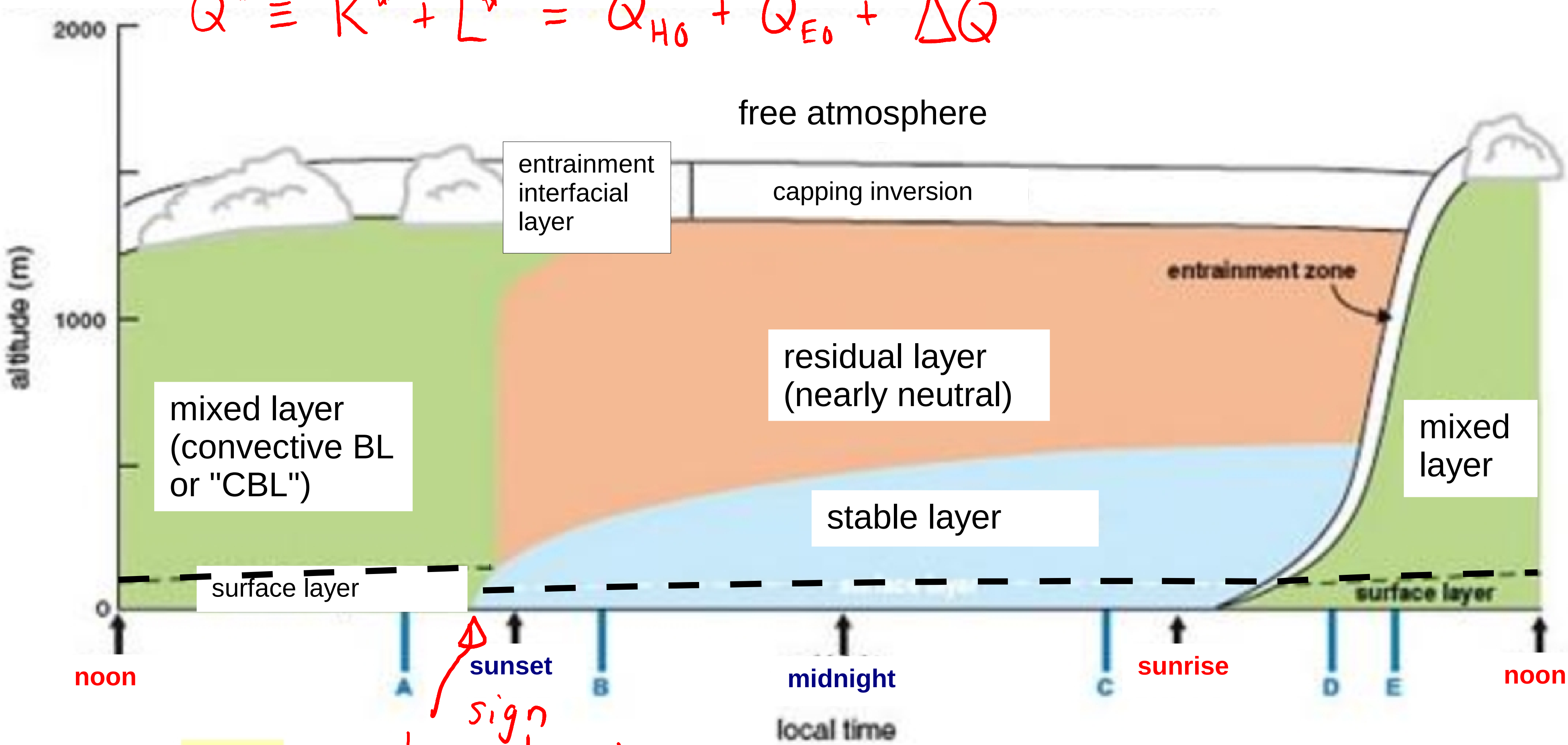
$$u' \approx v' \approx w' = \mathrm{O}[\mathrm{m\ s^{-1}}]$$



eas471_ABL_ASL.odp
JDW 15 Mar. 2016

Setting the ASL within the atmospheric boundary layer (ABL)

$$Q^* \equiv K^* + L^* = Q_{H0} + Q_{E0} + \Delta Q$$

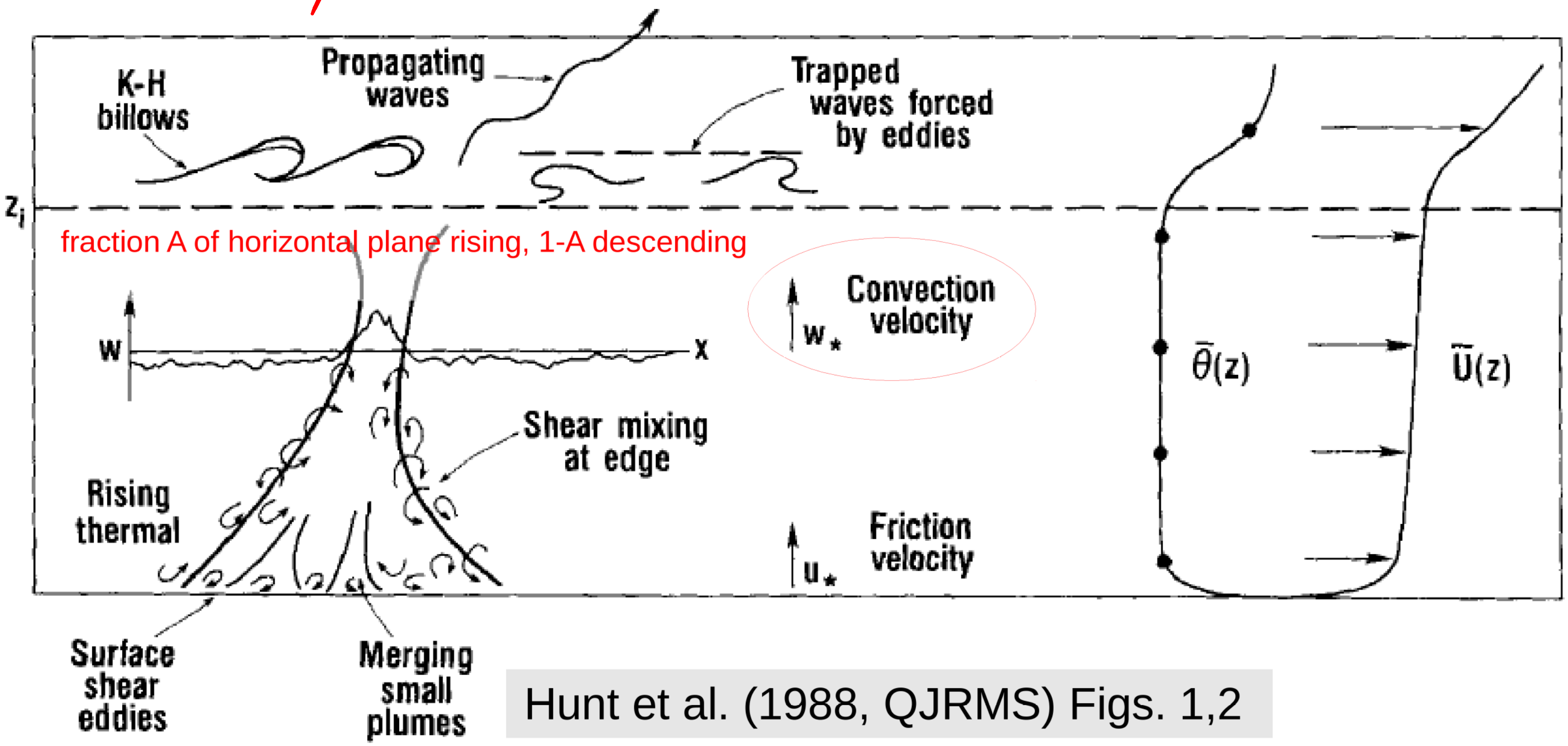


Boundary Layer Meteorology

MOST stems from dimensional analysis guided by measurement, intuition and guesswork: it is justified by its *utility* (cannot be "proven", i.e. not an exact implication or consequence of the Navier-Stokes and other consv. eqns)

roughness sublayer } extends from surface to 2-5 times the height of the roughness elements and includes the canopy layer. Within the roughness sublayer the flow is three-dimensional... MO scaling doesn't apply.

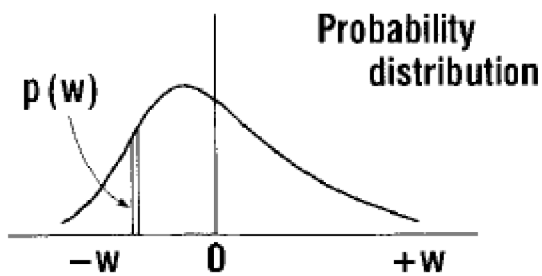
Idealized daytime ABL



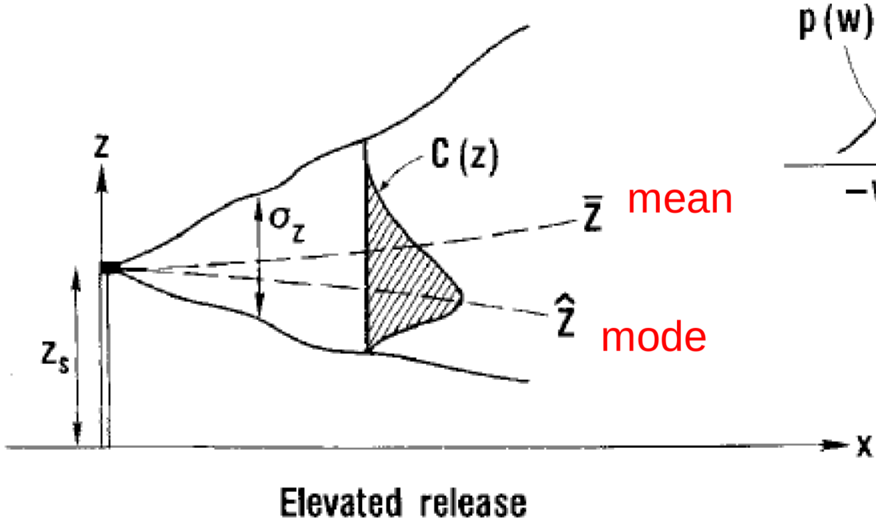
Hunt et al. (1988, QJRMS) Figs. 1,2

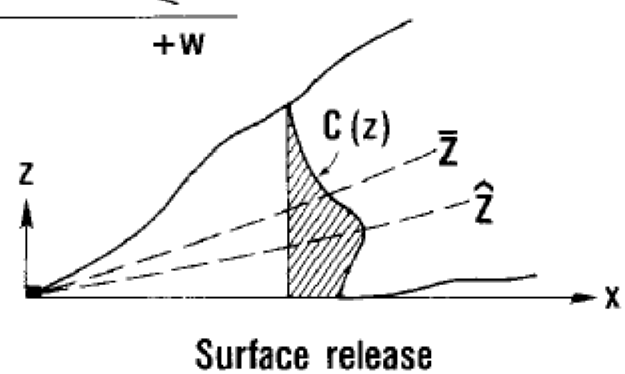
well mixed profiles except in surface layer

Figure 1. Schematic diagram of the flow, temperature and eddy structure of the convective boundary layer.

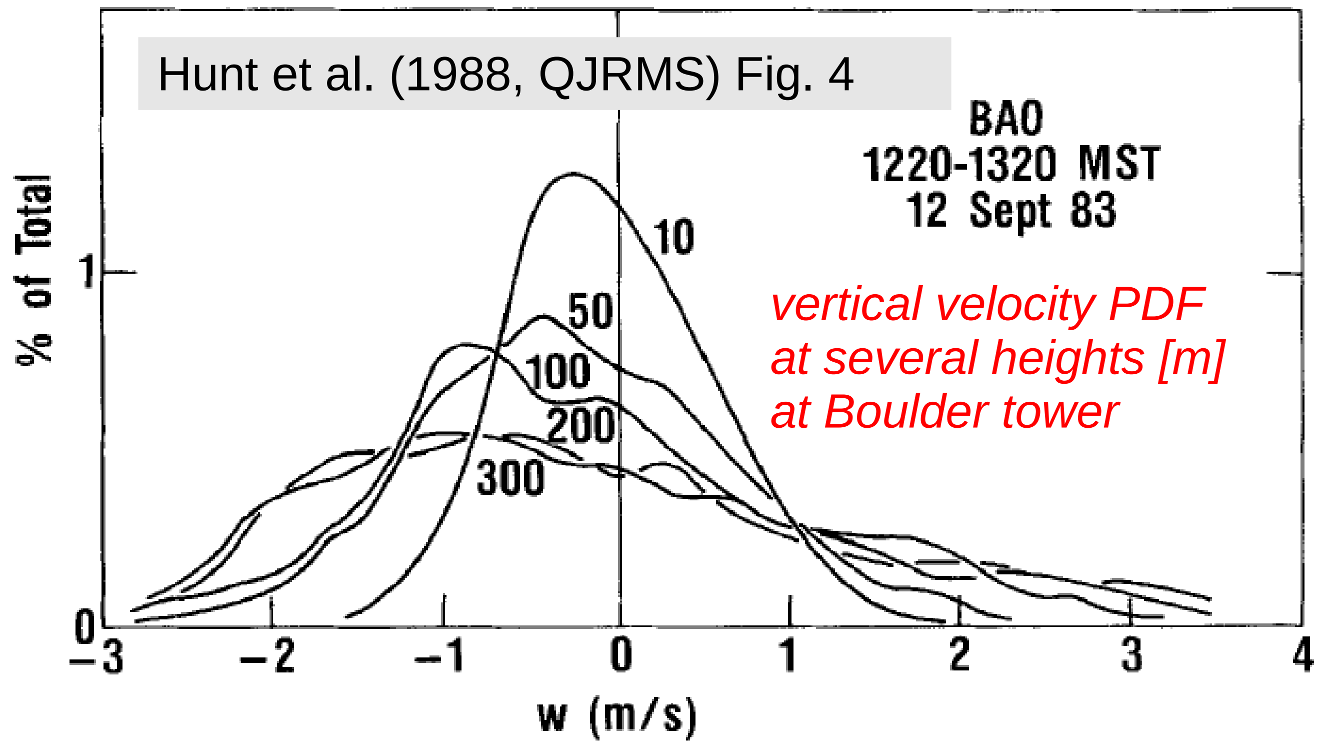


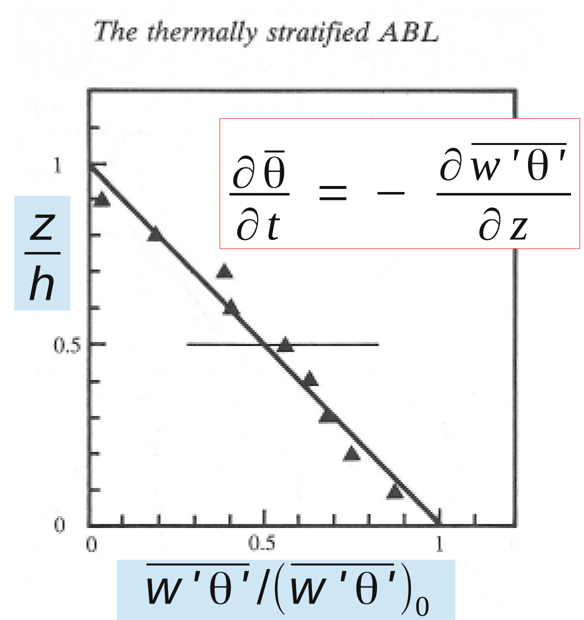
- CBL a "windless convection layer"
- skew PDF of *w*





CBL velocity statistics & mixed layer similarity theory





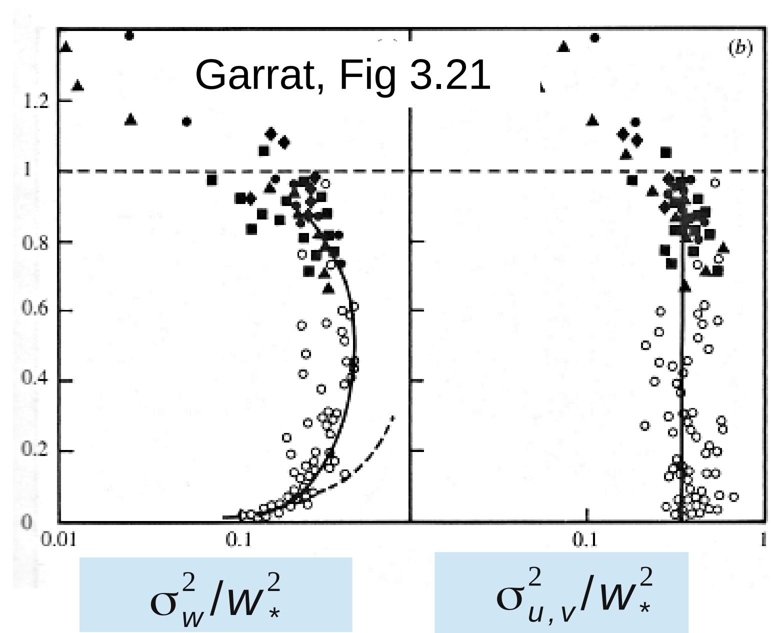
Garratt, Fig 6.13

Mixed-layer turbulence scales: *δ*, *w**, *θ**

$$w_* = \left[\frac{g}{\theta_0}\, \delta\, \left(\overline{w'\theta'} \right)_0 \right]^{1/3}$$

$$\theta_* = -\left(\overline{w'\theta'} \right)_0 / w_*$$

$$\frac{z}{\delta} \equiv \frac{z}{h}$$



Mixed-layer similarity scales successfully normalize turbulence statistics, but no similarity theory has provided a reliable formulation for the mean profiles, which are often "unmixed"

Reynolds-averaged governing equations for ABL (Garratt's Sec. 2.3)

- Decompose every variable into sum of Mean + Fluctuation $w = W + w'$

- Apply the averaging operation to the Navier-Stokes equations → Reynolds' equations

$$\frac{\partial U}{\partial t}+\frac{\partial}{\partial x}\left(U^2+\overline{u'u'}\right)+\frac{\partial}{\partial y}\left(UV+\overline{u'v'}\right)+\frac{\partial}{\partial z}\left(UW+\overline{u'w'}\right)=\frac{-1}{\rho}\frac{\partial P}{\partial x}+fV+\nu\nabla^2 U$$

non-divergence

Viscous friction

Posit horizontal homogeneity of *statistics* – then indicated terms vanish, leaving:

no evap/cond.

no radiative divergence

$$\frac{\partial U}{\partial t}=-\frac{\partial\overline{u'w'}}{\partial z}+f\left(V-V_G\right)$$

$$\frac{\partial\overline{\theta}}{\partial t}=-\frac{\partial\overline{w'\theta'}}{\partial z}$$

(corresponding heat budget)

"turbulent friction" (divergence of Reynolds stress). Fluctuations u' and w' are (anti-) correlated – so $\overline{u'w'}$ is a downward flux of u-momentum

"Geostrophic deficit" (balances to zero in free atmos) Large scale pressure gradient defines the Geostrophic wind (so $f\,V_G$ is just a substitution)

- Companion eqn. $$\frac{\partial V}{\partial t}=-\frac{\partial\overline{v'w'}}{\partial z}-f\left(U-U_G\right)$$

Turbulent Kinetic Energy (TKE) equation

- TKE defined $$k = \frac{1}{2}\left(\overline{u'u'}+\overline{v'v'}+\overline{w'w'}\right) \equiv \frac{1}{2}\left(\sigma_u^2+\sigma_v^2+\sigma_w^2\right)$$

- by manipulating the Navier-Stokes eqns. (under the Boussinesq approx.), the TKE eqn for a <u>horizontally-homogeneous</u> layer is:

$\epsilon = \nu \overline{\frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j}}$

$$\frac{\partial k}{\partial t} = \underbrace{-\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z}}_{\text{shear production}} + \underbrace{\frac{g}{\theta_0}\overline{w'\theta'}}_{\text{buoyant prodn}} \underbrace{- \frac{\partial}{\partial z}\overline{w'\left(\frac{p'}{\rho} + \frac{u'u'+v'v'+w'w'}{2}\right)}}_{\text{pressure transport + turbulent transport}} \underbrace{- \epsilon}_{\text{viscous dissip'n}}$$

- shear production large in surface layer due to strong near-ground shear
- buoyant production dominates above the surface layer
- at $|z/L| = 1$, shear and buoyant production have equal magnitude
- wherever the heat flux is negative (i.e. in inversion layers) buoyancy suppresses turbulence
- the transport term often modeled as "diffusion"
- viscosity damps the motion of the smallest eddies, converting their kinetic energy to heat
- TKE (k) and its dissipation rate (ϵ) key variables in most turbulence models

Form of the TKE equation for a horizontally-homogeneous layer under first-order closure

MO $\frac{k_v z}{u_*}\frac{\partial \bar{u}}{\partial z} = \phi_m \longrightarrow 1$ (neutral)

$\therefore\ u_*^2 = k_v u_* z \frac{\partial \bar{u}}{\partial z}$

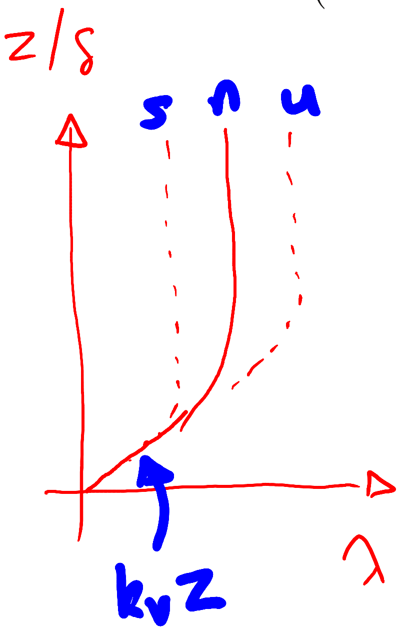
eddy viscosity

$\therefore\ \overline{u'w'} = - k_v u_* z \frac{\partial \bar{u}}{\partial z}$

- invoke

$$(\overline{u'w'}, \overline{v'w'}) = -K\frac{\partial}{\partial z}(U, V)$$

$$\overline{w'\theta'} = -K_H \frac{\partial \bar{\theta}}{\partial z}$$



$$\epsilon = \frac{(c\,k)^{3/2}}{\lambda}$$

$\lambda=\lambda(z)$ an empirical "length scale", c a tunable constant

$k_v z$

$$K = (c\,k)^{1/2}\ \lambda(z) \qquad \text{and} \qquad K_H = K/P_r$$

calibrate this to give u_*

generally

$$K^{mo} = \frac{k_v u_* z}{\phi_m\left(\frac{z}{L}\right)}$$

- result:

$$\frac{\partial k}{\partial t} = K\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] - \frac{g}{\theta_0} K_H \frac{\partial \bar{\theta}}{\partial z} + \frac{\partial}{\partial z}\left(K_k \frac{\partial k}{\partial z}\right) - \frac{(c\,k)^{3/2}}{\lambda(z)}$$

shear production — buoyant prodn — diffusion — dissipation

- ratio k/ε is effectively a turbulence "time scale"

Delage exploit this eqn plus those for U, V, θ in his time-dependent model of the nocturnal ABL – to follow in a later lecture