Modelling an idealized nocturnal (stable) boundary-layer ("NBL" or "SBL")

– *in what sense* idealized? Cloudless, unsaturated, horizontally homogeneous

and neglects radiative divergence $\nabla \cdot \vec{R}$

From Stull (1988), *An Intro. To Boundary Layer Meteorology* (see also Garratt's Fig. 6.1)

eas471_SBL_Delage.odp
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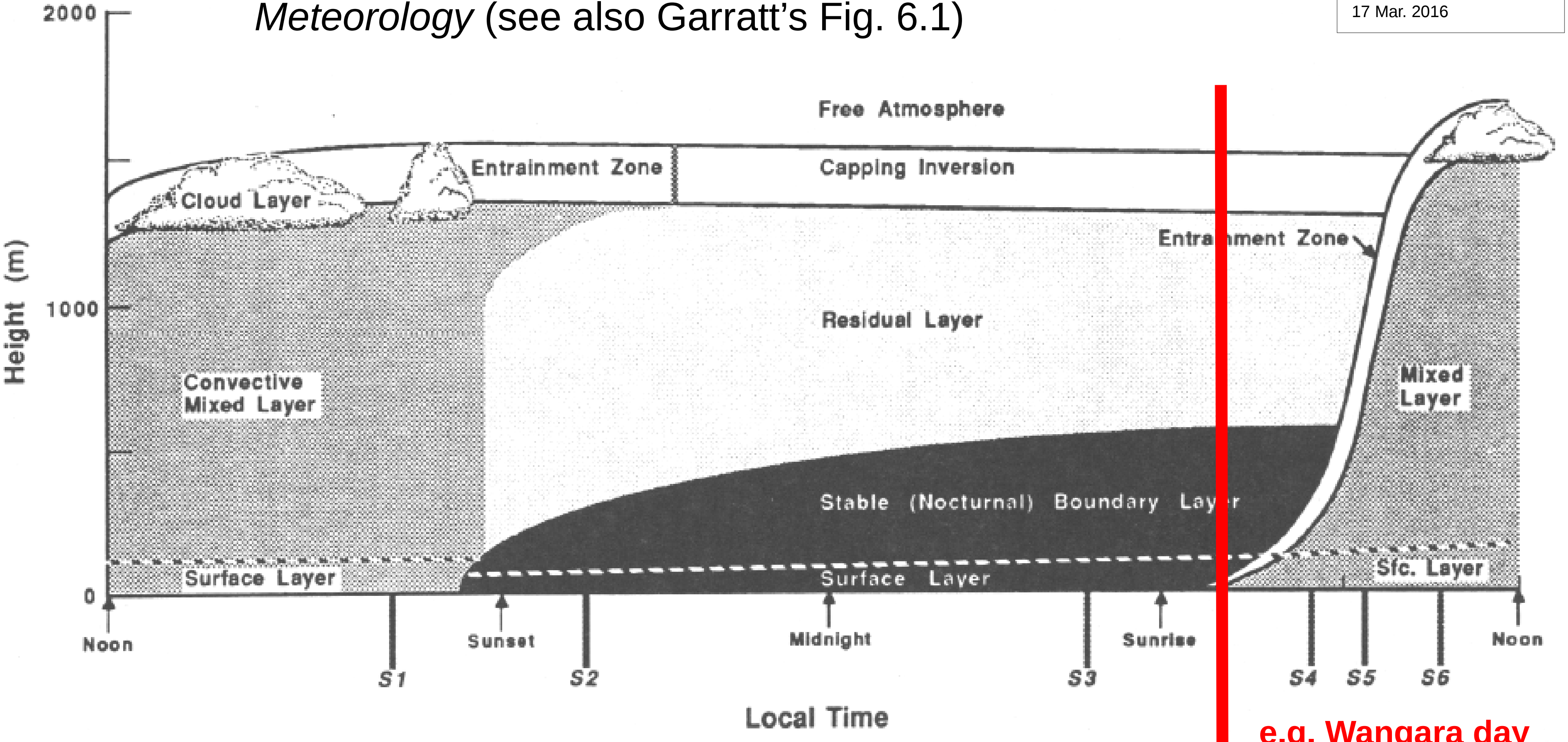
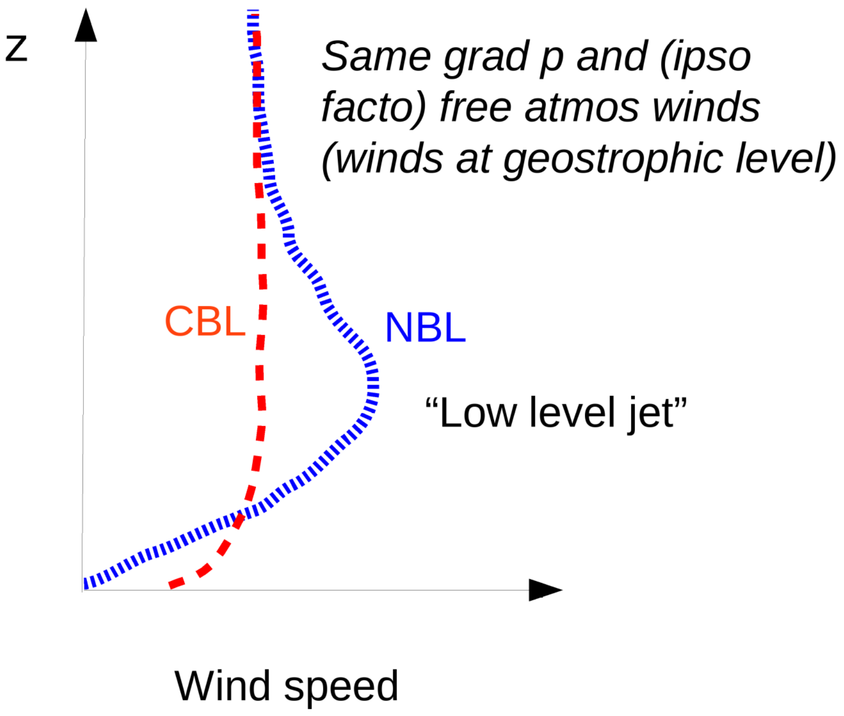
17 Mar. 2016



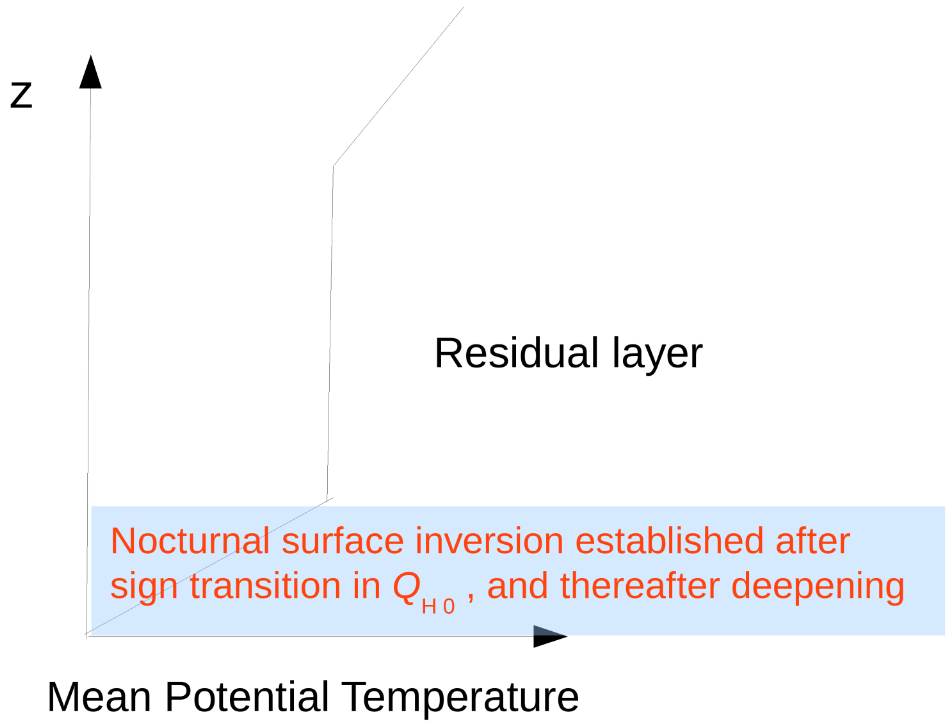
e.g. Wangara day 33 at 0900

Fig. 1.7 The boundary layer in high pressure regions over land consists of three major parts: a very turbulent mixed layer; a less-turbulent residual layer containing former mixed-layer air; and a nocturnal stable boundary layer of sporadic turbulence. The mixed layer can be subdivided into a cloud layer and a subcloud layer. Time markers indicated by S1-S6 will be used in Fig. 1.12.



Same grad p and (ipso facto) free atmos winds (winds at geostrophic level)

“Low level jet”



Nocturnal surface inversion established after sign transition in Q_{H0} , and thereafter deepening

Weakened friction hints at possibility of inertial oscillations in horizontal velocity

$$\left.\begin{aligned} \frac{\partial U}{\partial t} &= -\frac{\partial \overline{u'w'}}{\partial z} + f\,(V - V_G) \\ \frac{\partial V}{\partial t} &= -\frac{\partial \overline{v'w'}}{\partial z} - f\,(U - U_G) \end{aligned}\right\} \quad \frac{\partial^2 U}{\partial t^2} = + f\,\frac{\partial V}{\partial t} = -f^2\,U + f^2 U_G$$

turbulence damped out?

Energetics of the NBL – perspective of the TKE equation & surface energy budget

- surface energy budget results in surface cooling

$$Q^* \equiv \underset{\downarrow \, 0}{K^*} + \underset{L^*<0}{L^*} = Q_{H0} + \underset{\downarrow \, 0}{Q_{E0}} + \underset{\downarrow \, 0}{Q_G} < 0$$

(idealization – dry surface)

- thus $Q_{H0} < 0$, sensible heat is extracted from the base of the layer in conductive/convective contact with the surface

- TKE budget: (horiz homog form: in general, have advection terms and other production terms

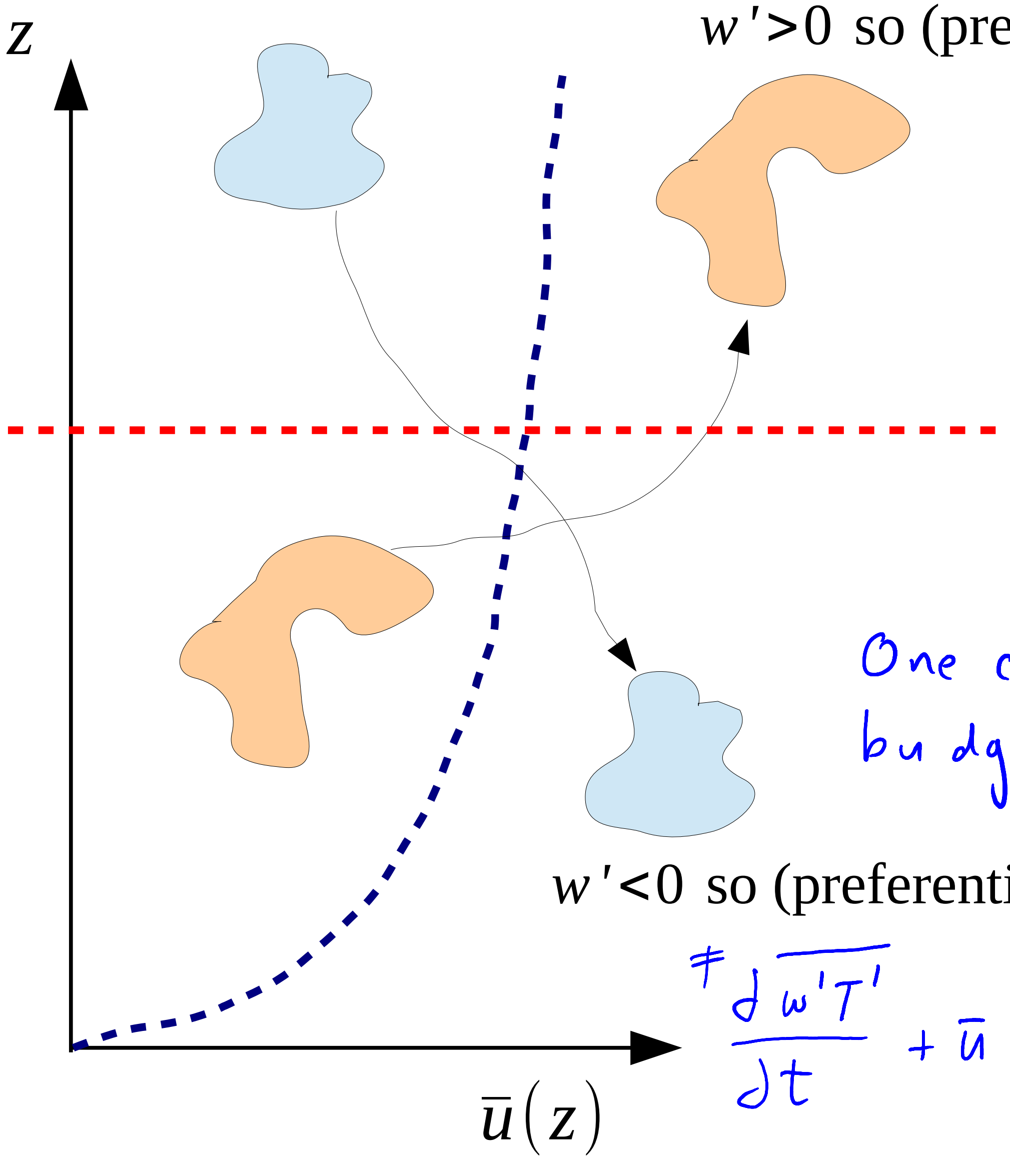
$$\frac{\partial k}{\partial t} = \underbrace{-\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z}}_{\text{shear production}} + \underbrace{\frac{g}{\theta_0}\overline{w'\theta'}}_{\text{buoyant prodn}} \underbrace{- \frac{\partial}{\partial z}\overline{w'\left(\frac{p'}{\rho_0} + \frac{u'u'+v'v'+w'w'}{2}\right)}}_{\text{pressure transport + turbulent transport}} \underbrace{- \epsilon}_{\text{viscous dissip'n}}$$

- as daytime winds die down, shear production is reduced; and because the **layer is stably stratified buoyant production is negative**, offsetting what (little?) shear production continues

- thus turbulence dies down to a low level – unless a strong free atmos. wind sustains shear production and overcomes buoyant destruction of TKE, so as to sustain mixing and limit the strength of the inversion

- and/or unless heavy cloud cover prevents rapid sfc cooling by longwave radiation

Reminder about the physical mechanism behind the velocity covariances (eddy momentum fluxes)



$w'>0$ so (preferentially) $u'<0$

$w'<0$ so (preferentially) $u'>0$

Furthermore, taking this as a daytime scenario such that the orange parcel (originating near ground) is warm, we can also see that rising parcels will carry positive T' so that (accordingly) $\overline{w'T'}>0$

One can derive exact (but unclosed) budget eqns for these covariances ‡

$$‡\ \frac{\partial \overline{w'T'}}{\partial t} + \bar{u}\frac{\partial \overline{w'T'}}{\partial x} + \ldots = -\overline{w'^2}\frac{\partial \bar{\theta}}{\partial z} + \frac{g}{T_0}\overline{T'^2}$$

$$+ \ldots \quad - \frac{\partial}{\partial z}\overline{w'w'T'} + \ldots$$

In terms of shear production, then, we see that $-\overline{u'w'}\frac{\partial \bar{u}}{\partial z}$ is positive

$$-\frac{1}{\rho_0}\overline{w'\,\partial p'/\partial z}$$

Energetics of the NBL – perspective of the velocity variance equations ($\overline{w'w'} \equiv \sigma_w^2$ etc.)

By manipulating the Navier-Stokes eqns. (under the Boussinesq approx.), the variance budget eqns for a horizontally-homogeneous layer are:

$$\frac{\partial \sigma_u^2}{\partial t} = -2\,\overline{u'w'}\,\frac{\partial U}{\partial z} - \frac{\partial}{\partial z}\,\overline{w'u'u'} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial u'}{\partial x}} - \epsilon_{uu}$$

$$\frac{\partial \sigma_v^2}{\partial t} = -2\,\overline{v'w'}\,\frac{\partial V}{\partial z} - \frac{\partial}{\partial z}\,\overline{w'v'v'} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial v'}{\partial y}} - \epsilon_{vv}$$

$$\frac{\partial \sigma_w^2}{\partial t} = \underbrace{2\,\frac{g}{\theta_0}\,\overline{w'\theta'}}_{\text{buoyant prodn}} - \underbrace{\frac{\partial}{\partial z}\,\overline{w'\left(\frac{2p'}{\rho_0} + w'w'\right)}}_{\text{turbulent (+ press.) transp.}} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial w'}{\partial z}} - \underbrace{\epsilon_{ww}}_{\text{viscous dissip'n}}$$

(small)

redistribution

$$\frac{\partial}{\partial t}\,\frac{\sigma_u^2+\sigma_v^2+\sigma_w^2}{2} = \dots + \frac{2}{\rho_0}\,\overline{p'\underbrace{\left[\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}\right]}_{\text{redistribution terms sum to zero in TKE eqn}}} - \overset{\text{"}\varepsilon\text{"}}{\frac{\epsilon_{uu}+\epsilon_{vv}+\epsilon_{ww}}{2}}$$

shear and buoyant production, turbulent and pressure transport

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redistribution

(small)

L* < 0 → Q* < 0 → Q_H< 0 → buoyant suppression of the vertical motion (thus) TKE, effective in the energy-containing range of scales; *w'* fed by inter-component transfer (redistribution) alone; lack of energy in *w'* limits heat and (downward) momentum transport by turbulent convection; light winds (measurement challenge), turbulence may be intermittent; gravity waves; ratio of buoyancy (gT'/T_0) to inertial (u'^2/L) forces becomes large, so slight topographic irregularities can result in drainage flows (three-dimensional and intermittent) – see Wyngaard's textbook Eqn. (12.20) where veloc. field parallel to gently sloping sfc contains buoyancy terms

An interesting cycle of intermittency can occur (Van de Wiel et al., J.Atmos.Sci. 67, 2010)

$$R_i^g = \frac{g}{\theta_0}\frac{\partial\theta/\partial z}{(\partial U/\partial z)^2}$$

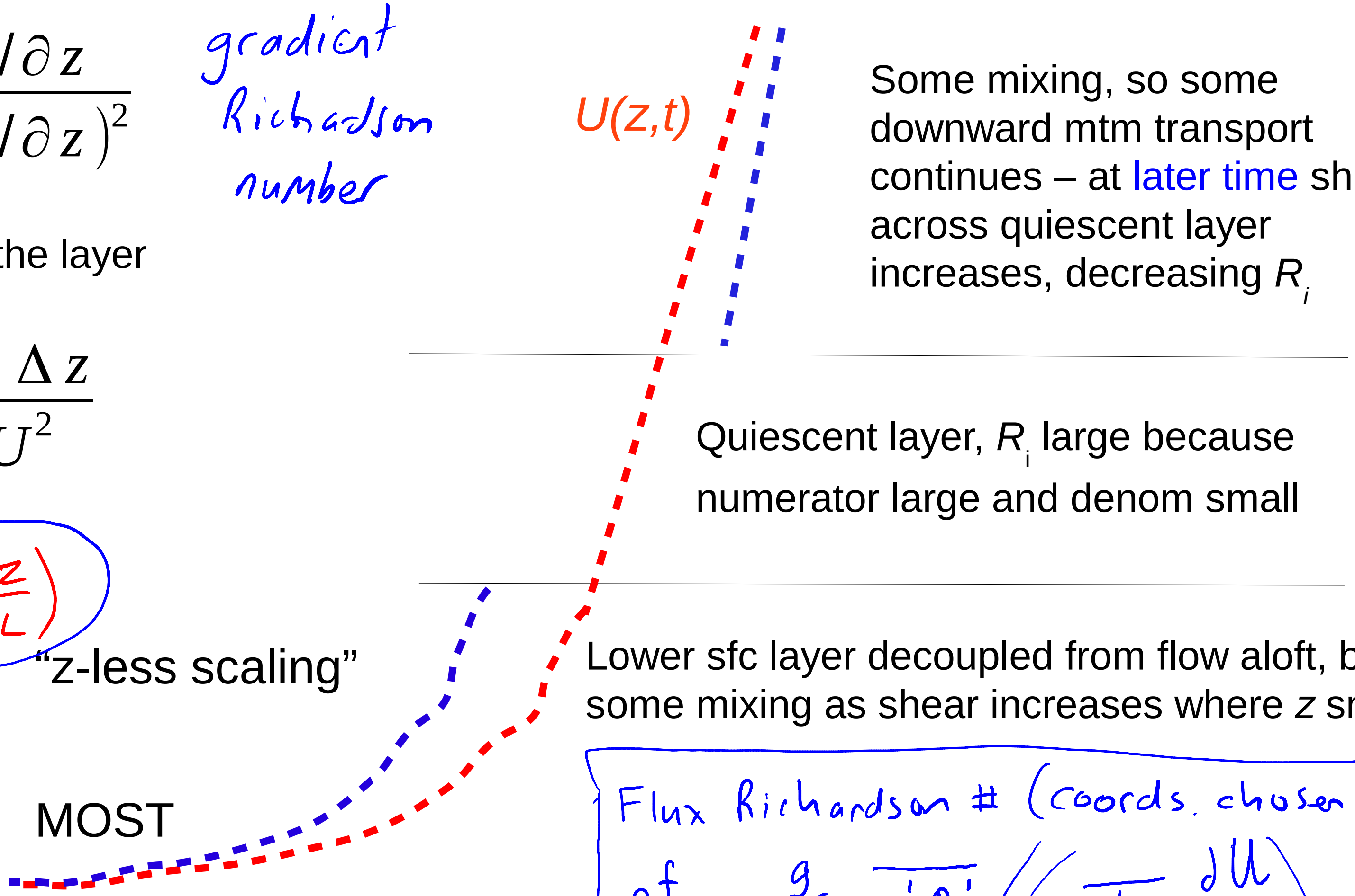
gradient Richardson number

or as bulk index for the layer

$$R_i = \frac{g}{\theta_0}\frac{\Delta\theta\ \ \Delta z}{\Delta U^2}$$

$$\boxed{\frac{\partial U}{\partial z} = \frac{u_*}{k_v z}\phi_m\left(\frac{z}{L}\right)}$$

shear is strongest near ground



Some mixing, so some downward mtm transport continues – at later time shear across quiescent layer increases, decreasing R_i

Quiescent layer, R_i large because numerator large and denom small

Lower sfc layer decoupled from flow aloft, but some mixing as shear increases where *z* small

In MOST $\frac{K_h}{K_m} = \frac{1}{S_c} = \frac{k_{vh}u_* z/\phi_h}{k_v u_* z/\phi_m}$

$= \phi_m/\phi_h$

Critical value of R_i to suppress turbulence surely not universal, but of order 0.1; textbooks cite obs. suggesting about 0.2

Flux Richardson # (coords. chosen so $V=0$)

$$R_i^f = \frac{g}{T_0}\overline{w'\theta'} \Big/ \left(+\overline{u'w'}\frac{\partial U}{\partial z}\right)$$

$$= \frac{g}{T_0}\ -K_h\,\partial\bar\theta/\partial z \Big/ -K_m\left(\frac{\partial U}{\partial z}\right)^2$$

$$= \boxed{\frac{g}{T_0}}\ \frac{K_h}{K_m}\ \boxed{\frac{\partial\bar\theta/\partial z}{(\partial U/\partial z)^2}}\quad R_i^g$$

Delage** (1974; QJRMS Vol. 100) 1-D model of dry SBL – turbulent transport (only)

1st order closure:

$$\overline{u'w'} = -K \frac{\partial \bar{u}}{\partial z}$$

$$-\frac{\partial \overline{u'w'}}{\partial z} = \frac{\partial}{\partial z}\left(K \frac{\partial \bar{u}}{\partial z}\right)$$

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \bar{u}}{\partial z}\right) + f(\bar{v} - V_g)$$

$$\frac{\partial \bar{v}}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \bar{v}}{\partial z}\right) - f(\bar{u} - U_g)$$

$$\frac{\partial \bar{\theta}}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial \bar{\theta}}{\partial z}\right)$$

$$K_h = K_m = K$$

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial k}{\partial z}\right) + K\left[\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right] - \frac{g}{\theta_{00}} K \frac{\partial \bar{\theta}}{\partial z} - \epsilon$$

**formerly of CMC Dorval; Yves Delage had much to do with ABL parameterization in CMC's NWP models

(neglects radiative divergence)

TKE budget of ideal ASL:

$k = \text{const.}$

$$0 = u_*^2 \frac{u_*}{k_v z} - \varepsilon$$

$$-\overline{u'w'} \frac{\partial \bar{u}}{\partial z}$$

Transp. term zero

<u>Closure</u>

$$K = \lambda(z) \sqrt{c_e\, k}$$ eddy diffusivity

effective velocity scale

$$\epsilon = (c_e\, k)^{3/2} / \lambda$$ TKE dissipation rate

$$\frac{1}{\lambda} = \frac{1}{k_v z} + \frac{1}{\lambda_\infty} + \frac{\beta}{k_v L}$$ length scale

reconcileable?

Recall that in context of Monin-Obukhov Similarity Theory (MOST):

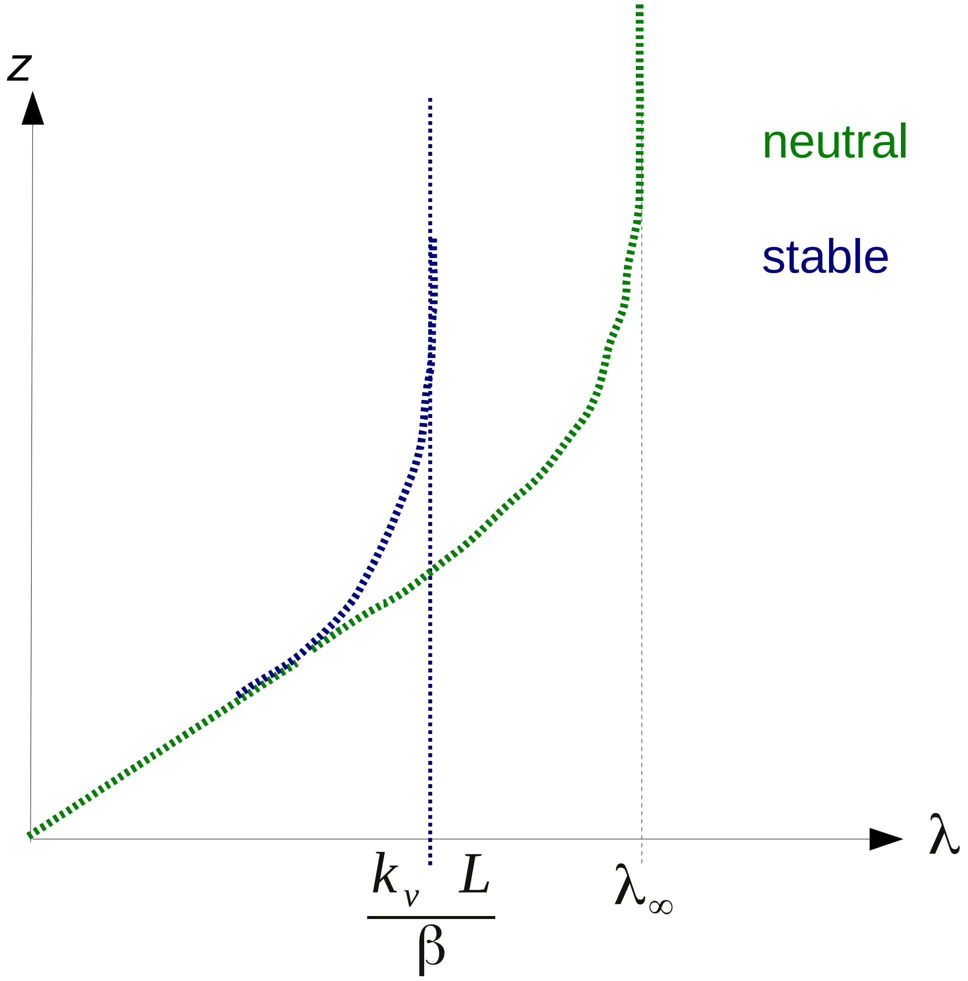
$$K_{m,k,v} = \frac{k_v\, u_*\, z}{\phi_{m,h,v}(z/L)}$$

Delage's imposed algebraic length scale

$$\frac{1}{\lambda(z)} = \frac{1}{k_v\ z} + \boxed{\frac{1}{\lambda_\infty}} + \boxed{\frac{\beta}{k_v\ L}}$$

limits λ in
neutral layer

limits λ in stratified layer

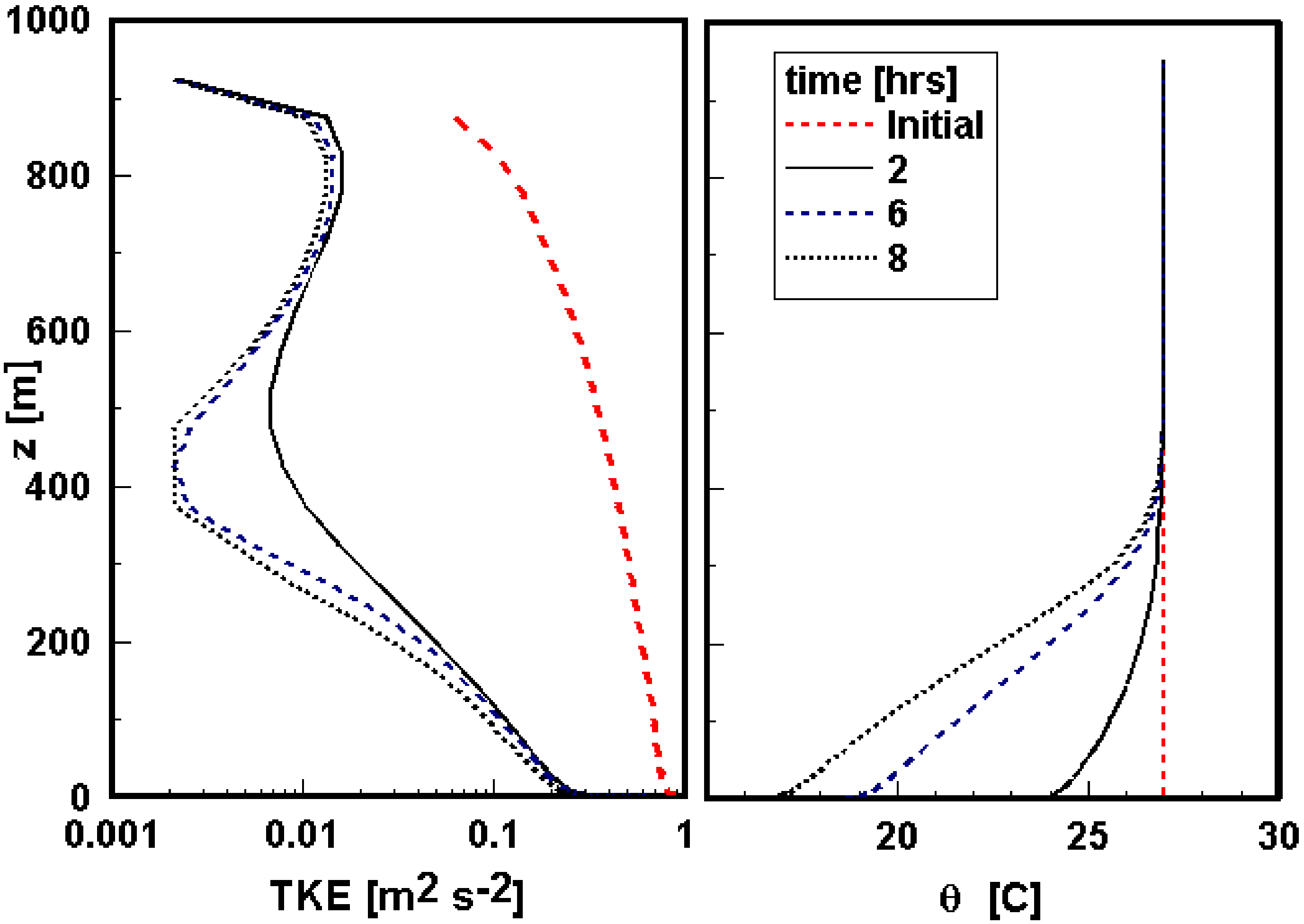


In the Regional Finite Element (RFE) model that preceded GEM, a more general stability-dependent profile was adopted, say $\lambda_e(z)$ ... then $\lambda(z,t)$ would "relax" towards this equilibrium value on a timescale τ

$$\frac{\partial \lambda}{\partial t} = \frac{\lambda_e(z) - \lambda}{\tau}$$

Initial condition: $\theta(z,0) = \theta_{00}$ and corresponding steady-state wind and TKE profiles from solution of these equations for the neutral state.

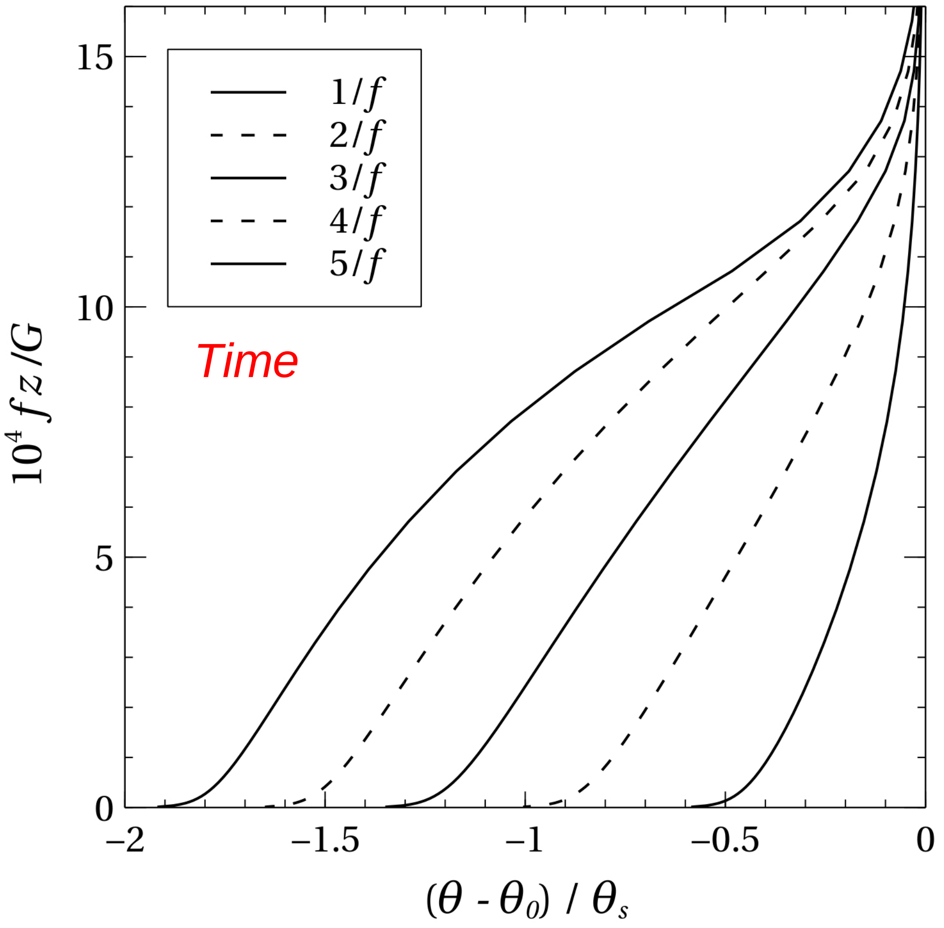
Forcing: “driven” by an imposed cooling trend in surface temperature



- intensifying surface-based inversion self-limits its own depth h_i
- depth h_t of surface-based mixing layer drops. Mixing continues in residual neutral layer aloft

Delage’s result for cooling rate, presented in dimensionless form. Case chosen corresponds to a strong geostrophic wind *G* such that the Rossby number

$$R_o = \frac{G}{z_0 \; f} = 10^7$$



Low-level jet develops in Delage simulation