Modelling an idealized nocturnal (stable) boundary-layer ("NBL" or "SBL")

- in what sense idealized? Cloudless, unsaturated, horizontally homogeneous and neglects radiative divergence $\nabla \cdot \mathbf{F}$

From Stull (1988), An Intro. To Boundary Layer Meteorology (see also Garratt’s Fig. 6.1)

The boundary layer in high pressure regions over land consists of three major parts: a very turbulent mixed layer; a less-turbulent residual layer containing former mixed-layer air; and a nocturnal stable boundary layer of sporadic turbulence. The mixed layer can be subdivided into a cloud layer and a subcloud layer. Time markers indicated by S1-S6 will be used in Fig. 1.12.
Inertial oscillation

\[ \frac{\partial U}{\partial t} = - \frac{\partial u'w'}{\partial z} + f(V - V_G) \]

\[ \frac{\partial V}{\partial t} = - \frac{\partial v'w'}{\partial z} - f(U - U_G) \]

\[ \frac{\partial^2 U}{\partial t^2} = f \frac{\partial V}{\partial t} = - f^2 U + f^2 U_G \]

Weakened friction hints at possibility of inertial oscillations in horizontal velocity

"Low level jet"

Nocturnal surface inversion established after sign transition in \( Q_{h0} \), and thereafter deepening

Residual layer
- surface energy budget results in surface cooling

\[ Q^* \equiv K^* + L^* = Q_{H0} + Q_{E0} + Q_G < 0 \]

- thus \( Q_{H0} < 0 \), sensible heat is extracted from the base of the layer in conductive/convective contact with the surface

- TKE budget: **horiz homog form**: in general, have advection terms and other production terms

\[
\frac{\partial k}{\partial t} = - \frac{u'w'}{\partial z} \frac{\partial U}{\partial z} - \frac{v'w'}{\partial z} \frac{\partial V}{\partial z} + \frac{g}{\theta_0} \frac{w'\theta'}{\partial z} - \frac{\partial}{\partial z} \left( \frac{p'}{\rho_0} + \frac{u'u' + v'v' + w'w'}{2} \right) - \epsilon
\]

- shear production
- buoyant prodn
- pressure transport + turbulent transport
- viscous dissipation

- as daytime winds die down, shear production is reduced; and because the **layer is stably stratified** buoyant production is negative, offsetting what (little?) shear production continues

- thus turbulence dies down to a low level – unless a strong free atmos. wind sustains shear production and overcomes buoyant destruction of TKE, so as to sustain mixing and limit the strength of the inversion

- and/or unless heavy cloud cover prevents rapid sfc cooling by longwave radiation
Reminder about the physical mechanism behind the velocity covariances (eddy momentum fluxes)

\[ u'(z) \]

\[ z \]

\[ w' > 0 \text{ so (preferentially) } u' < 0 \]

Furthermore, taking this as a daytime scenario such that the orange parcel (originating near ground) is warm, we can also see that rising parcels will carry positive \( T' \) so that (accordingly)

\[ w'T' > 0 \]

One can derive exact (but unclosed) budget eqns for these covariances

\[ w' < 0 \text{ so (preferentially) } u' > 0 \]

\[
\frac{\partial}{\partial t} \overline{w'T'} + \overline{u} \frac{\partial \overline{w'T'}}{\partial z} + \ldots = -\overline{w'^2} \frac{\partial \overline{T}}{\partial z} + \frac{g}{T_0} \overline{T'^2} + \ldots - \frac{\partial}{\partial z} \overline{w'w'T'} + \ldots
\]

In terms of shear production, then, we see that

\[-\frac{u' w'}{\partial z} \]

is positive

\[-\frac{\rho c \overline{w'} \partial \overline{T'}}{\partial z} \]
By manipulating the Navier-Stokes eqns. (under the Boussinesq approx.), the variance budget eqns for a horizontally-homogeneous layer are:

\[
\frac{\partial \sigma_u^2}{\partial t} = -2 \frac{\partial w' w'}{\partial z} \frac{\partial U}{\partial z} - \frac{\partial w' u' u'}{\partial z} + \frac{2}{\rho_0} p' \frac{\partial u'}{\partial x} - \epsilon_{uu}
\]

\[
\frac{\partial \sigma_v^2}{\partial t} = -2 \frac{\partial w' w'}{\partial z} \frac{\partial V}{\partial z} - \frac{\partial w' v' v'}{\partial z} + \frac{2}{\rho_0} p' \frac{\partial v'}{\partial y} - \epsilon_{vv}
\]

\[
\frac{\partial \sigma_w^2}{\partial t} = 2 \frac{g}{\theta_0} \frac{\partial w' \theta'}{\partial z} - \frac{\partial w' (\frac{2 p'}{\rho_0} + w' w')}{\partial z} + \frac{2}{\rho_0} p' \frac{\partial w'}{\partial z} - \epsilon_{ww}
\]

Buoyant production

Turbulent (+ press.) transport

Redistribution terms sum to zero in TKE eqn

Shear and buoyant production, turbulent and pressure transport

\[\frac{\partial}{\partial t} \frac{\sigma_u^2 + \sigma_v^2 + \sigma_w^2}{2} = \ldots + \frac{2}{\rho_0} p' \left[ \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} \right] - \frac{\epsilon_{uu} + \epsilon_{vv} + \epsilon_{ww}}{2}\]
Energetics of the NBL – perspective of the velocity variance equations ( \( \bar{w}'\bar{w}' \equiv \sigma_w^2 \) etc.)

By manipulating the Navier-Stokes eqns. (under the Boussinesq approx.), the variance budget eqns for a horizontally-homogeneous layer are:

\[
\frac{\partial \sigma_u^2}{\partial t} = -2 u'w' \frac{\partial U}{\partial z} - \frac{\partial}{\partial z} w'u'u' + \frac{2}{\rho_0} p' \frac{\partial u'}{\partial x} - \epsilon_{uu}
\]

\[
\frac{\partial \sigma_v^2}{\partial t} = -2 v'w' \frac{\partial V}{\partial z} - \frac{\partial}{\partial z} w'v'v' + \frac{2}{\rho_0} p' \frac{\partial v'}{\partial y} - \epsilon_{vv}
\]

\[
\frac{\partial \sigma_w^2}{\partial t} = 2 \frac{g}{\theta_0} w'\theta' - \frac{\partial}{\partial z} w' \left( \frac{2p'}{\rho_0} + w'w' \right) + \frac{2}{\rho_0} p' \frac{\partial w'}{\partial z} - \epsilon_{ww}
\]

- buoyant prodn
- turbulent (+ press.) transp. (small)
- redistribution
- viscous dissip'n

L* < 0 \rightarrow Q* < 0 \rightarrow Q_H < 0 \rightarrow \text{buoyant suppression of the vertical motion (thus) TKE, effective in the energy-containing range of scales; } w' \text{ fed by inter-component transfer (redistribution) alone; lack of energy in } w' \text{ limits heat and (downward) momentum transport by turbulent convection; light winds (measurement challenge), turbulence may be intermittent; gravity waves; ratio of buoyancy (} gT'/T_0 \text{) to inertial (} u'^2/L \text{) forces becomes large, so slight topographic irregularities can result in drainage flows (three-dimensional and intermittent) – see Wyngaard's textbook Eqn. (12.20) where veloc. field parallel to gently sloping sfc contains buoyancy terms}
An interesting cycle of intermittency can occur (Van de Wiel et al., J.Atmos.Sci. 67, 2010)

\[ R_i^{g} = \frac{g}{\theta_0} \frac{\partial \theta}{\partial z} \left( \frac{\partial U}{\partial z} \right)^2 \]

or as bulk index for the layer

\[ R_i = \frac{g}{\theta_0} \frac{\Delta \theta}{\Delta z} \frac{\Delta z}{\Delta U^2} \]

Quiescent layer, \( R_i \) large because numerator large and denominator small

Shear is strongest near ground

"z-less scaling"

Lower sfc layer decoupled from flow aloft, but some mixing as shear increases where \( z \) small

Critical value of \( R_i \) to suppress turbulence surely not universal, but of order 0.1; textbooks cite obs. suggesting about 0.2

\[ R_i^{f} = \frac{g}{T_0} \frac{\overline{w' \theta'}}{\left( \overline{u' w'} \right)} \]

\[ = \frac{g}{T_0} - \frac{K_h}{K_m} \frac{\overline{\theta'}}{\overline{\theta}} / -K_m \left( \frac{\partial U}{\partial z} \right)^2 \]
Recall that in context of Monin-Obukhov Similarity Theory (MOST): \( K_{m,k,v} = \frac{k_v u_* z}{\phi_{m,h,v}(z/L)} \)
In the Regional Finite Element (RFE) model that preceded GEM, a more general stability-dependent profile was adopted, say \( \lambda_e(z) \) ... then \( \lambda(z,t) \) would "relax" towards this equilibrium value on a timescale \( \tau \)

\[
\frac{\partial \lambda}{\partial t} = \frac{\lambda_e(z) - \lambda}{\tau}
\]
Initial condition: $\theta(z,0) = \theta_0$ and corresponding steady-state wind and TKE profiles from solution of these equations for the neutral state.

Forcing: “driven” by an imposed cooling trend in surface temperature

- intensifying surface-based inversion self-limits its own depth $h_i$
- depth $h_t$ of surface-based mixing layer drops. Mixing continues in residual neutral layer aloft
Delage's numeric solution

Delage's result for cooling rate, presented in dimensionless form. Case chosen corresponds to a strong geostrophic wind $G$ such that the Rossby number

$$R_o = \frac{G}{z_0 f} = 10^7$$