Exercise – the steady state surface layer TKE budget according to MOST

$$\frac{\partial k}{\partial t} = 0 = \underbrace{-\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z}}_{\text{Shear production, } \boldsymbol{P}_S} + \overbrace{\frac{g}{\theta_0}\overline{w'\theta'}}^{\text{Buoyant production, } \boldsymbol{P}_B} - \underbrace{\epsilon}_{\text{Dissipation (conversion of TKE to heat)}} \overbrace{- \frac{\partial}{\partial z}\overline{w'\left(\frac{p'}{\rho} + \frac{u'u'+v'v'+w'w'}{2}\right)}}^{\text{Turbulent \& pressure transport, } \boldsymbol{T}_T}$$

Assume the coordinate system is chosen to make V=0 throughout the surface layer, and neglect the term T_T (known as the assumption that the TKE budget is in "local equilibrium").

Adopting the assumptions and/or definitions inherent to MOST (including the definition of the Obukhov length L), prove that the TKE dissipation rate can be written

$$\frac{k_v\, z\, \epsilon}{u_*^3} = \phi_\epsilon\left(\frac{z}{L}\right)$$

where** $\phi_\epsilon\left(\frac{z}{L}\right) = \phi_m\left(\frac{z}{L}\right) - \left(\frac{z}{L}\right)$

** $\phi_m\left(\frac{z}{L}\right) = \frac{k_v\, z}{u_*}\frac{\partial U}{\partial z}$ is the MO function for the dimensionless wind shear

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