

We shall consider the treatment of atmospheric dispersion in the horizontally-uniform atmospheric surface layer whose statistics were the subject of Assignment 2 (note: to proceed you will have to determine the roughness length z_0 corresponding to your optimal solution for u_*, L).

Alice Ou: forward simulation using zeroth-order model

Write a computer algorithm to compute the mean concentration at $x = 100$ m, $z = (0, 2, 10)$ m due to a continuous ground-level line source of passive tracer at $x = x_s = 0$, $z = z_s = z_0$. The strength of the source is $Q = 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$.

Assume particle position (X, Z) evolves in time with steps

$$dX = \bar{u}(Z(t)) dt \quad (1)$$

$$dZ = \frac{\partial K}{\partial z} dt + \sqrt{2K} d\xi \quad (2)$$

where $d\xi$ is a random number drawn from a Gaussian distribution $N(0, dt)$, ie. with zero mean and variance dt , where the timestep dt should be “small”. The mean wind profile $\bar{u}(z)$ is that which you have computed in Assignment 2, ie. the theoretical profile that corresponds to your optimal u_*, L . The eddy diffusivity should be specified as:

$$K = \frac{1}{S_c} k_v u_* z \quad (3)$$

where $k_v = 0.4$ is the von Karman constant, and S_c is the turbulent Schmidt number. Assume $S_c = 0.63$, and examine your solution for several values of

the timestep, $\Delta t = 0.1, 1, 10$ s. Perform a calculation of the total horizontal flux and check against source strength. Estimate concentration in detectors whose dimension $\Delta x = \Delta z = 0.1$ m.

The zeroth-order LS model is equivalent to an eddy-diffusion treatment. Therefore please compare your solution with the analytical K-theory solution given as eqns (17.31 - 17.33). Interpret your comparison, remembering that the analytic solution (which is valid for the case $S_c = 0.63$) neglects the influence of instability.

John Postma: simulation using 1st-order model

On the basis of your choice of 1st-order LS model, estimate the mean concentration and concentration variance at $z = 2$ m due to a continuous (and perfectly steady) ground-level *area* source of passive tracer at ground level extending upwind from the detection point a distance (fetch) of $X = 50$ m. The strength of the source is $Q = 10^{-7} \text{ kg m}^{-2} \text{ s}^{-1}$.

Examine the sensitivity of your solution to inclusion/exclusion of the streamwise fluctuation U' and (with U' retained) to the inclusion /exclusion of the covariance $-u_*^2$.