EAS 572: Assignment 4: Lagrangian Simulation of Dispersion in the ASL

0.1 Passive Scalar

Write a Lagrangian stochastic algorithm to calculate the mean concentration field $C = C(x, z)$ downwind from a continuous line source at $x = y = 0, z = h_s$ in the horizontally-uniform surface layer. Apply this model to calculate the concentration profile at fetch $x = 100m$ for Project Prairie Grass run 57 (Tables 1, 2).

Assume particle position (X, Z) evolves in time with velocity (U, W) where

$$U(Z(t)) = \overline{u}(Z(t)) \tag{1}$$

(assume the micrometeorological profiles $U(z), T(z)$ are those of Monin-Obukhov similarity theory) and where W evolves in time accoriding to the unique, 1-d, first-order, well-mixed model for Gaussian inhomogeneous turbulence, ie.

$$\begin{aligned} dW &= a\ dt + \ b\ d\xi \\ a &= -\frac{C_0\ \epsilon(z)}{2{\sigma_w}^2(z)}\ W + \ \frac{1}{2}\frac{\partial {\sigma_w}^2}{\partial z}\ \left(\frac{W^2}{{\sigma_w}^2} + \ 1\right) \\ b &= \sqrt{C_0\ \epsilon(z)} \end{aligned} \tag{2}$$

where $C_0\epsilon$ should be specified according to:

$$\begin{aligned} \frac{2\ {\sigma_w}^2}{C_0\ \epsilon} &= T_L(z) = \frac{0.5\ z}{\sigma_w}\ \left(1 - \ 6\frac{z}{L}\right)^{\frac{1}{4}},\ L < 0 \\ \frac{2\ {\sigma_w}^2}{C_0\ \epsilon} &= T_L(z) = \frac{0.5\ z}{\sigma_w}\ \left(1 + \ 5\frac{z}{L}\right)^{-1},\ L > 0 \end{aligned} \tag{3}$$

It may be interesting to vary the constant a (= 0.5 in above equations) so as (in effect) to control the turbulent Schmidt number S_c: if $S_c = 1$ then one requires $a \sim 0.32$.

Table 1: Normalized cross-wind integrated concentration $\frac{z_0 u_* \chi}{k_v Q}$ observed at distance $x = 100m$ from the source (height $h_s = 0.46m$) in Project Prairie Grass run 57.

$z[m]$	$\frac{z_0 u_* \chi}{k_v Q}$
17.5	1.5E-6
13.5	6.6E-6
10.5	1.56E-5
7.5	3.51E-5
4.5	7.9E-5
2.5	1.25E-4
1.5	1.53E-4
1.0	1.62E-4
0.5	1.70E-4

0.2 Heavy Particles

Modify your LS model to obtain the "Settling Sticky Fluid Element" model of heavy particle trajectories, by adding a gravitational settling velocity. Simulate the observed pattern of crosswind-integrated deposition $D_0(x)$ observed (Table 3) in Walker's "Trial C" of the Suffield bead diffusion trials, for which $u_* = 0.44m/s, z_0 = 0.025m, L = 341m, z_{src} = 15m, w_g = 0.4m/s, \tau_p = 0.06s$.

Table 2: Micrometeorological data for Project Prairie Grass run 57.

$z[m]$	$U, [m/s]$	$T, [C]$
16	9.89	33.54
8	8.79	33.76
4	8.24	33.91
2	7.2	34.11
1	6.42	34.19
0.5	5.56	34.33
0.25	4.69	34.52
0.12		34.61

Table 3: Normalised cross-wind-integrated deposition D_0 $[mg\ g^{-1}\ m^{-1}]$ versus downwind distance for Suffield Trial C.

$x[m]$	27.4	45.7	73.2	100.6	128	201.2	402.3	804.6
D_0	0.003	0.011	0.96	5.76	5.92	2.83	0.2	0.074