EAS572, "The Atmospheric Boundary Layer" Final Exam 7 Dec. 2010

Professor: J.D. Wilson Time available: 150 mins Value: 35%

***Notes***: Indices $j = (1, 2, 3)$ are to be interpreted as denoting respectively the $(x, y, z)$ components, e.g. $\vec{u} \equiv u_j \equiv (u_1, u_2, u_3) \equiv (u, v, w)$. The summation convention applies for repeated alphabetic subscripts (e.g. $u_j u_j$). Symbols $p_R, \rho_R, T_R, \theta_R$ represent pressure, density, temperature and potential temperature of the reference state. Abbreviations: ABL, Atmospheric Boundary Layer; CBL, Convective Boundary layer; SBL, Stable Boundary Layer; NBL, Nocturnal Boundary Layer; ASL, Atmospheric surface layer; NSL, neutral surface layer. Flows should be considered horizontally-homogeneous, unless otherwise stated.

## Multichoice ($22 \times \frac{1}{2}\% = 11\%$)

1. "Relative dispersion" refers to the growth of a plume or puff ____ whereas "absolute dispersion" refers to growth ____
   (a) relative to stationary coordinates; relative to its instantaneous centre-line/centre of mass
   (b) relative to its instantaneous centre-line/centre of mass; relative to stationary coordinates ✓✓
   (c) in homogeneous turbulence; in inhomogeneous turbulence
   (d) in inhomogeneous turbulence; in homogeneous turbulence
   (e) due to turbulent convection; due to the combined effects of turbulent convection and molecular diffusion

2. The mixed-layer scales are ABL depth $\delta$, the convective velocity scale $w_* = \left( \frac{g}{\theta_R} \, \delta \, \left(\overline{w'\theta'}\right)_0 \right)^{1/3}$, and temperature scale $\theta_* = -\left(\overline{w'\theta'}\right)_0 / w_*$. These are useful for scaling ____
   (a) mean profiles ($\overline{u}$, $\overline{\theta}$, etc.) in the entire ABL
   (b) mean profiles ($\overline{u}$, $\overline{\theta}$, etc.) in a very unstable CBL, but only above the surface layer
   (c) turbulence statistics in the entire ABL
   (d) turbulence statistics in a very unstable CBL, but only above the surface layer and beneath the entrainment layer ✓✓
   (e) both mean profiles and turbulence statistics, whatever the stratification

3. In the nocturnal boundary layer one sometimes observes a "residual layer," within which mean potential temperature ____
   (a) is approximately independent of height ✓✓
   (b) increases with increasing height
   (c) decreases with increasing height
   (d) decreases with decreasing height
   (e) increases during the night

4. The preferred criterion for depth $\delta$ of the atmospheric boundary layer would be a threhold value for ____ . Radiosonde profiles (of temperature and dewpoint) permit an adequate estimation of $\delta$ on those occasions when the sounding happens to feature ____

   (a) turbulent kinetic energy; a surface-based inversion beneath a well-mixed layer
   (b) wind speed; a residual layer above an unstable surface layer
   (c) turbulent kinetic energy; a layer that is well-mixed (as regards its potential temperature) beneath an elevated (capping) inversion ✓✓
   (d) wind speed; a surface-based inversion beneath a well-mixed layer
   (e) wind speed; a moist ground-based layer overlain by a much drier free atmosphere

5. In his 1915 paper on the mean potential temperature profile in the ABL, G.I. Taylor developed the equation (here given in slightly revised notation)

$$\frac{\partial \overline{\theta}}{\partial t} = \mathcal{W}\,\mathcal{D}\,\frac{\partial^2 \overline{\theta}}{\partial z^2} ,$$

   ($\mathcal{W}, \mathcal{D}$ free constants). Which of the following is **not** a legitimate objection to this formulation?

   (a) it neglects the radiative flux divergence
   (b) it neglects latent heating/cooling associated with change of phase of water
   (c) it applies only to a horizontally-homogeneous ABL
   (d) it treats the eddy diffusivity as height-independent
   (e) it neglects the vertical transport of heat by turbulent convection ✓✓

6. The Geostrophic wind is height-independent in an ABL that is ____ ; referring to Eqn. (4; Question 11), the magnitude of the zonal (i.e. $x$-wise) component of the Geostrophic wind ($|U_g|$) is ____ [Note: the Geostrophic wind is that which would result from a balance of the pressure and Coriolis forces, alone.]

   (a) barotropic; $(\rho_R f)^{-1}\,\partial\overline{p}/\partial y$ ✓✓
   (b) baroclinic; $(\rho_R f)^{-1}\,\partial\overline{p}/\partial y$
   (c) well-mixed; $(\rho_R f)^{-1}\,\partial\overline{p}/\partial y$
   (d) well-mixed; $(\rho_R f)^{-1}\,\partial\overline{p}/\partial x$
   (e) stationary; $(\rho_R f)^{-1}\,\partial\overline{p}/\partial y$

7. Boundary layer "roll" circulations are nearly ____ with the mean wind, and occur under ____ stratification of the ABL

   (a) parallel; weakly stable
   (b) parallel; weakly unstable ✓✓
   (c) perpendicular; weakly stable
   (d) perpendicular; weakly unstable
   (e) coincident; neutral

8. A circular patch of the ground has radius $R = 10$ m and is emitting a passive gas at a steady and uniform but unknown rate $Q$ [$\mathrm{kg\,m^{-2}\,s^{-1}}$] into an undisturbed atmospheric surface layer. You would like to be able to measure or deduce $Q$, and have available whatever micrometeorological sensors you might wish to employ, including a sonic anemometer (path length $d = 10$ cm) and a fast point gas detector. Which of the (true) statements listed below is **irrelevant** to one's decision for or against employing eddy covariance at height $z_m$ at the centre of the patch to measure the flux directly above the source as $\overline{w'c'}$?

(a) by symmetry, the mean concentration profile over the centre of the plot is indifferent to (i.e. independent of) the mean wind direction ✓✓

(b) to ensure the flux is measured with adequate frequency response and spatial resolution, $z_m$ would need to satisfy $z_m \gg d$, thus one would need $z_m \gg 0.1$ m

(c) the vertical flux $\overline{w'c'}(z_m)$ cannot be independent of measurement height, in this situation

(d) the depth of the constant (gas) flux layer would grow with distance from the leading edge (perimeter) at a rate that might be as slow as 1 unit of depth for each 100 units of downwind distance, meaning one would need to choose $z_m < R/100$ (i.e. $z_m < 0.1$ m)

(e) a suitable model of atmospheric transport could supply a theoretical value for the dimensionless grouping $n = u_* \overline{c}(z_m)/Q$ where $u_*$ is the friction velocity and $\overline{c}(z_m)$ is mean concentration at $z_m$ over the plot centre; and the sonic could provide $u_*$ and any other data (such as Obukhov length $L$) needed by the atmospheric model to compute $n$. Accordingly, in lieu of using eddy covariance, $Q$ could be estimated as $Q = u_* \overline{c}/n$.

9. In a horizontally-uniform flow, assuming no in-situ production/destruction and with axes chosen such that $\overline{v} = \overline{w} = 0$, the Reynolds-averaged conservation equation for a passive tracer is

$$\frac{\partial \overline{c}}{\partial t} + \overline{u}\,\frac{\partial \overline{c}}{\partial x} = -\frac{\partial \overline{u'c'}}{\partial x} - \frac{\partial \overline{v'c'}}{\partial y} - \frac{\partial \overline{w'c'}}{\partial z}\,. \tag{1}$$

Eqn. (1) is often simplified on the assumption that

$$|\overline{u}\,\frac{\partial \overline{c}}{\partial x}| \gg |\,\frac{\partial \overline{u'c'}}{\partial x}|\,,$$

based on the stipulation that $\sigma_u/\overline{u}$ is ____ . That restriction has the consequence that, upon introducing an eddy diffusivity closure, streamwise ($x$) diffusion ____

(a) small; is retained, permitting tracer to spread *upwind*

(b) large; is retained, permitting tracer to spread *upwind*

(c) small; is neglected ✓✓

(d) large; is neglected

(e) of order unity; is more important than streamwise advection by the mean wind

10. In an ideal, stationary and horizontally-homogeneous surface layer, and neglecting molecular transport, the budget equation for the mean vertical flux of a passive scalar "$c$" is

$$\begin{array}{ccccccc} \overline{u}\,\dfrac{\partial \overline{w'c'}}{\partial x} & = & -\,\overline{w'u'_j}\,\dfrac{\partial \overline{c}}{\partial x_j} & - & \dfrac{\partial}{\partial x_j}\,\overline{w'u'_j c'} & - & \dfrac{1}{\rho_R}\,\overline{c'\,\dfrac{\partial p'}{\partial z}} + \dfrac{g}{\theta_R}\,\overline{c'\theta'} \\ I & = & II & & III & & IV \qquad V \end{array} \qquad (2)$$

(it has been assumed that the tracer field is stationary, i.e. the source is steady). If the further specifications are added that tracer emanates from a spatially uniform source on ground, that the ASL is neutral (i.e. adiabatic flow), and that the turbulent transport term is negligible, Eqn. (2) simplifies radically. The surviving terms are, or emerge from, these terms ____

(a) II, IV, V
(b) II, III, IV, V
(c) IV, V
(d) I, V
(e) II, IV ✓✓

11. The mean horizontal momentum equations in a horizontally-uniform ABL are

$$\frac{\partial \overline{u}}{\partial t} = -\frac{\partial \overline{p}/\rho_R}{\partial x} - \frac{\partial \overline{u'w'}}{\partial z} + f\,\overline{v}\,, \qquad (3)$$

$$\frac{\partial \overline{v}}{\partial t} = -\frac{\partial \overline{p}/\rho_R}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z} - f\,\overline{u}\,, \qquad (4)$$

$$I = II \qquad III \qquad IV$$

where the $x$-axis is defined such that $\overline{u}$ is the zonal and $\overline{v}$ the meridional velocity (viscous momentum transfer has been neglected; $f\,[\mathrm{s}^{-1}]$ is the Coriolis parameter). The term representing "turbulent friction" is ____ . If all transport terms were eliminated, the resulting coupled equations would sustain an undamped motion named the ____

(a) IV; inertial oscillation
(b) III; boundary layer roll
(c) IV; boundary layer roll
(d) III; inertial oscillation ✓✓
(e) I; Geostrophic wind

12. In many numerical models of the atmospheric surface layer the eddy viscosity is parameterized $K = \lambda\sqrt{\alpha\,k}$, where $k$ is the turbulent kinetic energy and the length scale $\lambda$ is specified such that in the neutral limit $\lambda = k_v\,z$ ($k_v$ the von Karman constant). The tunable constant $\alpha$ is therefore

(a) the value of $u_*^2/k$ appropriate to a neutral surface layer ✓✓
(b) the value of $K(u_*\lambda)^{-1}$ appropriate to a neutral surface layer
(c) the value of $k$ appropriate to a neutral surface layer
(d) the value of $(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)\,/2$ appropriate to a neutral surface layer
(e) the value of $k/u_*^2$ appropriate to a neutral surface layer

13. G.I. Taylor's expression for the rate of growth (along axis $z$) of a puff of tracer that is initially ($t = 0$) concentrated at the origin may be written

$$\frac{d\sigma_z^2}{dt} = 2\,\sigma_w{}^2 \int\limits_0^t R(\xi)\,d\xi$$

where $R(\xi) \le 1$ is the Lagrangian velocity autocorrelation function, a function of the lag time $\xi$. If the travel time $t$ is very much less than the Lagrangian integral time scale

$$\tau_L \equiv \int\limits_0^\infty R(\xi)\,d\xi$$

then the standard deviation of puff size (in this "near field" of the source) is ____

(a) $\sigma_z = \sigma_w\,\sqrt{t}$
(b) $\sigma_z = \sigma_w\,t$ ✓✓
(c) $\sigma_z^2 = \sigma_w^2\sqrt{t}$
(d) $\sigma_z = \sigma_w^2\sqrt{t}$
(e) $\sigma_z^2 = 2\sigma_w^2\,t$

14. The mean convective flux density of a passive scalar is $\overline{u_j\,c}$, and is linked (see Eqn. 1 given earlier) to the field of mean concentration. An *Eulerian* theory or model of dispersion (such as an advection-diffusion equation emerging from eddy-diffusion closure) in effect formulates approximations for ____ , while the *Lagrangian stochastic* approach makes approximation in regard to ____

(a) statistics of the concentration field; joint statistics of the velocity and concentration fields
(b) statistics of the velocity field; statistics of the concentration field
(c) mass conservation; the probability density function of the Eulerian velocity field
(d) joint statistics of the velocity and concentration fields; statistics of the concentration field
(e) joint statistics of the velocity and concentration fields; statistics of the velocity field ✓✓

15. Let $h_b(x)$ be the depth of an internal boundary-layer (IBL) growing from the leading edge (at $x = 0$) of a step change in surface Bowen ratio, and let subscript '1' denote properties of the flow far upstream from the discontinuity. A simple paradigm for the growth rate of the IBL assumes $\partial h_b/\partial x$ is proportional to ____

(a) $1/\overline{u}_1(L_1)$ where $L_1$ is the upstream Obukhov length
(b) $1/\overline{u}_1(\alpha\,h_b)$, where $\alpha$ is a constant
(c) $\sigma_{w1}/\overline{u}_1(\alpha\,h_b)$ ✓✓
(d) $z_{01}$, the upstream roughness length
(e) $k_1$, the upstream TKE

16. Consider a layer $z_m - \Delta z/2 \le z \le z_m + \Delta z/2$ in a horizontally-uniform plant canopy whose leaf area density profile is $a(z)$ $[\mathrm{m^2\,m^{-3}}]$, and let $\Delta Q_H$ $[\mathrm{W\,m^{-2}}]$ symbolize the difference between the sensible heat fluxes measured at top and bottom of this layer. Assume no phase changes of water are occcuring in the airstream. In the relationship

$$\Delta Q_H = a(z_m)\, \Delta z\, S$$

the quantity $S\,[\mathrm{W\,m^{-2}}]$ represents ____

(a) net radiation measured normal to the surface of a representative leaf in this canopy layer

(b) sensible heat flux density from leaf-to-airstream measured normal to the surface of a representative leaf in this canopy layer ✓✓

(c) latent heat flux density from leaf-to-airstream measured normal to the surface of a representative leaf in this canopy layer

(d) rate of heating of the layer by viscous dissipation of turbulent kinetic energy, viz. $\epsilon\,\rho\,\Delta z$

(e) rate of working by the canopy drag force in this layer

17. The mean wind profile in a plant canopy is often represented $\overline{u} = \overline{u}(h_c)\,\exp\left[\beta\,(z/H - 1)\right]$, where $h_c$ is canopy height. The sign of the extinction coefficient $\beta$ ____ and (theoretically) the numerical value of $\beta$ is ____ variables such as leaf area density ($a$), leaf drag coefficient ($c_d$) and canopy eddy viscosity ($K$)

(a) is positive; determined by ✓✓

(b) is positive; independent of

(c) is negative; independent of

(d) is negative; determined by

(e) can be positive or negative; independent of

18. Deep in a dense canopy all velocity variances and covariances normally are small relative to their values above the crop. Measurements have shown that the transport term $T = -\partial\,\overline{w'^3}/\partial z$ in the $\sigma_w^2$ ($\equiv \overline{w'^2}$) budget exports $w$-variance from the upper canopy to the lower canopy, where production is negligible, and thereby sustains the low level of $\sigma_w^2$ deep in the canopy. This transport is associated with large eddies (coherent structures) that make the dominant contribution to the mean shear stress $\overline{u'w'}$, and are known (in the terminology of quadrant analysis) as ____

(a) holes: [ $|u'w'| < H$]

(b) gusts (or sweeps): $[w' < 0, u' > 0]$ ✓✓

(c) ejections: $[w' > 0, u' < 0]$

(d) inward interactions: $[w' < 0, u' < 0]$

(e) outward interactions: $[w' > 0, u' > 0]$

19. Consider the aerodynamics of surface-layer flow encountering a long, thin, porous windbreak fence (height $h$) at perpendicular incidence. The steady-state governing equation for $\overline{u}$-momentum is

$$\begin{array}{cccccccc} \overline{u}\,\dfrac{\partial\,\overline{u}}{\partial x} & + \;\overline{w}\,\dfrac{\partial\,\overline{u}}{\partial z} & = & -\dfrac{1}{\rho_R}\dfrac{\partial\,\overline{p}}{\partial x} & -\;\dfrac{\partial\,\overline{u'^2}}{\partial x} & -\;\dfrac{\partial\,\overline{u'w'}}{\partial z} & +\;S_u \\ I & II & = & III & IV & V & VI \end{array}$$

where the momentum sink $S_u \equiv -k_r\,\overline{u}\,|\overline{u}|\,\delta(x-0)\,s(z-h)$ vanishes except at the barrier (i.e. it is localized to $x=0$, $z/h \leq 1$ by the delta-function $\delta$ and unit step function $s$). Upstream from the barrier term $III$ is ____ ; downstream it is ____ . In view of the upward deflection of mean streamlines over the barrier and the (related) by-passing jet aloft, it can be expected that the leeward recovery of the velocity (commencing beyond about $x/h=4$) is effected predominantly by term(s) ____ [Note: conventional thinking is that streamwise gradients of normal stresses, i.e. of diagonal elements of the Reynolds stress tensor, exert minimal impact on disturbed shear flows.]

(a) adverse; favourable; $III$

(b) favourable; adverse; $VI$

(c) favourable; favourable; $V$

(d) adverse; adverse; $II$, $V$ ✓✓

(e) adverse; adverse; $I$, $V$

20. The vertical profile of the kinematic shear stress $\overline{u'w'}$ within a plant canopy, when normalized by its canopy-top value $-u_*^2 \equiv \overline{u'w'}(h)$, is relatively invariant during days of appreciable wind, whereas under the same conditions profiles of normalized heat flux density $\overline{w'T'}(z)/\overline{w'T'}(h)$ vary markedly throughout the day. This is because

(a) Monin-Obukhov similarity theory applies for momentum but not heat transport

(b) the canopy is a "constant flux layer" for momentum, but not for heat

(c) the canopy is a "constant flux layer" for heat, but not momentum

(d) the source distribution for heat depends on solar elevation, leaf water status and other varying factors ✓✓

(e) in a plant canopy, the probability density function for vertical velocity is markedly non-Gaussian

21. Y. Delage adopted an eddy viscosity/diffusivity closure $K = \lambda \sqrt{\alpha\, k}$ to model the development of the horizontally-homogeneous nocturnal boundary layer (NBL) from an initially neutral state. The turbulent kinetic energy $k$ was computed as

$$\begin{array}{ccccccc} \dfrac{\partial k}{\partial t} & = & \dfrac{\partial}{\partial z}\left(K\,\dfrac{\partial k}{\partial z}\right) & + \; K\left[\left(\dfrac{\partial \overline{u}}{\partial z}\right)^2 + \left(\dfrac{\partial \overline{v}}{\partial z}\right)^2\right] & - \; \dfrac{g}{\theta_R}\,K\,\dfrac{\partial \overline{\theta}}{\partial z} & - \; \dfrac{(\alpha\, k)^{3/2}}{\lambda} & , \\ I & = & II & III & IV & V & \end{array}$$

where

$$\frac{1}{\lambda} = \frac{1}{k_v z} + \frac{1}{\beta\, L} + \frac{1}{\lambda_{\max}} \; ,$$

$k_v$ is the von Karman constant, $L$ is the Obukhov length, and $(\alpha, \beta, \lambda_{\max})$ are tunable coefficients chosen such that if $|L| = \infty$ then $K = k_v u_* z$ wherever $z \ll \lambda_{\max}$ ($u_*$ being the friction velocity based on surface shear stress).

The flux Richardson number is the ratio ____ and viscous dissipation is represented by term ____

(a) IV/III; V ✓✓
(b) III/II; I
(c) IV/II; V
(d) IV/III; I
(e) II/I; V

22. Referring again to Delage's closure, in the initial (neutral) state the maximum value for the length scale is limited to ____ while as the strength of the nocturnal inversion develops (i.e. with increasingly positive $\partial\overline{\theta}/\partial z$) eventually it is limited to ____

(a) $\lambda_{\max}$; $k_v\, z$
(b) $\lambda_{\max}$; $\beta\, L$ ✓✓
(c) $k_v\, z$; $\lambda_{\max}$
(d) $k_v\, z$; $\beta\, L$
(e) $\beta\, L$; $k_v\, \lambda_{\max}$

## Short answer $(2 \times 12\% = 24\%)$

***Instructions***: Please answer any **two** of the following questions. Use diagrams wherever they may be helpful. State any assumptions or simplifications you make.

1. **CBL & SBL**: Introducing appropriate symbols (e.g. $\delta$ for ABL depth) and resolving the surface layer, mixed-layer and entrainment interface layer, draw schematic profiles of mean potential temperature $\overline{\theta}(z)$, vertical heat flux density $\overline{w'\theta'}(z)$, and turbulent kinetic energy $k(z)$ for the summertime, late-afternoon, convective boundary layer (CBL). Discuss (making reference to the surface energy budget) the mechanisms driving the subsequent development of a nocturnal boundary layer, explaining (with the help of diagrams) why and how the profiles of $\overline{\theta}$, $\overline{w'\theta'}$, $k$ evolve away from their late-afternoon state. Explain one or more mechanisms or phenomena that have been hypothesized as cause for the *intermittency* sometimes observed in the nocturnal boundary layer. What factors render scientific description of the very stable ABL much more difficult than a CBL or NBL?

2. **Atmospheric Dispersion**: Consider the task of computing the downwind field of mean concentration $\overline{c}(x,z)$ caused by a steady line source of passive tracer (at $x = 0$ and height $z = h$) oriented perpendicular to the mean wind $\overline{u} = \overline{u}(z)$ in a horizontally-homogeneous atmospheric surface layer. Compare and contrast

   a. the "Gaussian plume" model, for which the concentration field is given analytically as:

$$\overline{c}(x,z) = \frac{q}{\sqrt{2\pi}\,\sigma_z\,U}\left[\exp\left(-\frac{(z-h)^2}{2{\sigma_z}^2}\right) + \exp\left(-\frac{(z+h)^2}{2{\sigma_z}^2}\right)\right] ,$$

   where $U$ is a (constant) wind speed and $\sigma_z = \sigma_z(x)$ is a tunable function whose *theoretical* value is $\sigma_z = \sqrt{2\,K\,x/U}$.

   b. the zeroth-order Lagrangian stochastic trajectory model

$$dZ = \frac{\partial K}{\partial z}\,dt + \sqrt{2\,K\,dt}\;r ,$$
$$dX = \overline{u}(Z)\;dt ,$$

   where $r$ is a standardized Gaussian random number (zero mean, unit variance).

   Address the comparative scientific fidelity of these two models; the assumptions implicit within them; and their validity (or invalidity) very close to the source.

3. **Disturbed flows**: Assume a surface layer flow along the $x$-axis encounters a disturbance at $x = 0$, in the form **either** of [a.] a step decrease in the surface Bowen ratio $(Q_{H0}/Q_{E0})$ without change in the total thermodynamic energy input $Q_{H0} + Q_{E0}$ $(\equiv Q^* - Q_G)$ at the surface; **or** [b.] a thin, porous windbreak of height $h$ and resistance coefficient $k_r$ standing perpendicular to the mean wind. Whichever case you choose to discuss, please assume constancy of all statistics along the crosswind ($y$) axis (i.e. two-dimensional flow).

   a. ***Either***: Discuss the downwind evolution of the fields of mean temperature $\overline{T}(x, z)$ and mean specific humidity $\overline{q}$, assuming the upwind flow is unstably stratified (upwind surface heat flux density $Q_{H01} > 0$ and Obukhov length $L_1 < 0$) and completely dry ($Q_{E01} = 0$; $\overline{q} = 0$ for all $z$ at $x \leq 0$), and that the surface fluxes $Q_{H02}$, $Q_{E02}$ over the downstream surface ($x > 0$) are unvarying (note: physically, this is an oversimplication). Give diagrams illustrating a sequence of vertical profiles of $\overline{T}$, $\overline{w'T'}$, $\overline{q}$ and $\overline{w'q'}$ at several downwind distances, and capturing the idea of a developing internal boundary layer (IBL) and, within it, a fully-adjusted equilibrium layer within which $\rho_R c_p \overline{w'T'} = Q_{H02}$ and $\rho_R L_v \overline{w'q'} = Q_{E02}$ ($L_v$ is the latent heat of vapourization). A numerical model of this flow, if adopting second-order closure, would involve closed conservation equations for which set of statistics?

   b. ***Or***: Describe (in qualitative terms, with assistance of diagrams) the effect of the windbreak on the fields of mean wind speed and turbulent kinetic energy (you may assume the upwind surface layer is neutrally stratified). Draw a transect of relative mean wind speed $\overline{u}(x, z)/\overline{u}_0(z)$ along a transect through the windbreak at $z/h = 1/2$, and extending over $-10 \leq x/h \leq 50$. Referring to the momentum equation given with multichoice Question (19), explain the mechanisms that control the shape of the relative windspeed curve.

   A highly simplified TKE equation for this flow

$$\overline{u}\,\frac{\partial \overline{k}}{\partial x} = -\overline{u'w'}\,\frac{\partial \overline{u}}{\partial z} - \epsilon - S_k \tag{5}$$

   (where $\epsilon$ is viscous dissipation) contains a localized sink term (co-located with the windbreak) due to interaction of the turbulence with the windbreak, viz.

$$S_k = k_r\,\overline{u}\,\left(2\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)\,\delta(x - 0)\,s(z - h)\ ,$$

   where $k_r$ is the windbreak resistance coefficient. Interpret the pattern of TKE (i.e "quiet zone" and "wake zone") relative to this TKE budget.

4. **Micro-meteorology of a plant canopy**: Consider a horizontally-uniform plant canopy (height $h$) whose area density profile

$$a(z) = 0 \quad , \quad z/h \leq 1/3 \ ,$$
$$\quad\;\; = a_c \quad , \quad 1/3 < z/h \leq 1 \ ,$$

consists of a trunk space (negligible leaf area) below a dense crown; assume the leaf area index of this canopy (LAI $= (2/3)\, a_c\, h$) is sufficiently large that negligible solar radiation penetrates into the trunk space.

Assuming a coordinate system chosen to ensure $\overline{v} = \overline{w} = 0$, and introducing appropriate symbols (such as "$u_*$" for the friction velocity based on canopy top shear stress $u_* = \sqrt{-\overline{u'w'}(h)}$, draw schematic profiles of *normalized* mean wind velocity $\overline{u}$, kinematic shear stress $\overline{u'w'}$, kinematic heat flux density $\overline{w'T'}$, and vertical velocity variance $\overline{w'^2}$ that will be appropriate to a windy summer afternoon. Referring to your diagrams where appropriate, discuss the features of wind in plant canopies that render this flow very different from the wind in the inertial sublayer (i.e. layer described by Monin-Obukhov similarity theory). Explain the occurrence of "ramps" in the time trace of the temperature fluctuation $T'(t)$ in a canopy.

Assuming that canopy drag is parameterized by a sink in the $\overline{u}$-momentum equation, viz.

$$0 = -\frac{\partial\, \overline{u'w'}}{\partial z} - c_d\, a\, \overline{u}^2 \ ,$$

discuss the connection between the mean wind profile $\overline{u}(z)$ and the profile of shear stress.