EAS572, “The Atmospheric Boundary Layer” Exam 13 Dec., 2005

Professor: J.D. Wilson Time available: 150 mins Value: 35%

Instructions: Organization of your answers is important to ensure clarity. Use diagrams wherever they may be helpful and especially if they demonstrate your interpretation of the situations/questions as posed here verbally. Be sure to state any assumptions or simplification you make. ***No page limit applies.***

1 Ideal atmospheric surface layer

1.1 Mean momentum equation

In the ideal (horizontally-uniform, stationary) ASL, and with restriction to heights above (any) canopy layer, the mean horizontal momentum equations reduce to

$$\frac{\partial \overline{u'w'}}{\partial z} = + f \; (\overline{v} - V_G)$$

$$\frac{\partial \overline{v'w'}}{\partial z} = - f \; (\overline{u} - U_G)$$

where U_G, V_G are the components of the Geostrophic wind (viz. $V_G = \frac{1}{\rho_0} \frac{\partial \overline{p}}{\partial x}$, etc).

One may choose to orient the axes such that $\overline{v} = 0$ at (and $\overline{v} << \overline{u}$ near) some reference level, say $z = 5$ m. Making reference to these equations, comment on how you would interpret the fact that the surface layer is often stated to be a “constant stress layer”. It may be useful to relate your discussion to the challenge of *measuring* the shear stress and the likely level of accuracy attainable.

How could the above equations be altered (in a heuristic way) to parameterize the influence on the momentum budget of a (horizontally-uniform) plant canopy having drag area density $A(z)\,[\mathrm{m}^{-1}]$? What would be the consequent nature of the shear stress profile?

1.2 Monin-Obukhov Similarity Theory

Succinctly describe the Monin-Obukhov similarity theory, defining its domain of validity, its key heuristic postulates (as to which variables ‘control’ the turbulence in the ideal ASL), and giving some illustrative examples of the type of formulae it provides.

Using appropriate (ie. MO) scaling, and assuming weakly unstable stratification, sketch surface layer profiles of: $\overline{u}$, $\overline{\theta}$ (mean potential temperature), $\overline{u'w'}$, $\overline{w'\theta'}$ (mean vertical heat flux density), $\sigma_u^2 \equiv \overline{u'^2}$. Note: these need not be quantitatively accurate sketches, but they should convey the main character of the profiles

1.3 Budget of streamwise variance

Further simplify the budget equation

$$\frac{\partial \overline{u'^2}}{\partial t} = 0 \quad = \quad -2\, \overline{u'w'}\, \frac{\partial \overline{u}}{\partial z} - \frac{\partial \overline{w'u'^2}}{\partial z} - \frac{2}{\rho_0}\, \overline{u'\, \frac{\partial p'}{\partial x}}$$
$$+ \quad \nu\, \nabla^2\, \overline{u'^2} - 2\nu\, \overline{\frac{\partial u'}{\partial x_j}\, \frac{\partial u'}{\partial x_j}}$$

Assume local equilibrium, neglect molecular transport of variance, and re-name the viscous dissipation term ϵ_{xx}. Comment on the significance of the remaining terms, in particular identifying the “redistribution” term and its action.

2 Windbreak flow

The following questions pertain to the situation of a neutrally-stratified surface-layer flow (scaling parameters u_{*0}, z_0) encountering a perpendicularly-oriented, infinitely-long, porous fence (parameters the fence height h and resistance coefficient k_r, and the angle of incidence of the mean wind $\overline{\theta} = 0$). Let the fence define the origin $x = 0$ of the x-axis and lie parallel to (ie. along) the y-axis (thus by symmetry, everywhere $\overline{v} = 0$). A suitable $\overline{u}$-momentum equation, here written in advection form, is

$$\overline{u}\, \frac{\partial \overline{u}}{\partial x} + \overline{w}\, \frac{\partial \overline{u}}{\partial z} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} - \frac{\partial \overline{u'w'}}{\partial z} - \frac{\partial \overline{u'^2}}{\partial x} - k_r\, \overline{u}\, |\overline{u}|\, \delta(x-0)\, s(z-H)$$

where the terms have their usual meaning.

- What dimensionless parameters control this flow?
- Specify (giving *formulae*) appropriate inflow (ie. far upwind) profiles $\overline{u}_0(z)$, $\overline{w}_0(z)$, $(\overline{u'w'})_0(z)$, $\sigma_{u0}^2 \equiv (\overline{u'^2})_0(z)$
- Reduce the momentum budget to its dominant terms in the near-fence region $|x|/h \leq 5$ (with explanation)
- Reduce the momentum budget to its dominant terms in the far wake $x/h > 10$ (with explanation)

- Sketch (qualitative) transects of $\overline{u}/u_{*0}$, $\overline{w}/u_{*0}$, $\overline{p}/(\rho_0 u_{*0}^2)$, k/u_{*0}^2 along streamlines that, at $x = 0$, cut the z-axis at $z/h \sim \frac{1}{2}, \frac{3}{2}$. Note: k is the turbulent kinetic energy.

3 Lagrangian stochastic model for fluid element paths in turbulence

3.1 A technical detail

Suppose X is a stochastic variable that evolves in time t. Suppose further that its evolution were modelled by changes over discrete timesteps Δt according to

$$\begin{aligned} \Delta X &= K + n\, \Delta t\, d\xi \\ X &= X + \Delta X \end{aligned}$$

where $K, n, \Delta t$ are constants, and $d\xi \in N(0,1)$, ie. $d\xi$ is a random impulse drawn from a Normal distribution with zero mean and unit variance.

Let x be the phase-space variable corresponding to X. Deduce the transition density function $p(x, t + \Delta t |\, x_0, t)$ corresponding to the stochastic equation above.

3.2 The well-mixed constraint

In your own words, explain the meaning and significance of the "well-mixed constraint", with specific reference to first-order, multi-dimensional LS models for fluid element trajectories having the form

$$\begin{aligned} dU_i &= a_i\,(\mathbf{X}, \mathbf{U}) \;+\; b\, d\xi_i \\ dX_i &= U_i\, dt \end{aligned}$$

where $b = b(\mathbf{X})$. Note that the Fokker-Planck equation corresponding to this model is

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x_i}\,(u_i p) - \frac{\partial}{\partial u_i}\,(a_i p) + \frac{b^2}{2}\,\frac{\partial^2 p}{\partial u_i\, \partial u_j}$$

Your response should make reference to the transition density function $p(x_i, u_i, t{+}\Delta t |\, x_{i0}, u_{i0}, t)$ and the Eulerian velocity PDF $g_a()$.