EAS572, "The Atmospheric Boundary Layer" Final Exam 14 Dec., 2007

Professor: J.D. Wilson Time available: 150 mins Value: 35%

Note: Relevant equations, definitions and data are given at the back of the exam.

Multichoice ($22 \times \frac{1}{2}\% = 11\%$)

1. A flow with 'high turbulence intensity' would be characterized by:
   - (a) $k \ll 1$
   - (b) $k/\overline{u} \ll 1$
   - (c) $k \gg \sigma_u^2 + \sigma_v^2 + \sigma_w^2$
   - (d) $\sigma_u/\overline{u} \gtrsim 1$
   - (e) $\sigma_u/\overline{u} \ll 1$

2. The sum of the sensible and latent heat flux densities at the surface, $Q_{H0} + Q_{E0}$, can be regarded as the surface source strength [$\mathrm{W\,m^{-2}}$] for which scalar property?
   - (a) mean temperature, $\overline{T}$
   - (b) mean potential temperature, $\overline{\theta}$
   - (c) mean dewpoint temperature, T_d
   - (d) mean equivalent temperature, $\overline{T}_{eq} \equiv \overline{T} + \overline{e}/\gamma$
   - (e) mean saturation deficit at the surface, $\overline{e}(z_0) - e_S(\overline{T}(z_0))$

3. Let $h_b(x)$ be the depth of an internal boundary-layer (IBL) growing from the leading edge (at $x = 0$) of a step change in surface Bowen ratio, and let subscript '∞' denote properties of the flow far upstream from the discontinuity. The simplest paradigm for the growth rate of the IBL assumes $\partial h_b/\partial x$ is proportional to:
   - (a) $1/\overline{u}_\infty(z_{0\infty})$
   - (b) $1/\overline{u}_\infty(\alpha\, h_b)$, where α is a constant
   - (c) $\sigma_{w\infty}/\overline{u}_\infty(\alpha\, h_b)$
   - (d) $z_{0\infty}$
   - (e) k_∞

4. In a neutrally-stratified and horizontally-homogeneous ASL ("hhNSL"), the aerodynamic resistance r_a between two levels z_1 and $z_2 > z_1$ is ____

   (a) $r_a = (z_2 - z_1)\, u_*^{-1}$

   (b) $r_a = \int_{z_1}^{z_2} (k_v\, u_*\, z)^{-1}\, dz$

   (c) $r_a = u_*\, (z_2 - z_1)^{-1}$

   (d) $r_a = \ln(z_1/z_2)$

   (e) $r_a = u_*^{-1}$

5. The property $k\,\epsilon^{-1}$ serves in many turbulence closure models as a/an

   (a) eddy diffusivity

   (b) eddy flux

   (c) eddy flux divergence

   (d) turbulence timescale

   (e) Richardson number

6. In the (parameterized) budget equation for $\overline{u'^2}$, a term in $(\,\overline{u'^2} - \ 2k/3\,)$ models ____ and tends to ____ the turbulence

   (a) turbulent transport; isotropize

   (b) buoyant production; stratify

   (c) shear production; energize

   (d) pressure-strain correlation ("redistribution term"); dissipate

   (e) pressure-strain correlation ("redistribution term"); isotropize

7. Random variable x is uniformly distributed on $-1 \le x \le 1$. Its variance $\overline{x'^2}$ is

   (a) 1/5

   (b) 1/4

   (c) 1/3

   (d) 1/2

   (e) 2/3

8. In an adiabatic, incompressible turbulent flow, e.g. neutrally-stratified surface layer flow about a windbreak, the mean pressure field is determined by a Poisson equation

$$\frac{1}{\rho_0}\,\nabla^2\overline{p} = -\,\frac{\partial \overline{u}_i}{\partial x_j}\,\frac{\partial \overline{u}_j}{\partial x_i}\;-\;\overline{\frac{\partial u_i'}{\partial x_j}\,\frac{\partial u_j'}{\partial x_i}}$$

This is an "elliptic" partial differential equation (i.e. highest-order spatial derivative of the dependent variable is the curvature, and features on all spatial axes), implying

(a) an obstacle can perturb the upwind pressure field

(b) the pressure field is not affected by obstacles introduced into the flow

(c) the pressure field is determined fully by the velocity statistics

(d) eddy viscosity closure is legitimate

(e) eddy viscosity closure is not legitimate

9. In 'advection form' and without approximation, the continuity equation reads

(a) $\frac{\partial\rho}{\partial t} + \vec{u}\cdot\nabla\rho = 0$

(b) $\frac{\partial\rho}{\partial t} = -\,\nabla\cdot(\,\rho\,\vec{u}\,)$

(c) $\frac{d\rho}{dt} = 0$

(d) $\frac{\partial\rho}{\partial t} + \vec{u}\cdot\nabla\rho = \;-\,\rho\,\nabla\cdot\vec{u}$

(e) $\nabla\cdot\vec{u} = 0$

10. In a Cartesian coordinate system the gradient ('grad') operator is $\nabla \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$. Representations of the Laplacian operator ∇^2 valid in Cartesian axes include

(a) $\hat{i}\frac{\partial^2}{\partial x^2} + \hat{j}\frac{\partial^2}{\partial y^2} + \hat{k}\frac{\partial^2}{\partial z^2}$

(b) $\frac{\partial^2}{\partial x_i\,\partial x_j}$

(c) $\frac{\partial}{\partial x_j}\,\frac{\partial}{\partial x_i}$

(d) $\nabla\cdot\nabla \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

(e) $\frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial x_i} + \frac{\partial}{\partial x_j}\right)$

11. In high Reynolds number turbulence, the mechanism for passing kinetic energy from larger to smaller scales of motion is

(a) viscous dissipation

(b) shear reduction

(c) vortex stretching

(d) turbulent transport

(e) diffusion

12. One of the conditions for validity of the Boussinesq approximation is that $L \ll c^2/g$, where c is the speed of sound and g the acceleration due to gravity. Here L represents

   (a) the Obukhov length
   (b) the Kolmogorov length
   (c) a characteristic scale for the horizontal extent of the motion field
   (d) a characteristic scale for the vertical extent of the motion field
   (e) the molecular mean free path

13. Under the Boussinesq approximation and neglecting certain small Coriolis terms, the vertical momentum equation is

$$\frac{\partial w}{\partial t} + \vec{u} \cdot \nabla w = -\frac{1}{\rho_R} \frac{\partial \tilde{p}}{\partial z} + g \frac{\tilde{T}}{T_R} + \nu \nabla^2 w$$

   Here $\tilde{p}, \tilde{T}$ represent

   (a) total pressure and Kelvin temperature
   (b) mean (Reynolds-averaged) total pressure and Kelvin temperature
   (c) instantaneous fluctuation in total pressure and Kelvin temperature
   (d) instantaneous deviations of pressure and temperature from hydrostatic, adiabatic reference state
   (e) pressure and temperature of the hydrostatic, adiabatic reference state

14. 'Inflection point instability' refers to the existence of a level (say, z_p) across which the curvature of the wind profile changes sign. The ASL wind profile has this characteristic

   (a) near the top of a plant canopy
   (b) at the top of the roughness sublayer layer
   (c) within the mixed layer
   (d) in neutral stratification
   (e) in unstable stratification

15. The flux and gradient Richardson numbers can be related to z/L as $R_i^f = \frac{z}{L} \frac{1}{\phi_m(z/L)}$ and $R_i^g = \frac{z}{L} \frac{\phi_h(z/L)}{\phi_m^2(z/L)}$. In neutral stratification the Richardson number evaluates to

   (a) infinity
   (b) minus infinity
   (c) zero
   (d) unity
   (e) minus unity

16. Consider a uniform, managed forest of height h_c having a pruned (branchless, leafless) trunk space that spans $0 \le z \le h$. The source distribution for water vapour is multi-modal due to soil evaporation at $z = 0$, non-transpiring trunk space, and leaf transpiration in the tree crown. Typically the sources are vertically separated by a distance of order d (displacement length), and the occurrence of a counter-gradient flux at the base of the crown reflects a regime where

   (a) turbulence length scale $\ell \ll d$: observation point is in the *far* field of sources

   (b) turbulence length scale $\ell \ll d$: observation point is in the *near* field of sources

   (c) turbulence length scale $\ell \sim d$: observation point is in the *far* field of sources

   (d) turbulence length scale $\ell \sim d$: observation point is in the *near* field of sources

   (e) the eddy diffusion paradigm is useful

17. The vertical profile of the kinematic shear stress $\overline{u'w'}$ within a plant canopy, when normalized by its canopy-top value $-u_*^2 \equiv \overline{u'w'}(h_c)$, is relatively invariant during days of appreciable wind, whereas under the same conditions profiles of normalized heat flux density $\overline{w'T'}(z)/\overline{w'T'}(h_c)$ vary markedly throughout the day. This is because

   (a) Monin-Obukhov similarity theory applies for momentum but not heat transport

   (b) the canopy is a "constant flux layer" for momentum, but not for heat

   (c) the canopy is a "constant flux layer" for heat, but not momentum

   (d) the source distribution for heat depends on solar elevation, leaf water status and other varying factors

   (e) in a plant canopy, the probability density function for vertical velocity is non-Gaussian

18. The advection-diffusion equation

$$\frac{\partial \overline{c}}{\partial t} + U \frac{\partial \overline{c}}{\partial x} = K_x \frac{\partial^2 \overline{c}}{\partial x^2} + K_y \frac{\partial^2 \overline{c}}{\partial y^2} + K_z \frac{\partial^2 \overline{c}}{\partial z^2}$$

   ($U =$ const.) governs the ensemble-averaged growth rate of a puff of material released at the origin at $t = 0$, for which scenario the initial condition is $c(\mathbf{x}, 0) = q\,\delta(\mathbf{x}-\mathbf{0})\,\delta(t-0)$. It is a characteristic of "diffusion" that the standard deviation of the puff size (σ_x, taking as example the spread along the x-direction) behaves as

   (a) $\sigma_x \propto (K_x t)^2$

   (b) $\sigma_x \propto K_x t$

   (c) $\sigma_x \propto \sqrt{K_x t}$

   (d) $\sigma_x \propto \sqrt{t/K_x}$

   (e) $\sigma_x \propto K_x^{1/2} t^{1/3}$

19. The Gaussian plume model that finds frequent application in air pollution studies is founded on the steady-state form of the above equation. It treats the σ_y, σ_z (plume widths) as

   (a) empirical functions of downstream distance and atmospheric stability
   (b) properties prescribed by the theory, i.e., those values (which will depend on $K_y, K_z, x/U$) which arise from solving the equation to obtain a multi-dimensional Gaussian distribution
   (c) to be evaluated by a modern Lagrangian stochastic model
   (d) dependent on distance from the source, but not atmospheric stability
   (e) dependent on atmospheric stability, but not distance from the source

20. In G.I. Taylor's expression

$$\frac{d\sigma_z^2}{dt} = 2\,\sigma_w{}^2 \int_0^{\xi=t} R(\xi)\, d\xi$$

for the rate of growth of a puff in homogeneous turbulence, the function $R(\xi)$ is

   (a) the velocity structure function (ξ a separation in space)
   (b) the Lagrangian velocity autocorrelation function (ξ the lag time)
   (c) the Eulerian velocity autocorrelation function
   (d) the Lagrangian velocity spectrum (ξ the frequency)
   (e) the Eulerian velocity spectrum (ξ the wavenumber)

21. The equation

$$0 = -\overline{u'w'}\,\frac{\partial \overline{u}}{\partial z} + \frac{g}{T_0}\,\overline{w'T'} - \frac{\partial}{\partial z}\,\overline{w'\ (p'/\rho_0 +\ e'\ )} - \epsilon$$

expresses conservation of ____ , assuming ____

   (a) mean vertical momentum ($\rho_0 \overline{w}$); neutral stratification
   (b) $k = (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)/2$; horiz. homogeneity and stationarity
   (c) $k = (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)/2$; neutral stratification, horiz. homogeneity and stationarity
   (d) TKE dissipation rate ϵ; horiz. homogeneity and stationarity
   (e) $\overline{u'w'}$; neutral stratification and horiz. homogeneity

22. With respect to the above equation, if the flow is in "local equilibrium" then

   (a) $0 = \overline{u'w'}\ \partial\overline{u}/\partial z = \frac{g}{T_0}\ \overline{w'T'}$
   (b) $\epsilon = -\overline{u'w'}\ \ \partial\overline{u}/\partial z + \frac{g}{T_0}\ \overline{w'T'}$
   (c) $\epsilon = 0 = -\overline{u'w'}\ \ \partial\overline{u}/\partial z$
   (d) $\epsilon = \ -\ \overline{w'\ (p'/\rho_0 +\ e'\ )}$
   (e) $\epsilon = \ -\ \frac{\partial}{\partial z}\ \overline{w'\ (p'/\rho_0 +\ e'\ )}$

Short answer $(4 \times 6\% = 24\%)$

Instructions: Please answer any **four** of the following seven questions. Organization of your answers is important to ensure clarity. Use diagrams wherever they may be helpful and especially if they demonstrate your interpretation of the situations/questions as posed here verbally. Be sure to state any assumptions or simplification you make. ***No page limit applies.***

A. **Explain** the conceptual basis, content, utility and limitations of the Monin-Obukhov similarity theory.

B. The TKE dissipation rate

$$\epsilon \approx 15\,\nu\,\overline{\left(\frac{\partial u'}{\partial x}\right)^2}$$

and commonly ϵ is evaluated by means of differentiating a time series of streamwise velocity $u'(t)$, on the assumption that

$$\frac{\partial u'}{\partial x} \approx -\frac{1}{\overline{u}}\frac{\partial u'}{\partial t} \tag{1}$$

where $\overline{u}$ is the only non-zero component of the mean wind. **Explain** why eq.(1) is known as the "frozen turbulence hypothesis" and, making reference to $(\sigma_u,\, \sigma_v,\, \sigma_w,\, \overline{u})$, outline under what condition(s) the approximation may be valid

C. Suppose the mean wind direction, aligned with and defining the x-axis, is normal to an infinitely long, narrow, rectangular concrete channel (depth d, width X). At the bottom of the channel a liquid is evaporating at a uniform but unknown rate Q [$\mathrm{kg\,m^{-2}\,s^{-1}}$]. Upwind from the channel, where the surface roughness length is z_0, the ASL is undisturbed and characterized by friction velocity u_* and Obukhov length L. Except at the base of the channel, the source strength for this particular gas is zero (well-mixed background).

Now suppose you have a laser gas detector which you have set up to measure the line average gas concentration $\langle c \rangle$ along a line normal to the channel (i.e. parallel to the x-axis) and tangent to the ground surface (i.e. the beam runs along $z/z_0 \sim 1$), and that (furthermore) you average this signal for (say) 30 minutes to obtain the time average $\overline{\langle c \rangle}$. You would like to be able to infer Q from $\overline{\langle c \rangle}$ and other relevant variables.

Perform a dimensional analysis suggesting (as specifically as you are able) the form of the relationship between $\overline{\langle c \rangle}$ and Q.

D. Let $\overline{u}$ be the component of the mean wind oriented along the x-axis, and assume an ASL that is in every respect invariant along the perpendicular (y) coordinate. It is easy to show that if the wind profile is

$$\overline{u} = \frac{u_*}{k_v} \ln\left(\frac{z}{z_0}\right) \tag{2}$$

then

$$\int_{z_0}^{h} \overline{u}(z)\, dz = \frac{u_*}{k_v}\, [h\, (\ln(h/z_0) - 1) + z_0]$$

Evaluate the difference between the heat flux density $\overline{w\,T}(h)$ measured at height $h = 2$ m and the surface heat flux $\overline{w\,T}(z_0)$, under the approximation that *the wind profile is horizontally-invariant* and given by eq.(2), but given that there is a constant and height-independent streamwise temperature gradient

$$\frac{\partial \overline{T}}{\partial x} = \text{const.} = -\,0.001 \text{ [K m}^{-1}\text{]}$$

Assume the friction velocity $u_* = 0.4\,\mathrm{m\,s^{-1}}$ (so that $u_*/k_v = 1$) and that $z_0 = 0.01$ m. Base your calculation on a simplified[1] heat conservation equation

$$0 = -\frac{\partial \overline{u}\,\overline{T}}{\partial x} - \frac{\partial \overline{w\,T}}{\partial z}\,,$$

whose height integral

$$\overline{w\,T}(h) - \overline{w\,T}(z_0) = -\int_{z_0}^{h} \frac{\partial\, \overline{u}\,\overline{T}}{\partial x}\, dz$$

further simplifies (under the stated conditions) to

$$\overline{w\,T}(h) - \overline{w\,T}(z_0) = -\frac{\partial\,\overline{T}}{\partial x} \int_{z_0}^{h} \overline{u}(z)\, dz$$

[1]Simplifications: steady state; symmetry along y; streamwise eddy heat flux neglected; no volumetric heat production/destruction (i.e. no phase changes, no radiative divergence).

E. **Calibrate** the power law mean wind profile

$$\overline{u}(z) = U_r \left(\frac{z}{z_r}\right)^m$$

to the semi-logarithmic profile for the neutral surface layer

$$\overline{u}(z) = \frac{u_*}{k_v} \ln\left(\frac{z}{z_0}\right)$$

by evaluating U_r, m in terms of u_*, k_v, z_0. The availability of two parameters allows one to impose two conditions: please evaluate U_r, m so that that both wind speed and wind shear are correctly reproduced, at the (arbitrary) reference height z_r.

F. **Explain** the aerodynamics of windbreak flow, taking the idealized case of an infinitely long, porous barrier (height h) set perpendicular to the mean wind in a neutrally-stratified surface-layer. Explain what mechanisms determine the characteristic shape of the relative windspeed curve. Note: with reference to the $\overline{u}$-momentum equation given as data, here the source term $S = -k_r\, \overline{u}\ \mid \overline{u} \mid\ \delta(x-0)\ s(z-h)$ where 's' is a step function. As a first approximation, you may neglect vertical advection by the mean flow, and the streamwise gradient in $\overline{u'^2}$.

G. A common assumption for the mean streamwise momentum equation within a horizontally-uniform plant canopy (height h_c) is

$$0 = \frac{\partial \tau}{\partial z} - c_d\, a\, U^2$$

where $U(z) = \overline{\langle u \rangle}$ is a suitably-defined mean velocity (time average of a spatial average), $\tau = -\overline{\langle u'w' \rangle}$ is (minus) the kinematic momentum flux, c_d is a drag coefficient, and a [$\mathrm{m^2\,m^{-3}}$] is leaf area density. Assuming $c_d a$ to be independent of height, and adopting an eddy viscosity closure

$$\begin{aligned} \tau &= K \frac{\partial U}{\partial z} \\ K &= \ell^2 \frac{\partial U}{\partial z} \end{aligned}$$

(where $\ell =$ const. is the mixing length, treated as height independent), **derive** the exponential canopy wind profile

$$U(z) = U(h_c)\ \exp\left[\beta\left(\frac{z}{h_c} - 1\right)\right]$$

and express the extinction coefficient β in terms of other named variables. Deduce also the profile of the momentum flux $\tau(z)$.

Symbols, Definitions, Data, Equations

Symbols: reference density and reference (Kelvin) temperature (ρ_0, T_0) (alternatively designated ρ_R, T_R); roughness length z_0; boundary-layer depth δ; friction velocity u_*; turbulent kinetic energy k; variance of (e.g.) vertical velocity $\sigma_w^2 \equiv \overline{w'^2}$; TKE dissipation rate ϵ; kinematic viscosity of air ν; von Karman constant k_v; pressure p; vapour pressure e; saturation vapour pressure (at temperature T) $e_S(T)$; psychometric constant γ (for a parcel at temperature T with vapour pressure e, the ratio e/γ equals the hypothetical increase in temperature due to release of latent heat, assuming all the water vapour was to be isobarically condensed).

- Assuming steady state, and homogeneity along the y direction, the Reynolds equation for mean streamwise momentum is

$$\overline{u}\,\frac{\partial \overline{u}}{\partial x} + \overline{w}\,\frac{\partial \overline{u}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} - \frac{\partial \overline{u'^2}}{\partial x} - \frac{\partial \overline{u'w'}}{\partial z} + S$$

  where the source term S parameterizes interaction with obstacles and is typically treated as $S \propto -\overline{u}\ \mid \overline{u} \mid$

- The 'surface energy balance' on a reference plane at the base of the atmosphere is expressed by the equation

$$Q^* = Q_H + Q_E + Q_G$$

  where all fluxes are in [W m^{-2}]. Sign convention: Q^* the net radiation, positive if directed towards the surface; Q_H, Q_E the sensible and latent heat fluxes, positive if directed from the surface towards the atmosphere; Q_G the 'soil' heat flux, positive if directed from the surface into ground/lake/ocean.