6.2 WIND TRANSPORT IN AN IDEALISED URBAN CANOPY

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1. INTRODUCTION

At an intermediate stage in the dispersion of a plume in a city, inhomogeneity of the wind field may remain an important factor, even though the details of the distribution of buildings may be redundant. We examine a simple numerical model as a potential means to quickly generate a non-uniform urban flow field, and we study the significance of the spatial-inhomogeneity of the (modelled) flow for plume dispersion, using a Lagrangian stochastic (LS) model to simulate tracer trajectories.

The idea is to compute the large-scale spatial gradients in wind statistics, on length scales comparable to the building size, gradients we can characterize (eg.) in terms of a spatial mean-velocity variance Σ_u^2 , defined by*

$$\Sigma_u^2 = \left\langle \left(\bar{u}(x,y,z) - <\bar{u}> \right)^2 \right\rangle \qquad (1)$$

We ask, is Σ_u^2 accurately calculated by this particular model?, and, irrespective of the answer, whether in urban flows Σ_u^2 is generally big enough, in relation to the turbulent kinetic energy (TKE), to warrant computation of the 3d wind field, and input to a dispersion model?

2. FLOW MODEL

We compute the 3-d fields of mean velocity ($\bar{u}$,$\bar{v}$,$\bar{w}$) and TKE (k) within an idealized domain that encloses one or more buildings, and connects to neighboring domains via the assumption of periodicity, on intervals L_x,L_y. Buildings (height H) are represented as porous, and assigned a (fictitious) drag coefficient x area density product, C_d A. We invoke eddy viscosity $K \propto \lambda\, k^{1/2}$, where the lengthscale $\lambda = \max(\lambda_i , \lambda_o)$, the inner and outer scales being

$$\frac{1}{\lambda_i} = \frac{1}{k_v\, z} + \frac{1}{\lambda_c} \quad , \quad \frac{1}{\lambda_o} = \frac{1}{k_v\, (z - d)} + \frac{1}{L_\infty} \qquad (2)$$

Here k_v=0.4 is von Karman's constant, and λ_c is a canopy "shear length scale,"

$$\lambda_c = \sqrt{k(H)} \left(\frac{\partial S}{\partial z} \right)_H^{-1} \qquad (3)$$

where S is mean horizontal windspeed. This closure has proven useful in uniform and disturbed plant/forest canopies (Wilson et al., 1998, hereafter WFR; Wilson & Flesch, 1999), and our numerical procedure follows those descriptions.

*Single point time averages, such as of x-wise velocity u, will be denoted by an overbar, thus $\bar{u}$. The spatial average of $\bar{u}$ is written as <$\bar{u}$>.

3. SIMULATION OF AN "URBAN CANOPY"

Raupach et al. (1986; hereafter RCL) reported wind statistics, and Legg et al. (1986) the temperature field due to a line source of trace heat, within the "Tombstone Canopy," a staggered array of vertical bars (H=60mm high, Y_0=10mm wide, X_0=1mm thick), arranged in a diamond pattern on the floor of a wind tunnel (cross-stream spacing L_y=60mm, along-stream spacing ½ L_x=44mm). Figure (1) gives velocity contours for this canopy, calculated with resolution $\Delta x=\Delta y=\Delta z$=2mm. Inputs were a prescribed displacement length d/H = 0.01; closure parameters c=μ=1.0, α=0 (see WFR); and C_dA=1000m^{-1}. Contrary to what was hoped, solutions were *not* independent of this last (fictitious) parameter.

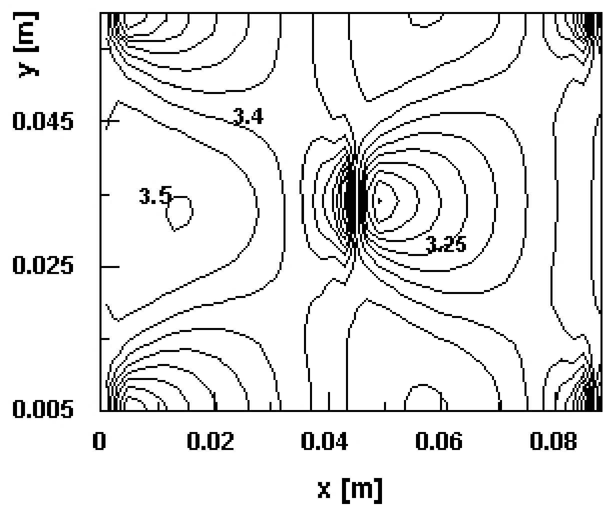

Figure 1. Contours of mean streamwise velocity $\bar{u}$ [m/s] at *z/H*=1 in the periodic Tombstone Canopy.

Model $\bar{u}$ contours at *z/H*=1somewhat resemble those measured by RCL (their Fig 4), but although the spatial *ordering* of speeds was encouraging, $\bar{u}(z)$-profiles compared poorly with those measured (RCL Fig.3a, stations A-J). Spatial variation in the canopy is rather *small* (RCL Fig.3a), despite the fact that the frontal area index $\lambda_f = 2\,(H X_0)/(L_x L_y)$ = 0.23 implies a near maximally-rough canopy, in terms of its effective drag coefficient. From the model, at *z/H*=½ the index (Σ_u^2 / <k>) << 1, suggesting that inhomogeneity of the mean fields may not be very important for dispersion in this flow.

The model's spatial-mean profiles <$\bar{u}$>, <k> (not shown) were in poorer agreement with the observations than those provided by the *1-d* form of the flow model (Wilson et al., 1998), but success of that *1-d* model is largely ordained by one's having provided a drag coefficient *derived* from the observed profiles of mean velocity and shear stress.

In the next section we depend on the flow model only in that it provides us a gridded 3-d flow field having a *plausible* degree of inhomogeneity, relative to the Tombstone Canopy observations.

4. DISPERSION MODEL

To explore the consequences of resolving/neglecting horizontal-inhomogeneity in urban flow, we calculated dispersion from the line source in the Tombstone Canopy, using Thompson's (1987) well-mixed, 3-dimensional, LS model for trajectories in Gaussian, inhomogeneous turbulence (in our modelled flow there is no basis to assume other than Gaussian velocity pdf's; and there is evidence, Sawford, 1999, that this LS model is at least as good a choice as any other).

Normal components of the stress tensor V^{ij} were deduced from the modelled TKE by assuming the same partitioning as in neutrally stratified, horizontally-uniform atmospheric surface layer flow, where $k/u_*^2 = ½ (c_u^2 + c_v^2 + c_w^2)$; tangential components were calculated from the mean shear $\partial \bar{u}^i/\partial x^j$ and eddy viscosity. Flow statistics (and their spatial gradients) were (optionally) interpolated from gridpoints to the particle's equivalent position in the fundamental cell (interpolation was based on the eight radii from the particle to nearest gridpoints).

Thomson's criteria for LS models do not limit the "permissible" spatial variability of flow statistics; in principle, arbitrary profiles are accommodated, provided only that the timestep is appropriately small. "Continuity" of a modelled flow field is a contradiction in terms, and so it is not clear what criteria must be imposed. In practise, these complex (generalised) Langevin equations may generate "rogue trajectories" terminating with impossible velocities, or (in the 3d urban field) trajectories that "go dormant" in regions of tiny time- and velocity-scales. It is unclear how *legitimately* to prevent these sorts of difficulties without ad hoc interventions. May one insist, for example, that in the limit $|U_i| \rightarrow \infty$ (where U_i is the Lagrangian velocity *fluctuation*), the conditional mean acceleration a_i should act to decrease the magnitude of the fluctuation? For the present model that would imply that the effective timescale should obey

$$T_L < \left(U_i \frac{\partial \ln \sigma_i}{\partial x_i} \right)^{-1} \qquad (4)$$

(σ_i the velocity standard deviation for direction *i*, with no summation implied). The "stability intervention" mentioned below was this: if (for any i) $|U_i|>10\sigma_i$ *and* the inequality (4) was untrue, we *reset* all velocity fluctuations by random choice from a Gaussian with the given σ_i. We found it very helpful to be able to view the trajectories, so as to observe whether they appeared feasible. "Bad" trajectories typically culminated in floating-point errors.

5. PROVISIONAL CONCLUSIONS

1) Representation of solid buildings as porous, in order to simplify calculations, does not appear to be a useful idea.

2) Rogue trajectories resulted if we neglected terms in the Langevin equations involving *horizontal* derivatives (like $\partial\sigma_u^2/\partial x$; "locally-homogeneous" LS model). This was so even *with* the stability intervention.

3) The inverse $(V^{-1})^{ij}$ of the stress tensor needed to be calculated *after* interpolation of V^{ij} to the particle (if both V^{ij} and $(V^{-1})^{ij}$ are individually interpolated, their product is unlikely to be the identity matrix).

4) In our experience stability intervention was necessary even if no terms in the LS model were neglected. Is this a problem of the LS model itself, or one's implementation of it? Does it imply, eg., that velocity statistics provided were unphysical or mutually inconsistent?

5) Prohibition of particle entry into tombstones, by reflection or absorption, was no remedy for rogue or dormant trajectories.

6) Continuing reduction of the timestep (as small as dt=0.0005 T_L) did not prevent the above difficulties

7) For this (fairly high-resolution, modelled) urban flow field, interpolation of flow statistics to the particle greatly increased trajectory computation time, with undetectable impact on the mean tracer concentration field.

7) None of our simulations of tracer dispersion in the modelled Tombstone canopy matched the data as well as those reported by Flesch and Wilson (1992), which were based on the horizontally-uniform reduction of this same LS model, driven with (formulae fitted to) measured, horizontally-invariant flow statistics. Probably this is symptomatic of (our present) imperfectly-computed horizontal-mean flow properties, but it surely also implies that our (not wholly unrealistic) representation of spatial variability in the flow did not have a positive impact.

In conclusion, even though the Tombstone canopy is very rough, inhomogeneity is not so serious (in terms of its implication for dispersion on the scale examined) as to warrant the step of calculating an *imperfect* 3d flowfield, for provision to a dispersion model. In this example, good estimation of the spatial mean fields ($\langle\bar{u}\rangle$, $\langle k\rangle$) from measurements or from a 1-d model, seems more important than attempting to represent the horizontal inhomogeneity.

REFERENCES

Flesch & Wilson,1992: *Bound. Lay. Meteo.* 61.
Legg, Raupach & Coppin,1986: *Bound. Lay. Meteo.* 35.
Raupach, Coppin & Legg,1986:*Bound.Lay. Meteo.* 35, 39
Sawford,1999: *Bound. Layer Meteo.*, 93.
Thomson, 1987: *J. Fluid Mech.* 180.
Wilson & Flesch, 1999: *Agric. Forest Meteo.* 93.
Wilson, Finnigan & Raupach,1998: *Q.J.R.Met.Soc.* 124.