Calculation of winds disturbed by an array of fences

John D. Wilson a,∗, Eugene Yee b

a Department of Earth & Atmospheric Sciences, University of Alberta, Edmonton, Alberta, Canada T6G 2E3
b Defense Research Establishment (Suffield), Medicine Hat, Alberta, Canada

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Abstract

It is known that when an isolated porous windbreak is represented by a sink in the mean momentum equations, even the simplest turbulence closures lead to reasonably good simulations of the mean wind close to the barrier, irrespective of whether or not sources are introduced in the turbulent kinetic energy (TKE) and dissipation rate (ϵ) equations. However, unless the barrier is considered to dissipate TKE in addition to opposing the mean flow, the pattern of turbulence is poorly simulated. Here we examine the performance of simple Reynolds-averaged Navier–Stokes (RANS) wind models for the case of a windbreak network, relative to the field experiment of McAneney and Judd (15 porous fences, height h = 2 m, spaced at Dx/h = 6 along the mean wind). All closures we considered gave mediocre predictions even for the mean flow, contrasting with the case of an isolated barrier; while, just as for the isolated barrier, prediction of the TKE in a windbreak network hinges on ambiguous choices in the treatment of TKE (and ϵ) sources at the barriers.

The additional turbulence generated by a sequence of barriers implies that a proper representation of the Reynolds stress is more critical than in the case of an isolated barrier, near which the pressure-gradient force dominates. This surely accentuates the importance of the turbulence closure, and our results may imply that no existing RANS turbulence closure is adequate for this type of flow. However, the problem could alternatively stem from the treatment of sinks parameterising the flow–fence interaction, not only in the TKE and ϵ-equations (where perforce such terms are heuristic), but even in the momentum equations.

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1. Introduction

In some regions of the world, for example, parts of New Zealand and France, arrays of windbreaks are vital in specialized agriculture and horticulture. For almost a century, and perhaps even longer, the scientific study of windbreaks has been motivated by the search for the most effective types of and usages of windbreaks, for a diverse range of benefits including reduced soil erosion, higher canopy temperatures, or reduced fruit motion. Early work is reviewed by van Eimern et al. (1964), and a more up-to-date survey is given by Brandle et al. (1988).

Wind and micro-climate models express in the most general way our understanding of disturbed flows, for in principle all particular cases lie within their compass. Standard turbulence models have proven able to simulate quite well the mean flow and (not so unambiguously) the turbulent kinetic energy (TKE) about a long, isolated, porous windbreak standing on flat ground (Wilson, 1985; Wang and Takle, 1995; Packwood, 2000; Wilson et al., 2001). But this article
will present evidence that may imply the aerodynamics of multiple porous barriers eludes existing (Reynolds-averaged Navier–Stokes (RANS)) models.

Early attempts to extract a general understanding of the principal aerodynamic effects of windbreaks were empirical, field and wind tunnel experiments contributing many valuable qualitative insights. In parallel with the experimental work, some scientists treated the windbreak as a problem in turbulent fluid mechanics, and appropriately so, for an isolated windbreak constitutes one of the canonical disturbances of an ideal atmospheric surface layer (others being the step change in surface roughness, and the step change in surface energy-flux densities \( Q^*, Q_L, Q_H \).

As early examples, both Tani (1958) and Kaiser (1959) correctly treated a windbreak as a source of mean velocity-deficit \( \Delta u \), but they treated \( \Delta u \) as a passive scalar. This means that the spatial distribution of \( \Delta u \) was controlled by turbulent diffusion along the vertical, and advection by the approach flow \( \bar{u}(z) \), i.e. terms that are non-linear in the velocity disturbance were ignored. These treatments also missed the essential influence of the pressure field, which is responsible for the displacement of the location of minimum mean velocity downwind from the windbreak. Continuing experiments, steadily improving in their fidelity (e.g. Raine and Stevenson, 1977), emphasized the deficiencies of the early numerical treatments, although perhaps the theoretical studies helped make experimentalists more aware of the requirements for similitude, and the value of giving an aerodynamic characterization of the windbreak itself, rather than previous qualitative, visual descriptions (‘very dense’, ‘permeable’, or ‘thin from top to bottom’).

With growing accessibility of computing power, and the incursion of numerical fluid mechanics into micro-meteorology, e.g. Taylor’s (1970) early treatment of local advection over surface temperature changes, came more plausible simulations of windbreak flows, using RANS models. Using the \( k-\epsilon \) closure, Durst and Rasogi (1980) calculated the effects of a solid barrier, requiring that mean velocity vanish at the barrier. Also using the \( k-\epsilon \) closure, Hagen et al. (1983) treated a porous fence, by imposing a measured mean velocity profile in the immediate wake of the fence.

Wilson (1985) reverted to the representation of a porous barrier as a momentum sink, involving the (easily measurable) resistance coefficient \( (k) \) of the material, and showed that good predictions of the mean wind field near an isolated windbreak could be obtained. This was the case irrespective of the complexity of the turbulence closure, presumably due to the dominating influence of the pressure gradient near the fence. However, further downwind, where the pressure gradient has relaxed and is less important than the turbulent shear stress, these simulations, regardless of closure, slightly underestimated the rate of recovery of the wind: thus Wilson (1985) concluded that ‘one could not with confidence simulate the more complex problem of a windbreak network’.

Nevertheless similar calculations by Wang and Takle (1995) did not manifest this problem. Wilson and Mooney (1997) sought to discover why not, and thought the better performance of Wang and Takle’s model may have been ascribable to a too-shallow computational domain. However, it is probably a fruitless distraction to focus unduly on small differences between simulations, especially when observations themselves are scarce and carry their own uncertainty.

The question posed here is this: may we say that today’s RANS models satisfactorily describe the wind in an array of windbreaks? In what follows a tentative answer will be given, from a comparison of simulations of the wind in an array of fences with the corresponding observations of McAneney and Judd (1991).

2. Experiments on arrays of windbreaks

In view of the practical importance of windbreak arrays, the paucity of experimental data relating specifically to their aerodynamic performance is surprising. van Eimern et al. (1964) noted that ‘systematic experiments for this problem demand numerous parallel belts of the same kind on an even, uniform ground, which has so far only been possible in the wind tunnel and at small belts’. They questioned ‘whether the results gained with a single belt are applicable, and which is the best interval for the belts’ and observed that ‘a simple application of the results gained before cannot be enough, because a system of several belts alters the roughness of the land . . . An extensive system of wind protection leads to a new equilibrium of the flow owing to the greater roughness of the surface’.
Seguin and Gignoux (1974) nicely brought out this latter point in a comparison of the mean wind profile (at \( z \leq 12 \text{ m} \)) observed downwind of a succession of hedges (height \( h = 8 \text{ m} \)) spaced at intervals \( D_x/h = \) 4–5, against the corresponding profile amid a large open area (airport). They showed that in the lee of the hedges the neutral wind profile displayed a ‘kink’, demarcating an inner region (or internal boundary layer (IBL)) from an outer region. The inner region was characterized by a small roughness length and a local friction velocity \( u'_f \) that was reduced relative to the reference value \( u'_r \) at the airport. However, the mean velocities above the inner region implied an apparent roughness length that was large (of order \( h/10 \)), and a friction velocity \( u'_f \) exceeding the reference value. Iqbal et al. (1977) systematically studied the roughness and displacement lengths of the equilibrium wind profile developed by a series of equi-spaced, non-porous barriers in a naturally-developed wall shear layer within a wind tunnel. Varying the windbreak spacing \( D_x/h \) from 1 to 20, they determined that maximum roughness occurs with \( D_x/h = 3 \).

Judd et al. (1996) carried out a wind tunnel simulation of the flow through porous windbreaks standing in a model plant canopy \( (h/h_c \approx 3, \text{ where } h_c \text{ is crop height}) \), the barriers being spaced at intervals of \( D_x/h = 6 \) or 12. In the case \( D_x/h = 6 \), the array consisted of a total of seven windbreaks, three upwind and three downwind from the central break, about which measurements were focused. According to Judd et al., the upstream members of an array of windbreaks may be considered as ‘an additional upstream roughness’ having two main consequences: ‘an overall decrease in windspeed . . . due to upwind (windbreaks)’; and ‘an increase in ambient turbulence leading to a decrease in the shelter efficiency of any one break in a multiple array relative to an equivalent single break’. These findings are consistent with earlier studies of windbreak arrays, the increased levels of turbulence having been noted by Woodruff and Zingg (1955), who in the context of their wide-ranging interest in wind erosion, reported mean windspeed and turbulence intensity within a flow domain defined by four consecutive porous fences (field and wind tunnel observations).

From their study, Judd et al. concluded that the ‘non-local’ shelter afforded by an array as a whole dominates the reduced effectiveness of any one of its members due to enhanced turbulence (which causes faster recovery of the velocity in the wake of any one member of the array), observing ‘windspeeds within a multiple array that are smaller than those behind a single windbreak, given identical external conditions’. However, the increased turbulence is a serious limitation to the use of shelterbelts for some purposes, and particularly in the context of plant or fruit movement.

McAneney and Judd (1991) measured mean wind (using cup anemometers) and turbulence (using a one-dimensional sonic anemometer) in their unique field study of the windflow about a series of 15 very long, parallel fences (their ‘aeolian array’). The site was flat and unobstructed (roughness length \( z_0 = 0.007 \text{ m}, \text{ displacement height } d = 0.045 \text{ m} \)), and data were selected for near-neutral stratification, with the mean wind direction within \( 10^\circ \) of normal to the fences. The fences were composed of a porous, plastic windbreak cloth, had height \( h = 2 \text{ m} \) \((h/z_0 = 290)\), and were spaced along-wind at intervals \( D_x/h = 6 \).

McAneney and Judd concluded with the observation that ‘in a practical sense, the data clearly demonstrate the difficulty of simultaneously reducing both mean windspeeds and turbulence at crop levels using repeated windbreaks at conventional horticultural spacings’.

The above survey covers virtually all existing observations useful in judging models and theories of micro-meteorological flow in an array of porous barriers. Of these, the data of McAneney and Judd have been chosen as a criterion for the simulations to follow. This is because the wind tunnel study of Judd et al. (1996) is more complex, due to having included a plant canopy, and more especially because of other uncertainties that would have arisen in attempting a simulation, e.g. due to the fact that some members of the windbreak array actually stood, not in the canopy, but in the inflow (metal tombstones) or outflow (gravel) regions of the flow.

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\[^1\] Patton et al. (1998) reported an LES study of this experiment, approximating the repeated windbreaks by assuming flow through a single fence in a domain terminated by periodic along-wind boundary conditions. Patton et al. concluded that ‘mean fields from the numerical and physical simulations show striking resemblances’, but that velocity variances were not simulated quite so well. Thus, LES, though computationally far more demanding than RANS, does provisionally appear promising for the calculation of disturbed flows.
3. Governing equations for numerical simulations of winds in an array of fences

Because this paper hopes to draw a general inference on the capability of micro-meteorological models for highly disturbed flows, it is necessary that the models investigated should include representative closures that span the range available, i.e., several variants of eddy viscosity closures, as well as stress closures.

As will become clear, primarily we parameterize the interaction of the wind with a fence (or fences) by means of a sink in the momentum equations. But while there is some evidence that this step is satisfactory, the corresponding sources and sinks that (could) appear in the various supporting equations (such as equations for the TKE and TKE dissipation rate, if required by the closure) are much less certain. It is known, however, that these choices are crucial: without a TKE sink, for example, simulations do not produce the region of reduced turbulence in the lee of a porous fence (Wilson, 1985; Packwood, 2000).

3.1. Symmetries, assumptions and the key variables

We let $x$ denote a coordinate aligned with the mean wind, and let the windbreak(s) be infinitely long and aligned cross-wind, i.e. along the cross-wind ($y$) direction. Assume the surface layer approaching the windbreak(s) is neutrally-stratified, and that the wind is stationary, i.e. the vertical flux of momentum relative to these means, aligned cross-wind, i.e. along the cross-wind ($y$) wind, and let the windbreak(s) be infinitely long and of porous fences, numbered $k=1, \ldots, N$. A corresponding sink is included in the $u$-momentum equation, but its effect has been found to be negligible, whether in the case of a single fence or an array.

From Eq. (1) it is obvious that the velocity fluctuations impact the mean velocity $\bar{u}_j$ through the Reynolds stress $\bar{uu}_j$, but turbulence also impacts the mean velocity through its influence on the mean pressure. In models, the feedback of the computed turbulence field on the computed mean velocity depends in detail on the closure assumptions.

RANS wind simulations couple Eq. (1) with the vertical momentum equation, the incompressible continuity equation, and a “turbulence closure” whose role is to provide the impact of the unresolved (turbulent) eddies on the mean flow. Before outlining the closures tested here, we first consider the impact of porous barriers on velocity fluctuations.

3.2. Mean momentum equations and momentum sinks

The governing equation for $\bar{u}$ is:

$$\frac{\partial}{\partial x} \left( \bar{u}^2 + \bar{w}^2 + \bar{\rho} \right) + \frac{\partial}{\partial z} \left( \bar{u} \bar{w} + \bar{w} \bar{\rho} \right) = S_k,$$

where $\bar{\rho}$ is the disturbance in mean kinematic pressure, generated by the interaction of the wind with obstacles.

The LHS is simply the divergence of the mean flux ($\bar{F}_u$) of $u$-momentum, and so Eq. (1) could be written symbolically as:

$$\frac{\partial \bar{u}}{\partial t} = -\nabla \cdot \bar{F}_u + S_k. \quad (2)$$

In reference to a small cube of airspace, the term $\nabla \cdot \bar{F}_u$ measures the net rate of transport of $\bar{u}$-momentum across the walls, while $S_k$ represents production or destruction of momentum inside the cube. In the present case the momentum sink $S_k$ represents the drag of the $N$ fences, and may be parameterized as:

$$S_k = -k_i \bar{u} \sqrt{\bar{u}^2 + \bar{\rho}^2} s(z-h) \sum_{k=1}^{N} \delta(x-x_{k}), \quad (3)$$

where $k_i$ is the (dimensionless) resistance coefficient of the fence(s), $s(z-h)$ is a dimensionless step function ($1$ for $z\leq h$ and $0$ for $z>h$) and the delta-functions specify the $x$-wise locations ($x_{k}$) of fences, numbered $k=1, \ldots, N$. A corresponding sink is included in the $\bar{w}$-momentum equation, but its effect has been found to be negligible, whether in the case of a single fence or an array.

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3.3. TKE sources arising from interaction of the wind with a fence

If a porous barrier is treated as a momentum sink, in lieu of imposing correct boundary conditions on the true (complex, three-dimensional) geometry and fully resolving the flow at the fence, an important consequence is that there are additional turbulent...
kinetic energy sources and sinks. Firstly, the drag forces convert kinetic energy of the mean flow (MKE) to turbulent kinetic energy in some range of scales of motion ('wake scales', WKE); and secondly the fluctuating drag extracts energy from the eddies, re-depositing it in the wake scales (a spectral-transfer mechanism which acts in parallel with the normal vortex-stretching mechanism).

In vegetation canopies wake scales of motion are usually considered to be 'small', and thus rapidly dissipated. Although it is sometimes possible to detect a hump or peak in the power spectrum, reflecting the deposition of energy into wake scales, measurements reviewed by Seginer et al. (1976) led them to regard "eddies generated by branches and leaves as only a small perturbation on the main spectrum" for the C_dA product, postponing consideration of whether "wake scales" of motion actually are unimportant, and whether spectral division is appropriate.

Then the corresponding SKE sinks in the individual variance-budget equations would be (omitting the localising functions):

\[ \epsilon_{xz} = 4k_{xz}u'w'^2 + \frac{3}{2}\epsilon_{zz}, \]
\[ \epsilon_{yy} = 2k_{yy}u'^2 + \frac{3}{2}\epsilon_{zz}, \]
\[ \epsilon_{zz} = 2k_{zz}u'^2 + \frac{3}{2}\epsilon_{zz}. \]

Wilson’s treatment of the stress tensor departed from the Launder et al. (1975; LRR) closure it derived from, in several respects. Normal components of the stress tensor were considered as representing the SKE band (only); canopy sinks were included, and a closure coefficient was arbitrarily modified. The usual \( \epsilon_{zz} \)-equation was not carried, \( \epsilon_{zz} \) being parameterized directly using the equilibrium form shown above; of course the closure then relied on an imposed length scale.

Ayotte et al. (1998; AFR) gave a turbulence closure for plant canopies differing in several respects from that of Wilson (1988), and we have performed some calculations in line with their scheme. As with Wilson’s closure the dissipation rate is split into two terms, but to prescribe \( \epsilon_{zz} \) AFR retain (and merely modify) the usual \( \epsilon_{zz} \)-equation.

To estimate the instantaneous drag forces, AFR replaced Wilson’s projections \( (uu, uv, uw) \) with the rigorous form \( (u\bar{u}[i], v\bar{u}[i], w\bar{u}[i]) \), where \( \bar{u}[i] \equiv (u_{ik})^{1/2} \) is the magnitude of the instantaneous velocity vector \( \bar{u} \). Then the instantaneous drag terms are:

\[ S_{uy} = -k_{uy}[u_{j}u_{j}]^{1/2}f(z-h) \sum_{k=1}^{N} \delta(x-x_{ik}). \]

To facilitate ensemble-averaging, one may ‘linearize’ the non-linear portion of the instantaneous drag term
as follows:

\[
\begin{align*}
&\left([\bar{u}_j + u'_j][\bar{u}_j + u'_j]\right)^{1/2} \approx M \left(1 + \frac{\bar{u}_j u'_j}{M^2} + \frac{u'_j u'_j}{M^2}\right) (\bar{u}_i + u'_i) \\
&\approx M \left(1 + \frac{\bar{u}_j u'_j}{M} (\bar{u}_i + u'_i) \right) \\
&\approx M (\bar{u}_i + u'_i) + \frac{\bar{u}_j u'_j}{M} u'_j.
\end{align*}
\]

where \( M \equiv (\bar{u}_j u'_j)^{1/2} = \sqrt{\bar{u}^2 + \bar{w}^2} \) is the magnitude of the mean velocity vector, and we have neglected all terms involving products of turbulent velocity fluctuations of order two or greater. Then we recover a source term:

\[
S_i = -k_i \bar{u}_i M s(z-h) \sum_{i=1}^{N} \delta(x - \chi_k),
\]

in the th mean momentum equation [cf. Eq. (3)]. Correspondingly, we retrieve a source/sink term in the transport equation for TKE that has the following form, assuming that the Reynolds stresses are modeled using the Boussinesq \( K \)-closure (to follow):

\[
\epsilon_{ik} = k_i \left[ \frac{8M - 2K}{3} \frac{u'_j u'_j}{M} \frac{\bar{u}_i u'_i}{\bar{u}_i u'_i} \right] s(z-h) \sum_{k=1}^{N} \delta(x - \chi_k).
\]

This source/sink contribution to the TKE transport equation is obtained by the usual manipulation. The first term in the square brackets of Eq. (9) is always positive, and hence leads to an additional mechanism for the reduction of TKE at the barriers. However, the second term, which depends on the mean velocities and their gradients, has an indefinite sign and can act either as a source or sink in the TKE transport equation.

"Spectral division" is a heuristic approach to parameterizing turbulence in RANS models when a form drag term (distributed mean-momentum sink) appears in the momentum equations. In the present case of windflow about porous barriers, it is not clear that it is appropriate to consider that MKE (and large-scale TKE) lost to the form drag should reappear as (unimportant) small-scale TKE, i.e. there may well be large scales of WKE. Thus, the conceptual advantage of Wilson’s two-band TKE treatment is dubious and one might prefer to consider that \( k \) represents the entire spectrum of turbulent motion. In that case an MKE \( \rightarrow \) TKE source term (in its simplest guise, \( k_i \delta_k \)) must appear in its budget equation. Furthermore since fluctuating drag forces merely change the scale of the turbulent eddies, it would be wrong in principle to include the form drag sink \( \epsilon_{ik} \). As far as we know, no calculation of this type has ever captured the “quiet zone” in the lee of a windbreak; on the contrary, such calculations normally have the outcome that TKE in the lee of a barrier is increased, due to the conversion of MKE.

Finally, lest including a TKE sink in windbreak simulations still seems optimistic, please note that the application of rapid-distortion theory to the passage of a uniform, weakly turbulent \( \sigma_{u}/u \ll 1 \), confined stream through a fine mesh, as in a wind tunnel, agrees well with observations that all three components of the turbulence are damped (Batchelor, 1953). Of course, the tenets of rapid-distortion theory do not apply to the natural windbreak (high turbulence intensity, mean shear).

3.4. First-order closures

The Boussinesq eddy viscosity \( K \)-closure:

\[
\bar{u}'_i u'_j = -K \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) + \frac{2}{3} \bar{u}^2 k_{ij}.
\]

is often a useful approximation for the shearing stresses, e.g.:

\[
\bar{u}' u'_j = -K (s, z) \left( \frac{\partial \bar{w}}{\partial x_j} + \frac{\partial \bar{w}}{\partial x_i} \right).
\]

but is less satisfactory with respect to normal stresses, and so adjustments (that violate the \( ij \)-symmetry) are often introduced. We shall show simulations of wind about multiple windbreaks using the following variants of the \( K \)-closure.
3.4.1. Eddy viscosity of the approach flow ("$K_0$")
Wilson (1985) showed that, for a single barrier, quite good simulations of the mean wind (less good for the turbulence) result by setting:

$$K(x, z) = K_0(z) = k_w u_o z,$$

where $u_o$ is the friction velocity of the approach flow and $k_w$ is von Kármán’s constant ("$K_0$-closure"). Where we tested this treatment of the eddy viscosity, we simplified the momentum equations, viz.:

$$\frac{\partial}{\partial x} \left( \bar{u}^2 - K_a \frac{\partial \bar{u}}{\partial x} \right) + \frac{\partial}{\partial z} \left( \bar{u} \bar{w} - K_d(z) \frac{\partial \bar{u}}{\partial z} \right) = -\frac{\partial \bar{p}}{\partial x} + S_v,$$

(13)

where $K_a$ is a small “artificial diffusivity”. The rationale for an artificial diffusivity is to emphasize that the Boussinesq closure for the normal stresses is unrealistic; since $K_a$ is small, in effect the normal stress gradients are neglected in this treatment.

3.4.2. One-equation closure: imposed length scale ("$K_0$")

As what they regarded as the “simplest turbulence closure that will adequately describe changes to the mean wind field, particularly very close to ground, in flow over hilly, forested terrain”, Wilson et al. (1998; WFR) explored the classical first-order closure:

$$K(x, z) = \lambda(x, z)^{1/2},$$

(14)

where $\lambda$ is the turbulent kinetic energy, calculated from its simplified governing differential equation, and $\lambda(x, z)$ is an algebraic length scale. With the simplest formula for $\lambda$ capable of reproducing the flow structure in and above the canopy, this closure performed as well or better than earlier, more complex closures for in and above the canopy, this closure performed as well or better than earlier, more complex closures for

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3.4.3. Two-equation closures: standard $k-\epsilon$ model and variants

By far the most popular two-equation closure is the $k-\epsilon$ model in which model transport equations are solved for the TKE ($k$) and the viscous dissipation rate ($\epsilon$) from which can be formed a turbulence length scale $\lambda = k^{3/2}/\epsilon$ and an eddy viscosity:

$$K(x, z) = C_\lambda \frac{k^2}{\epsilon},$$

(20)

(Any number of heuristic adjustments of $\lambda$ could have been tried.)

For calculations testing this formulation of the eddy viscosity, we used Eq. (13), and the analogous form for a $\bar{u}$-equation, the associated TKE equation was:

$$\frac{\partial}{\partial x} \left( \bar{u} \frac{\partial k}{\partial x} - \bar{k} \frac{\partial \bar{u}}{\partial x} \right) + \frac{\partial}{\partial z} \left( \bar{u} \bar{w} - \bar{k} \frac{\partial \bar{u}}{\partial z} - \frac{\partial \bar{p}}{\partial x} \right) = K \left( \frac{\partial \bar{u}}{\partial x} \right)^2 - \epsilon,$$

(16)

where $K_a$ is the “artificial diffusivity” and $\sigma_k$ is the ratio of the eddy viscosity to the transport coefficient for TKE. The computed TKE field feeds back on the computed mean velocity field only through its impact on the “true” eddy viscosity applied to calculate the shear stress.

We have split the TKE dissipation rate as:

$$\epsilon = \epsilon_{cc} + \epsilon_{fd},$$

(17)

and parameterized $\epsilon_{cc}$ conventionally as:

$$\epsilon_{cc} = \frac{(C_{\mu} k)^{3/2}}{\lambda},$$

(18)

In conjunction with this $K_d$ closure, rather than use Eq. (9) we specified the TKE sink due to form drag by the simpler prescription:

$$\epsilon_{fd} = \alpha k \bar{u} k \bar{s}(z-h) \sum_{k=1}^{N} \delta(x - \xi_k),$$

(19)

where $\alpha$ is a constant of order 1 (Wilson, 1985). Note that where using this closure we consider that $k$ excludes wake-scale TKE, and so we do not include MKE $\rightarrow$ TKE conversion.

3.4.4. Three-equation closures: standard $k-\epsilon$ model and variants

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$$K(x, z) = C_\lambda \frac{k^2}{\epsilon},$$

(20)
where $C_{\mu}$ is one of five model constants. In this paper, the $k-\epsilon$ model will be used within a linear eddy viscosity framework, i.e. used with the Boussinesq linear constitutive relationship between stress and strain as expressed in Eq. (10), and without simplification of the mean momentum equations, in contrast with the previously-described $K_0$ and $K_1$ closures. Please note too that in our calculations $C_{\mu}$ always substitutes for $\epsilon$ in the definition of the eddy viscosity, except where otherwise stated.

The TKE is determined from:

$$\frac{\partial}{\partial t} \left( \frac{\sqrt{2}}{A} \right) + \frac{\partial}{\partial z} \left( \frac{\sqrt{2}}{A} \right) = P_K - \epsilon,$$

(21)

and $\epsilon$ from:

$$\frac{\partial}{\partial x} \left( \frac{\sqrt{2}}{A} \frac{\partial \epsilon}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\sqrt{2}}{A} \frac{\partial \epsilon}{\partial z} \right) = \frac{\epsilon}{2} (C_1 \rho - C_2 \epsilon).$$

(22)

In these equations, $P_K$ is the shear production term modeled within the Boussinesq eddy viscosity approximation as:

$$P_K = K \left[ \frac{\partial \epsilon}{\partial x} \right]^2 + \frac{\partial \epsilon}{\partial z} \frac{\partial \epsilon}{\partial x} \frac{\partial \epsilon}{\partial z} + \frac{\partial \epsilon}{\partial x} \frac{\partial \epsilon}{\partial z} \frac{\partial \epsilon}{\partial x} \frac{\partial \epsilon}{\partial z} \right].$$

(23)

It is simple to demonstrate that in a constant stress layer (e.g., log-law region of a wall-bounded turbulent flow) where (by assumption!) production and dissipation balance, Eq. (22) has the solution:

$$\epsilon = \frac{C_{\mu}^{1/3} k^{3/2}}{\kappa \bar{z}},$$

(24)

with the closure constants related by:

$$\kappa^2 = \beta \eta^{1/2} (C_{\mu} - C_{\mu}).$$

(25)

Comparing this result with Eq. (18), it is seen that $C_{\mu} = C_{\mu}^{1/3}$.

Different variants of the $k-\epsilon$ model arise depending on the different approaches used to determine the closure coefficients $C_{\mu}, \alpha_0, \alpha_1, C_{\mu}, \alpha_0, C_{\mu}$, and $C_{\mu}$. Here, two forms will be investigated.

The standard $k-\epsilon$ model ("$k-\epsilon_0$") employs values for the closure constants that have been arrived at by comprehensive data fitting for a wide range of "canonical" turbulent flows, with the "standard" values of the constants given by:

$$\epsilon = 0.09, \quad \alpha_0 = 1.0, \quad \alpha_1 = 1.3, \quad C_{\mu} = 1.44, \quad C_{\mu} = 1.92.$$  (26)

These values of the constants correspond to a compromise chosen to give the "best" performance for a range of flows, and it is not likely that minor and/or ad hoc adjustments to their values would significantly affect the predictive accuracy.

We shall also present a simulation of windbreak flow using a more recent version of the $k-\epsilon$ model developed by Yakhot et al. (1992) using the renormalization group (RNG) procedure. In the RNG model ("$k-\epsilon_{RNG}$"):

$$C_{\mu} = 0.085, \quad \alpha_0 = 0.72, \quad \alpha_1 = 0.72;$$

(27)

$$C_{\mu} = 1.42 - \frac{1}{1 + \beta}, \quad C_{\mu} = 1.68;$$

(28)

where

$$\beta = 0.012, \quad \eta = 4.377, \quad \eta = (P/K)^{1/3}.$$  (29)

This functional dependence of $C_{\mu}$ on the non-dimensional characteristic strain rate $\eta$ is an empirical result, the value of the adjustable constant $\beta$ having been set using near-wall turbulence data. It is this ad hoc adjustment of $C_{\mu}$ that is largely responsible for differences in performance between the standard and RNG models.

We shall report simulations using the ($k-\epsilon_0$) and ($k-\epsilon_{RNG}$) closures entirely without additional source or sink terms in the $k$ and $\epsilon$-equations, as discussed in the previous section (such simulations are designated "Model 1"). We also investigate two different methodologies for incorporating form drag source/sink terms...
in the model transport equations for $k$ and $\epsilon$ (cf. Eqs. (21) and (22)).

Results designated “Model 2” treat $k$ as if spectrally divided; such simulations resemble model “$K_k$”, but accept literally the Boussinesq closure for the normal stresses, and model $\epsilon_{cc}$ using the transport equation.

The inclusion of the earlier-derived source/sink term in the transport equation for TKE yields:

$$
\frac{d}{dx} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial x} - \frac{K}{\sigma_k} \frac{\partial \overline{k}}{\partial z} \right) + \frac{d}{dz} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial z} - \frac{K}{\sigma_k} \frac{\partial \overline{k}}{\partial x} \right) = P_k - \epsilon_{cc} - k_{\text{fd}} \left( \frac{8 K}{M \sigma_k} - \frac{2 K}{\sigma_k} \frac{\partial \widehat{\overline{k} \widehat{u} \widehat{w}}}{\partial z} + \frac{\partial \overline{\overline{k} \overline{u} \overline{w}}}{\partial x} \right) + C_{\text{cc}} \left( \sigma_k \frac{\partial \overline{k \sigma_k}}{\partial z} + \frac{\partial \overline{k \sigma_k}}{\partial x} \right)
$$

$$
\times \left( s(z-h) \sum_{l=1}^{N} \delta(s-x_{l}) \right). \tag{30}
$$

The total dissipation is split according to Eq. (17) and $\epsilon_{cc}$ is modeled as (AFR):

$$
P_{\text{MKE}} = \frac{1}{2} k_{\text{fd}} M s(z-h) \sum_{l=1}^{N} \delta(s-x_{l}). \tag{32}
$$

Hence, for Model 3, the TKE transport equation assumes the following form:

$$
\frac{d}{dx} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial x} - \frac{K}{\sigma_k} \frac{\partial \overline{k}}{\partial z} \right) + \frac{d}{dz} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial z} - \frac{K}{\sigma_k} \frac{\partial \overline{k}}{\partial x} \right) = P_k - \epsilon_{cc} - P_{\text{MKE}}
$$

$$
- k_{\text{fd}} \left( \frac{8 K}{M \sigma_k} - \frac{2 K}{\sigma_k} \frac{\partial \widehat{\overline{k} \widehat{u} \widehat{w}}}{\partial z} + \overline{\overline{k} \overline{u} \overline{w}} \frac{\partial \overline{k \sigma_k}}{\partial x} \right) + C_{\text{cc}} \left( \sigma_k \frac{\partial \overline{k \sigma_k}}{\partial z} + \frac{\partial \overline{k \sigma_k}}{\partial x} \right)
$$

$$
\times \left( s(z-h) \sum_{l=1}^{N} \delta(s-x_{l}) \right), \tag{33}
$$

where the eddy viscosity is defined here using the total dissipation $\epsilon$, i.e. $K \equiv C_{\text{cc}} k_{\text{fd}}^{2}/(\epsilon_{cc} + \epsilon_{id})$. Ayotte et al. advocate the use of the total dissipation (i.e. $\epsilon$) for the determination of the eddy viscosity in this equation and also in the transport equation for $\epsilon_{cc}$, arguing that this is required to model the “reduction of the lifetime of energy containing eddies compared to the ‘no-canopy’ case” that arises due to the interaction of the flow with the canopy.

The free-air dissipation $\epsilon_{cc}$ in Model 3 is modeled using the conventional transport equation for TKE dissipation rate (cf. Eq. (22)), with $\epsilon_{cc}$ replacing $\epsilon$ everywhere in the equation except in the determination of the eddy viscosity $K$. Hence:

$$
\frac{d}{dx} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial x} - \frac{K}{\sigma_k} \frac{\partial \overline{k}}{\partial z} \right) + \frac{d}{dz} \left( \frac{1}{\sigma_k} \frac{\partial k}{\partial z} - \frac{K}{\sigma_k} \frac{\partial \overline{k}}{\partial x} \right) = \frac{\epsilon_{cc}}{C_{\text{cc}}} \left( C_{\text{cc}} \partial \overline{k \sigma_k} \right) \tag{34}
$$

Note that in Eq. (31), the portion of the source/sink term in the TKE equation that has indefinite sign has been incorporated with the term multiplied by $C_{\text{cc}}$ (production of dissipation), whereas the portion of the source/sink term in the TKE equation that has a positive definite sign has been incorporated with the term multiplied by $C_{\text{cc}}$ (dissipation of dissipation).

Finally, $k-\epsilon$ simulations identified as “Model 3” are the $k-\epsilon$ analog of the second-order closure model proposed by Ayotte et al. (1998) for plant canopy flows. Here, the total dissipation is split according to Eq. (17) and $\epsilon_{id}$ is determined according to Eq. (9). In addition to the inclusion of $\epsilon_{id}$ as a sink term in the TKE transport equation, Model 3 includes the MKE $\rightarrow$ TKE transfer term as (AFR):

$$
P_{\text{MKE}} = \frac{1}{2} k_{\text{fd}} M s(z-h) \sum_{l=1}^{N} \delta(s-x_{l}). \tag{32}
$$

5. Rao–Wyngaard–Côté second-order closure

The Rao–Wyngaard–Côté (RWC) local advection model (Rao et al., 1974a,b; Bink, 1996; Wilson et al.,
In Eq. (35), no additional sinks or sources of $\epsilon$ and strain. Under neutral stratification, RWC consists of does not parameterize the "rapid" part of the pressure calculations the $\epsilon$ sink term $\epsilon_{cc}$ is a turbulence timescale, and for the present calculations the $\epsilon^2$ sink term $\epsilon_{cc}$ is given by (cf Eq. (5)):

$$\epsilon_{cc} = 4\hat{a}_e \bar{a}_{cc}(z - h) \sum_{i=1}^{N} \delta(x - x_i).$$  \hspace{1cm} (36)

The flow disturbance is "driven" by the sink terms $\epsilon_{cc}$ and $\epsilon_{ed}$ in the $\hat{u}$- and $\epsilon^2$-equations. Away from the fences, dissipation of TKE is entirely due to the normal vortex-cascade mechanism, and the RWC $\epsilon$-equation is:

$$\frac{\partial}{\partial x}(\alpha_{cc} - \alpha_0 \bar{a}_{cc}) \frac{\partial \epsilon}{\partial x} + \frac{\partial}{\partial z} \bar{a}_{cc} \frac{\partial \epsilon}{\partial z} = -2\epsilon_{cc} \hat{a} \bar{a}_{cc} \frac{\partial \epsilon}{\partial z} - \frac{\epsilon^2}{k} + 2\epsilon_{ed} \frac{\partial}{\partial x} \left( \frac{k}{\epsilon_{ed}} \frac{\partial \epsilon}{\partial x} \right),$$  \hspace{1cm} (37)

where $a = 1/2$ (and the summation convention applies). This is similar to the $\epsilon$-equations of other well-known closures, and provided the coefficients are appropriately chosen, for the equilibrium state it will result in:

$$\epsilon_{cc} = \frac{\epsilon_{cc}}{k} = \frac{\epsilon^2}{k}.$$  \hspace{1cm} (38)

Note that in application here of the RWC closure, no additional sinks or sources of $\epsilon_{cc}$ have been inserted due to the presence of the fences. For reasons of numerical accuracy, we transformed Eq. (37) into an equation for the product $(\epsilon_{cc})^2$.

From this section, it must be clear that there is ambiguity as to what are the proper sources in TKE and $\epsilon$-equations. It is for this reason that we have covered a range of choices, including closures that are too naïve even to contain those equations.

4. Previous simulations of a single fence

Before presenting simulations of a windbreak array, it is pertinent to remind the reader of the promising performance of RANS models for a single barrier.

4.1. Bradley and Mulhearn’s fence

Bradley and Mulhearn (1983; BM83) carried out an experiment on neutral flow through a very long, isolated, porous fence standing on uniform ground, parameters being $h = 1.2 \text{ m}$, $\rho = 0.002 \text{ m}$ ($h/\rho = 600$), and $k = 2$. Fig. 1 is a simulation of this experiment. The domain was $-10 \leq x \leq 100 \text{ m}$, $z \leq 50 \text{ m}$, and gridlengths were $\Delta x = 1 \text{ m}$, $\Delta z = 0.15 \text{ m}$ ($\Delta x/h = 0.83$, $\Delta z/h = 0.125$, i.e. very close to the resolution used for the simulations of a windbreak array which follow). Fig. 1 is similar to the result reported by Wilson et al. (2001, Fig. 2), and the differences are consequences of a minor change in the program: the $\hat{u}$-momentum sink specified by Eq. (3) involves the projection of the square of the mean velocity on the $x$-axis, whereas in the earlier program the sink was simply $-\hat{u} \hat{u}^2$.


4.2. Ellerslie fence

Wilson (1987) compared observations of the along-wind profile of mean ground-level pressure $p_0(x)$ upwind and downwind from a single porous plastic fence on bare ground ($h = 1.25 \text{ m}$, $\rho = 0.008 \text{ m}$, $h/\rho = 160$, $k = 2.4$), with a simulation using the LRR second-order closure. Modeled mean pressure was in reasonable agreement with the observations.
4.3. Surrey wind tunnel fence

Packwood (2000) compared simulations using a proprietary CFD code against others’ wind tunnel experiments, with two-dimensional solid and porous (23 and 50%) fences. Packwood considered that, regarding previous experiments on flow through a porous fence, “none give sufficiently detailed data of either the upstream or downstream flows to permit comparison with CFD modeling without some estimate of one or more of the inlet (upstream) quantities”. On first sight this appears a surprising statement because, for example, in the experiment of BM83, all necessary details of the (neutral) atmospheric surface layer flow are known, and the normalized mean velocity statistics were reported in detail. However, the comment is justified by Packwood’s careful focus on “determining a suitable resistance model for a given geometry where often only a crude estimate of the volumetric porosity is available”. That is, Packwood considers the aerodynamic parameterization of the BM83 fence to be incomplete, or ambiguous, for he correctly distinguishes between the drag of a porous screen “not in the presence of walls or immersed in a boundary layer”, versus the drag in situ of a real fence. This is potentially an important point, for Wilson (1985) had proceeded by assuming an adequate estimate of the resistance coefficient of a fence (in its atmospheric condition) could be had by blocking a uniform, non-turbulent stream with a (full-scale) section of the actual fence. We shall return to this point later.

Here focusing on the results for porous fences, in Packwood’s simulations the fence itself was represented as having a finite thickness, spanning about 3 “cells” of a total of about 200 cells along the streamwise axis (this means the region of favorable pressure gradient and flow acceleration within the material is explicitly represented, whereas in the Wilson (1985) approach the fence is represented by a delta-function source). The drag coefficient of the porous region was inferred from earlier measurements. Computations were reported using both a $k$-$\epsilon$ closure and, alternatively, a Reynolds stress closure; the standard $k$-$\epsilon$ closure is well known to over-predict production of TKE near stagnation points, thus in flows about bluff bodies, and so in Packwood’s computations, a modified $\epsilon$-equation was used.

Regarding prediction of the mean velocity field, Packwood found that both the closures he examined performed almost equally well (results were shown for
This is consistent with Wilson (1985) (who also considered a range of closures) and the hypothesis that the flow disturbance near the fence is dominated by the influence of the strong adverse pressure gradient. Predicted profiles of the Reynolds stress $\overline{u'w'}$ at $x/h = 1$ were about equally skilful, but at $x/h = 8$ the Reynolds stress closure gave clearly superior results; again, this is completely consistent with Wilson (1985).

Finally, Packwood noted that while in reality there is a region of reduced turbulence ("quiet zone") in the lee of an isolated porous fence, his simulations with the Reynolds stress closure over-predicted that region. Wilson (1985) had noted that the LRR closure does not capture the reduced turbulence in the quiet zone unless a TKE sink at the fence is included. Packwood does not state whether in his simulations the fences were taken to imply additional sources or sinks of TKE and $\epsilon$, but from his results it seems not.

Thus we conclude that there is a remarkable harmony of results across these two independent examinations of the competence of numerical fluid mechanics for the case of an isolated porous barrier, with (of particular relevance for this paper) consistent consequences for the inclusion (or otherwise) of fence drag modifications of the TKE and $\epsilon$-equations.

5. Simulations of the McAneney–Judd windbreak array

A sample of the windbreak cloth, provided by Dr. McAneney, was placed in a wind tunnel (Department of Mechanical Engineering, University of Alberta) in order to determine its resistance coefficient:

$$k_1 = \frac{\Delta p}{\mu U^2},$$

(39)

i.e. the pressure drop $\Delta p$ across the cloth, normalized by the scale $\mu U^2$, where $\mu$ is the air density, and $U$ is the windspeed in the tunnel. Results were $k_1 = 1.66$ (at $U = 11 \text{ m s}^{-1}$) and $k_1 = 1.74$ (at $U = 3.0 \text{ m s}^{-1}$).

5.1. Details of numerical method

For the simulations to be reported, the first of the 15 fences of the "aeolian array" was placed at $x/h = 0$, the 15th at $x/h = 84$, and the domain spanned $-25 \leq x/h \leq 150$. Along-wind resolution was $\Delta x = 1.5 \text{ m}$ ($\Delta x/h = 0.75$), thus a $h$ grid-point fell on each fence, with seven grid-points between each pair of fences. Vertical grid spacing was $\Delta z = 0.333 \text{ m}$ ($\Delta z/h = 0.167$), and the domain depth $z_{max}/h = 75$.

All the simulations except those involving the $k$-$\epsilon$ model used a staggered arrangement for velocity components and pressure on the grid (as in Wilson, 1985). The simulations with the $k$-$\epsilon$ model used a co-located, cell-centered procedure, i.e. all the dynamical variables were stored at the same set of grid points. The co-located arrangement of velocity components and pressure provokes checkerboard oscillations in the pressure field due to the velocity-pressure decoupling. To avoid this, the method of Rhie and Chow (1983) was used to interpolate cell face velocities from the nodal velocity values at the cell center. This non-linear interpolation essentially introduces a fourth-order "pressure diffusion" which smooths out the pressure oscillations.

Numerical method was a variation of SIMPLE (Patankar, 1980), as summarized by Wilson (1985). Within this scheme, the transport equations and the pressure-correction equations were solved sequentially and iterated to convergence. For all simulations except those involving the $k$-$\epsilon$ model, iterations continued until the whole-domain $u$-momentum budget was satisfied to within 1% of the total windbreak drag in the domain. For the simulations using the $k$-$\epsilon$ model, a solution was assumed to have converged when the sums (over the whole-domain) of the absolute cell residuals, normalized by the respective total fluxes at the inflow (undisturbed upstream) plane, fell below 0.1%.

For all closures we checked that the specified inflow profiles

$$\overline{w}(z) = \log \left( \frac{z}{c_0} \right),$$

$$k_0(z) = \frac{\mu c_0^3}{\epsilon \alpha},$$

$$\epsilon_{cell}(z) = \frac{\mu'_{cell}^2}{k_0^2},$$

(40)

This can be regarded as firmly established: solutions of momentum equations that entirely neglect perturbation Reynolds stresses capture well the mean wind effects to about $x/h = 5$ and even beyond.
were retained as the equilibrium solution, \( \tilde{u}(x, z) \approx \tilde{u}_0(z) \), etc. after many iterations with \( k_x = 0 \) (no barriers).

5.2. Results of simulations

Fig. 2a–c compare the windspeed \( \tilde{u}(x, z)/\tilde{u}_0(z) \), calculated by the models with that observed, at heights \( z/h = 1/2, 2 \). The high-resolution data at \( 37 \leq z/h \leq 41 \) stem from the values of \( \tilde{u}(x, z)/\tilde{u}_0(z) \) given by McAneney and Judd (1991, Table 1), from their observations between fences 7 (at McAneney and Judd (1991, Table 1), from their observations) in any case. For the population one infers from McAneney and Judd (1991) and which is slightly misleading, in that the interpolative pre-publication report on the experiment by McAneney, Judd and Astill states “windspeed varies only by 13% between fences 7 and 8”. This is far less than the modulation that is much too small relative to that observed, as noted above), and do not show the slow but definitely continuing recovery in mean windspeed that was observed along the array; the \((k-\epsilon)_{\mathrm{RNG}}\) (Model 2) calculation is marginally superior in the latter respect. At \( z/h = 2 \), none of the calculations capture the observed minimum windspeed between the third and fourth fences, and slow recovery further to leeward. Perhaps the most definite conclusion from Fig. 2 is that differences between the various predictions are large, but all are in accord with the observations: does this imply that, of turbulence closures available to us, none is adequate for this flow?

The periodicity of the modeled mean wind field beyond about the fifth fence suggests that, to simulate the wind in the interior of the array (say, between fences 4 and 10) it would be acceptable to simulate a reduced number of fences, using higher resolution. Such a simulation, using the RWC closure, is shown (as a dotted line) in Fig. 2c, and except downwind of the (five) fences, is essentially identical to the simulation of the full array (the difference is largely if not all due to the fact that the gridpoint \( z_j \) falling closest to \( z/h = 1/2 \) differs between simulations). This result proves two points: that the full-array simulations can be regarded as essentially grid-independent; and, that the influence of the leeward ten fences on the flow about the windward five is negligible (a point which is fairly intuitive, and which has been observed many times in earlier studies).

In addition to their observations of the pattern in the mean wind about the aeolian array, McAneney and Judd reported the along-wind profile of the standard deviation \( \sigma_w \), in vertical velocity: Fig. 3 compares simulated and observed values of \( \sigma_w \) between the fences. If the TKE sink is included, model predictions underestimate the spatial modulation in \( \sigma_w \) between the fences.

\footnote{If the three leeward most-high-resolution observations in Fig. 2 were rescaled to ensure self-consistency between McAneney and Judd’s Fig. 1 and Table 1, observed modulation would be more consistent with the models. A pre-publication report on the experiment by McAneney, Judd and Astill states “windspeed varies only by 13% between fences 7 and 8”. This is far less than the modulation one infers from McAneney and Judd (1991), and which has been plotted here in Fig. 2a–c.}
Fig. 2. (a) Simulated (lines) and observed (symbols) normalized mean windspeed \( U/U_0 \) within the ‘aeolian array’ of 15 windbreaks (McAneney and Judd, 1991). Parameters of the experiment were \( h = 2 \) m, \( z_0 = 0.007 \) m, \( k_r = 1.7 \), and \( U_0 \) is the mean windspeed observed upstream from the fence at \( z = h \). Computational resolution was \( \Delta x = 1.5 \) m, \( \Delta z = 0.333 \) m. Showing simulations with the simplest first-order closures. (b) As for (a). Simulations with \( k-\epsilon \) closures and several treatments of TKE sources; standard \( k-\epsilon \) model, except where labeled ‘RNG’. (c) As for (a and b). Comparing simulations using the standard \( k-\epsilon \) closure (Model 2) and the RWC closure. Computational resolution was \( \Delta x = 1.5 \) m, \( \Delta z = 0.333 \) m except for the fine-dotted line, which is a finer-resolution simulation (\( \Delta x = 0.75 \) m, \( \Delta z = 0.2 \) m) with the RWC closure for the case where only the first five windbreaks are included.
at $z/h = (0.5, 1)$. The only other plus of the simulations with TKE sink is that they correctly indicate strong suppression of vertical exchange occurs only behind the first one or two fences, and near ground.

5.3. Discussion

Taken overall, the RANS simulations given above are ambiguous, and quite wrong relative to (these) observations. On the other hand there have been many demonstrations that, for sufficiently simple disturbances of the surface layer, RANS models are skillful. In the majority of such studies, admittedly, the flow to be simulated is ‘known’ experimentally, and so the simulations ultimately published could be the end result of an exercise that amounts to little more than ‘fitting’ curves (the $n$th variation of the $n$th model) to observations on a graph (we do subscribe to this viewpoint; on the contrary we consider RANS is objectively skillful, for simple flow disturbances). Furthermore, where models have been thoroughly ‘tested’ against observations, there is a tendency for the disturbances to be of the simplest character, e.g. disturbances where geometrical regularities prevail.

Cowan et al. (1997) and Sini et al. (1996) noted the concern that a given standard CFD ‘package’, e.g. the $(k-\varepsilon)_0$ model, put in the hands of different users, throws up differing solutions of the same flow problem. This complicates the task of defining the inherent capability of RANS, but let us take the optimistic viewpoint that the present results, good or bad, reflect the inherent quality of the turbulence closure—not the numerical procedure, grid and domain choice, etc. Then why should it be that RANS performs quite well for a single fence and quite poorly for an array of fences? This is perhaps the more surprising given that Wilson and Flesch (1999) showed that the $K \propto \sqrt{L}$ closure gave a credible simulation of flow through an array of forest blocks and clearings.9

9 Regarding uncertainty in CFD, Roache (1997) distinguishes ‘verification’ as the assurance that one is ‘solving the equations right’ from ‘validation’ which has to do with assuring that one is ‘solving the right equations’. In this paper, we suggest from our comparison of simulations against observations that the model(s) probably are not valid, i.e. not based on the right equations—which is perhaps not very surprising, in view of the well-known difficulty of properly parameterizing the Reynolds stresses.

10 But here one must emphasize that, not knowing the effective drag coefficient $C_D\Delta h$ for their forest canopy strips, Wilson and Flesch treated this as a free parameter. Such a freedom, due to incomplete experimental data, is very common in comparison of micro-meteorological models and observations, and whenever exploited, must be seen as having compromised any claim to have objectively tested (or validated, Roache, 1997) a numerical model or theory.
5.4. Possible reasons for poor agreement of multi-fence simulation with data

Firstly, how much confidence should be placed in the observations? The cup anemometers would surely have overestimated the mean speed, particularly in any region of high turbulence intensity. Fig. 4 shows a calculated horizontal profile of the turbulence intensity \( \sigma_u(x, z)/\bar{u}(x, z) \) in the windbreak array, according to the RWC closure. At \( z/h = 0.5 \), the turbulence intensity exceeds 1/2 virtually everywhere, but still it is unlikely the cups should have overestimated by a factor of something like 50–200% (Wyngaard, 1981). Furthermore it is improbable that overspeeding could explain the pattern of the discrepancy between the observations and models at \( z/h = 2 \). However, there is reason to be suspicious of the reported between-fence modulation in mean speed, for the reason given earlier.

Secondly, could there be a problem with the primary parameterisation of the barriers, i.e. with the form of the imposed sinks in the momentum equations? Wilson et al. (1990) showed that inclusion of a localized momentum sink, neglecting the finite thickness of the mesh and the details of the flow within it,
Fig. 4. Calculated along-wind profile of the turbulence intensity in the aeolian array, at $z/h = (1/2, 2)$, according to the RWC closure.

is a valid alternative to imposition of proper boundary conditions on the complex surface of a porous barrier. The sink, $k_r u \sqrt{\bar{u}^2 + \bar{w}^2}$, is considered to estimate the mean (kinematic) back-pressure exerted locally by the barrier on the flow, in terms of the (height-variable) resolved mean wind "at" the barrier.

However, the resistance coefficient $k_r = k_r(\theta)$ is defined in reference to the passage of a laminar, uniform, confined stream through the (given) barrier, which is mounted at an angle $\theta$ with respect to the streamlines, and blocks the stream. Is it valid to carry over the numerical value of $k_r$ from this non-turbulent “reference flow” and apply it to obtain the mean back pressure in a turbulent, unconfined stream?

The time-variability of the wind vector, and the non-normality and non-constancy of its orientation relative to the barrier, are the factors compromising the representation. At the first of a series of porous barriers, the mean angle of incidence of the wind $\theta = \arctan(\bar{w}/\bar{u})$ at heights of order $h$ is small ($< 10^\circ$), but instantaneous values can be much bigger. Justification for the naïve imposition of a resistance coefficient on the mean flow comes a posteriori—values of $k_r$ deduced from shear-less laminar wind tunnel tests, when thus imposed in the mean momentum equations, lead to good agreement with field observations (Wilson, 1985). Surely this must imply that, even in the fluctuating real-world flow, the instantaneous pressure of the wind on an isolated fence can not be too different from $k_r \rho u \sqrt{u^2 + v^2 + w^2}$?

But what of the further complication (relative to the laminar reference flow) of barrier wakes impinging on downstream members of the array? Angles of attack on the second and later barriers probably exceed those at the first barrier, and turbulence intensities must be higher. Use of the same (constant) value of $k_r$ for all members of an array (of physically-identical windbreaks) can be justified only by Occam’s razor.

One means to investigate this uncertainty in the treatment we have used here, would be large eddy simulation (LES), assigning each windbreak a resistance coefficient $k_r(u, v, w)$ applied to the instantaneous (resolved) wind $(u, v, w)$, and varying realistically with angle of attack.

There is no need to repeat the discussion of our uncertainty relative to sources and sinks in the TKE and $\epsilon$-equations, other than to record again that consequences for the calculated mean flow do not seem sufficiently important to explain the collective discrepancy of the simulations relative to the observations.

Finally, if none of the above mechanisms explain the poor simulation, it seems necessary to conclude (tentatively) that the turbulence closures surveyed are all unrealistic. Is this at all plausible? Fig. 5 shows
Fig. 5. Calculated horizontal profile at $z/h = 1/2$ of each term in the $\bar{u}$-momentum equation, normalized on $u^*_0/h$. From the RWC closure corresponding to results shown in Figs. 2 and 3. The forces have been evaluated such that their sign indicates their contribution towards $\partial_t \bar{u}$; that is, negative terms exert a decelerating influence on the wind.

The calculated along-wind profiles of the individual terms in the $\bar{u}$-momentum budget, from the RWC closure. The fence drag term ($-k_u \sqrt{u^2 + w^2}$) has been lumped with the pressure gradient, and but for this, spikes of positive (favorable) pressure gradient would occur at (across) each barrier. The term in Fig. 5 labelled “Sum” is the residual imbalance, and it should vanish; it is non-zero only because the upwind finite difference here used to estimate the advection terms differs from SIMPLE’s more refined scheme for fluxes across control volume faces.

It is evident from Fig. 5 that near the first windbreak, along-wind advection $\partial_x \bar{u}$ and pressure gradient $\partial_x \bar{p}$ are the dominant terms. The pressure gradient remains important close to downwind fences, but is largely annulled by almost a mirror-image pattern in the force $\partial_x u' \bar{w}'$, which results from the imposed TKE sink. Vertical advection is nowhere very important. Most significantly, the shear stress divergence $\partial_z u' w'$ rises promptly downstream of the first windbreak to take the dominant place among these forces, except in the very near vicinity of each subsequent fence; everywhere it acts to re-accelerate the wind. Now whereas the pressure-gradient force is directly calculated by the numerical model (and seems to be in reasonable agreement with observations, Wilson, 1997), the shear stress is necessarily parameterized, this being the fundamental task of the ‘turbulence closure’ that is key to any RANS model. This is the reason for our hypothesis that the key problem with these poor simulations is the turbulence closure itself, rather than the treatment of momentum and TKE sinks.

6. Conclusion

The utility of micro- and agro-meteorology rests on knowledge of the wind. By some magic (often stemming largely from symmetry) the mess of detail that is turbulent flow must be squeezed into a manageable few robust relationships before a description of such interesting processes as, say, pollen transport,
begin. When we address a process on a landscape that is not ideally flat and unobstructed (i.e. most of the real world) we may assume that wind models will be able to provide the needed spatial pattern of wind statistics, rather than an extensive (and expensive) program of measurements. But if we may hazard a generalization from the results given here, then it would be very unwise to place trust in present RANS wind models as a stand-alone alternative to measurements, in addressing processes in very disturbed micro-meteorological flows. Some meteorologists consider we need only await a fine-enough resolution in the mesoscale models, which is more or less the same as to say, enough computing power to push large eddy simulation (LES) towards other than the very limited flows presently within its grip. But the merit of such models for disturbed flows very much remains to be demonstrated, and there is a profound difficulty in how to parameterize the increasingly dominant subgrid turbulence, very near ground.

A reassessment of RANS wind models is timely because, particularly from the necessity to improve long term flux estimates from imperfect sites, micro-meteorologists increasingly are forced to accept that the idealization of horizontal uniformity is too obviously limiting—or wrong—to be ignored. And it is also important because technological society depends on models (particularly dispersion models, which usually ignore or naively parameterize flow disturbance) for the regulation of industry and the environment.

In this paper, we perhaps have bumped against a boundary, between the do-able and the not do-able, for RANS models in micro-meteorology.

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We hope this somewhat inconclusive paper is not amiss in a volume which honours George Thurtell, from the perspective that George’s work has always been grounded in measurements, with a constant (and skeptical) attention to the limits of accuracy, of both measurements and modeling.

References


