AN IDEALIZED MEAN WIND PROFILE FOR THE ATMOSPHERIC BOUNDARY LAYER

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Abstract. We test a flexible, idealized mean wind profile for the lower atmosphere that can easily be matched to whatever wind observations may be available. Its intended function is to provide a 'best guess' wind profile from limited observations, e.g., for use in dispersion models, and to this end, following earlier authors, we have matched a Monin–Obukhov layer to a baroclinic Ekman layer.

To demonstrate the flexibility of the two-layer wind profile, we optimize its free parameters to provide best interpolative fits to a sample of multi-level wind profiles. These include model wind profiles extracted from the Canadian Global Environmental Multi-scale weather model (GEM), as well as experimental profiles from the Wangara experiment, and from an over-ocean dispersion experiment (LROD). In most cases the two-layer profile fit is satisfactory.

Keywords: Boundary-layer winds, Ekman layer, Wind estimation, Wind interpolation, Wind model, Wind profile.

1. Introduction

Prediction or diagnosis of atmospheric dispersion always hinges on knowledge of the winds. But while semi-empirical scaling laws have been given to represent turbulence statistics, as Wyngaard (1985) noted, there is no prospect for a widely-valid similarity theory of the *mean* winds at heights above the atmospheric surface layer ('ASL'). This is problematical, for on ranges beyond a few hundred metres, the detailed variation of the mean horizontal wind components (U, V) with height (z) greatly influences the pattern of dispersion from a point source (e.g., Wilson et al., 1993; Luhar, 2002). Kristensen (1984) cites an unattributed remark that 'anyone trying to compare experimental results with model predictions is going to be faced with the fact that the wind turns with height under all atmospheric conditions . . . and unless they model this turning, they will find that their measurements beyond a few kilometers from the source do not compare at all with the model'.

The *true* mean wind field in the atmospheric boundary layer (ABL) cannot easily be determined by measurement. It is highly sensitive to terrain variability, and to the inhomogeneity of the atmosphere (e.g., cloud or mesoscale circulations). In consequence it is (to most practical purposes) both unmeasurable and unpredictable. Nevertheless, in atmospheric modelling, whether in the context of emergency response or in more routine conditions, situations arise where we need to specify the wind field, either as an ideal, or more importantly as a 'best guess', to cover an actual situation, and to be built up from a very few observations.

One approach to providing a wind field in these circumstances is to turn to a numerical model that solves the momentum equations in their horizontally-uniform or (if necessary) more general form (e.g., André et al., 1978). In this paper we instead profit from the work of earlier authors, who have developed a simple, analytical, two-layer wind profile: we decompose the ABL into an inner Monin–Obukhov layer within which the wind direction is constant, and an overlying Ekman layer of finite depth, in which the mean wind varies both in direction and speed. This presumes horizontal homogeneity, and involves the customary notion of an ABL sharply distinguished from the 'free winds'. The result is a unique wind profile for given boundary-layer and surface-layer depths (δ, h_s), upper atmosphere wind (U_G, V_G and their shear), surface roughness (z_0) and surface heat flux (or Obukhov length L).

Brown (1974) noted that 'the idea of a two-layer model has a fairly long history' and that 'solutions for the Ekman layer have consisted on myriad concepts for eddy viscosity distributions'. It remains the case, unsurprisingly, that 'no general characteristic K distribution has been found'. Most solutions (Taylor, 1915; Brown and Liu, 1982; Bergstrom, 1986; Ulrich, 1993; Miles, 1994; Tan, 2001) have placed an infinitely-deep outer layer (upper boundary condition at $z = \infty$) on top of the inner layer, and it has been found that the chosen profile K(z) for the eddy viscosity affects the calculated magnitude of the Ekman pumping (vertical velocity at the top of the boundary layer) and the cross-isobar wind angle. Regarding the specification of the *inner* layer, only after Monin–Obukhov (MO) similarity theory had been developed and tested could this take the modern form.

Before describing the two-layer model, it is worthwhile to briefly distinguish it from another simple model, that of Garratt et al. (1982). These authors posited a simple *three*-layer model of (strictly) the unstable ABL: An MO surface layer; a well-mixed layer, within which U = const and $V = 0$; and a capping transition layer. Coupled equations for the (by assumption, height-independent) geostrophic deficit in the mixed layer were derived by height-integration of simplified* mean momentum equations, resulting in formulae that resolve the influences of advection (i.e., mean horizontal gradients), non-stationarity, baroclinicity (thermal wind), and entrainment (vertical transport across the transition layer). In the sense that it depicts the boundary layer with additional degrees of freedom (explicit advection, entrainment, etc.), this is a more general model than the present one; but it is less general in that it is restricted to the unstable case (convective boundary layer, CBL), and, furthermore, prohibits wind shear in the mixed layer.

* Normal stresses were neglected; advection was linearized; horizontal divergence $\partial_x U + \partial_y V$ was assumed constant.

2. Wind Model

We assume a stationary and horizontally-uniform ABL of depth δ, which we subdivide at the 'matching height' h_s into an inner Monin–Obukhov layer and outer Ekman layer. The purpose of the wind model is to provide the complete profiles of mean wind speed $S(z) = \sqrt{U^2 + V^2}$ and mean wind direction $\beta = \tan^{-1}(V/U)$, or equivalently of the components of mean velocity:

$$U = S\cos(\beta), \tag{1a}$$

$$V = S\sin(\beta). \tag{1b}$$

The ABL depth δ could be one of the 'givens', but otherwise it is an output of the algorithm, albeit not a very skillful one; diagnosing δ is not the objective of our idealized mean wind profile, and there are better criteria for its prescription.

In the outer layer, assuming that shear stresses may be parameterized by means of a height-independent* eddy viscosity K, the U- and V-momentum equations are:

$$K\frac{\partial^2 U}{\partial z^2} = -f\,(V - V_G)\,, \tag{2a}$$

$$K\frac{\partial^2 V}{\partial z^2} = f\,(U - U_G)\,, \tag{2b}$$

which represents a balance of the pressure gradient, Coriolis force and turbulent shear stress. We define the coordinate system such that U is oriented along the *constant* wind direction $\beta = 0$ of the inner layer; and we admit a thermal wind by writing:

$$U_G = U_{G0} + U_T z, \tag{3a}$$

$$V_G = V_{G0} + V_T z, \tag{3b}$$

where the origin for our z-axis is the ground. Note that four unknown constants will arise from the integration of Equation (2), to be supplied by the imposition of the boundary conditions.

Following earlier authors (e.g., Holton, 1979), by introducing the complex variable

$$W = U + iV, \tag{4}$$

* Please note that we are not contending that a constant K is 'correct', but seeking a simple and flexible interpolative/extrapolative formula to make rational use of incomplete wind measurements. It is well known that turbulent convection cannot in general be accurately modelled as a diffusion process, e.g., Corrsin (1974, Sec. 10).

(where $i = \sqrt{-1}$) we can reduce Equation (2) to a single equation:

$$K\frac{\partial^2 W}{\partial z^2} - \frac{if}{K}W = -\frac{if}{K}W_G \tag{5}$$

with

$$W_G = W_{G0} + W_T z = (U_{G0} + iV_{G0}) + (U_T + iV_T)\,z. \tag{6}$$

The general solution is

$$W = W_G + \alpha e^{\mu z}e^{i\mu z} + \beta e^{-\mu z}e^{-i\mu z}, \tag{7}$$

where α, β are complex constants ($\alpha = \alpha_R + i\alpha_I$; $\beta = \beta_R + i\beta_I$) to be obtained from the boundary or matching conditions, and $\mu = \sqrt{\frac{f}{2K}}$ (the length scale μ^{-1} is known as the 'Ekman depth').

For the inner layer we write

$$W_{MO} = U_{MO}(z) + i0, \tag{8}$$

where the Monin–Obukhov wind profile scales with the surface friction velocity u_{*0} and accounts for the effects of thermal stratification as expressed in the ratio z/L of height to the Obukhov length L. Note that W_{MO} is real, i.e., $V_{\mathrm{MO}} = 0$, by virtue of our alignment of the coordinate system with the surface wind, and the assumption the wind direction is constant in the MO layer. In reality, wind direction may turn rapidly with height in the roughness sublayer lying at the base of the ASL, e.g., in the case of a tall forest. The solution we give ignores that complication, so that if it is intended to use the solution over a rough surface, the height z must be interpreted as being measured relative to the zero-plane displacement.

We chose not to make use of measured temperatures above the surface layer, as (potentially) an additional constraint on a best fit wind (and temperature) model. This could have been done, presumably by assuming a linear decay of the flux of potential temperature with height and invoking an eddy diffusivity (proportional or equal to the constant eddy viscosity) for heat transport, but would have entailed specification of, at a minimum, the entrainment heat flux aloft, and the turbulent Prandtl number (extra degrees of freedom). We thought it unlikely such a step would result in systematically improved fitting of wind profiles.

2.1. Matching and Boundary Conditions

We require the wind speed and wind shear to be continuous across $z = h_s$ (since $V_{\mathrm{MO}} = 0$ this means $(\partial V/\partial z)_{h_s} = 0$), providing four constraints on the solution and the additional requirement that the Ekman-layer diffusivity

$$K = \frac{k_v u_* h_s}{\phi\left(\frac{h_s}{L}\right)}, \tag{9}$$

where k_v is the von Karman constant and $\phi(z/L)$ is the universal Monin–Obukhov function for momentum. At the top of the boundary layer the outer-layer wind $W = W_G$, providing a further two constraints. Appendix A gives the constraints in matrix form.

Physical variables appearing in the wind profile are: the MO-layer variables u_*, z_0, L, the depth δ of the ABL, and the 'free wind' U_{G0}, V_{G0} and its shear, the thermal wind components U_T, V_T. The matching height h_s is a further variable, which we may regard as artificial, though with the hope that its numerical value will be in accord with one's (loose) expectation that the depth of the ASL is of the order of $\delta/10$. Our solution stems from almost the same assumptions as those invoked in Brown's model (Brown, 1981, 1982; Brown and Liu, 1982), differing only in that we apply the upper boundary condition at $z = \delta$, and in that we do not include Brown's 'stratification-dependent secondary flow' (boundary-layer roll vortices). In his tests of the model, Brown neglected the secondary flow, just as we do (from our perspective its inclusion implies extra degrees of freedom) and prescribed the matching height h_s (Brown set $\lambda = \mu h_s = 0.15$).

3. Flexible Implementation to Accommodate the Gamut of Circumstances

For simplicity, let us suppose that the *given* information on the mean wind comprises a set of measurements $(z_j, U_{mj}(z_j), V_{mj}(z_j))$ for n levels, $j = 1 \ldots n$ that are arbitrarily distributed across the inner and outer layers. Lest this seem restrictive, we make the point that the range of possibilities for the *form* of the given wind information is too broad to attempt to describe; but note, for example, that if in fact one were provided with the inner layer scales u_*, L, z_0 one may at once generate wind data for several levels.

If an insufficient number of observations is provided, in theory it will not be possible to determine a uniquely-fitting wind profile. Conversely, if the problem is over-determined, in the sense that more data are provided than the minimum required to determine the unknowns in the solution, there is no problem: we fit the profile by requiring that the residual

$$R = \sum_{j=1}^{n} \frac{\left(U_j - U_{mj}\right)^2 + \left(V_j - V_{mj}\right)^2}{U_{mj}^2 + V_{mj}^2}, \tag{10}$$

take on its absolute minimum value; this we assume to occur for a *uniquely* best set of parameters (whereas non-uniqueness of a 'best fit' might arise if the number of given data was smaller that the number of solution parameters). Normalization on the square of the observed local windspeed ensures that the residual is not dominated by the observations of large windspeed, far aloft. And obviously, it would be a mistake to concentrate a too-large fraction of given inputs in a narrow layer, or entirely within one or the other of the nominally distinct solution layers. For best

TABLE I
The parameter space searched in order to determine a best-fit analytical wind profile.

Parameter	Possible values
u_* (m s^{-1})	0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.80, 0.90, 1.0, 1.2, 1.4
L (m)	1, 2, 5, 10, 25, 50, 100, 300, 5000, −300, −100, −50, −25, −10, −5, −2, −1
h_s (m)	5, 10, 15, 20, 25, 30, 40, 50, 60, 70, 80, 90, 100, 125, 150, 175, 200, 250, 300, 350, 400, 450, 500
δ (m)	25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700, 750, 800, 900, 1000, 1100, 1200, 1300, 1400, 1500, 1600, 1700, 1800, 1900, 2000, 2200
U_T, V_T (s^{-1})	−0.016, −0.008, −0.004, −0.002, −0.001, −0.0005, 0.0, 0.0005, 0.001, 0.002, 0.004, 0.008, 0.016

performance, supervision and informed judgement (meteorological knowledge) are essential in the fitting procedure.

We search for the ‘best fit’ using nested loops in all ‘active’ solution parameters (a solution parameter, such as the boundary-layer depth δ, in our terminology becomes ‘inactive’ if its value is regarded, on the basis of the given data, as already known). Note that it is possible that a term in measured-versus-modelled *temperature* errors may need to be included, if the given information for the surface layer includes more than one temperature measurement; in the latter case, individual terms contributing to the residual need to be made dimensionless, using velocity and temperature scales that reflect one’s confidence in the measurements (Argete and Wilson, 1989).

In the fitting exercises which make up this paper we have assumed a known z_0, and then searched the parameter space of Table I for a best-fit profile. While there are $\sim$ 60 million parameter combinations, supplementary conditions reduce the search space, e.g., we required that the implied surface heat flux density $Q_{H0} \leq$ 500 W m^{-2}. A reviewer suggested the surface-layer depth h_s ought to have been more stringently limited, perhaps $h_s \leq 150$ m (Brown and Liu, 1982, who also used nested loops in the model parameters to determine the wind profile, fixed the surface-layer or matching height as $\mu h_s = 0.15$).

4. Comparison of Two-Layer Wind Model Against ‘Data’

In the discussion below we cite three goodness-of-fit categories:

- ‘Excellent’: Height-average fractional error ($\Delta^{\%}S$) in windspeed satisfies $\Delta^{\%}S \leq 10\%$; height-average error (Δ^{β}) in mean wind direction satisfies $\Delta^{\beta} \leq \max\,[5^{\circ}, 10\%$ (profile span)].

- ‘Poor’: $\Delta^{\%} S > 10\%$; $\Delta^{\beta} > 20°$.
- ‘Intermediate’: Fitting neither of the above categories.

This is arbitrary, and a more formal assessment of skill could be contrived: but we think a sense of the effectiveness of the two-layer model is best gained from visual comparisons across a representative set of profiles.

4.1. Two-layer fits to the wind profiles of a global weather model

We examined the utility of the two-layer model by looking at a set of wind ‘observations’ from a numerical weather prediction model, the Canadian ‘Global Environmental Multiscale’ (GEM) model (Coté et al., 1998). GEM, with about 30 levels in the vertical, predicts wind profiles (S, β) that are fully four-dimensional, i.e., these are not necessarily profiles of the stationary, horizontally uniform atmosphere. The possibility that the GEM profiles may have been chosen in disturbed (e.g., frontal) regions implies (extra) degrees of freedom absent from the idealized, two-layer wind model; but on the other hand, relative to the true atmosphere, the GEM wind profiles are surely constrained (have fewer degrees of freedom), owing to GEM’s simplification of terrain, of unresolved scales of motion, of cloud and precipitation processes, etc. In short, while GEM observations are not real winds, they *should* represent something of the variety of conditions that are found in the real world.

From one GEM model run (time valid: 7 October 2001, 1200 UTC), we extracted 29 profiles over North America and Africa, all below latitude 60° N. The observations required to fit the two-layer profile were taken from the eight⋆ lowest model levels (the roughness length z_0 was also a given).

Figure 1 is an example of a complex wind profile (low level maximum in speed; large swing in wind direction with height) that the two-layer model fits very nicely. Three layers are identifiable:

- A logarithmic surface layer over $z_0 \leq z \leq h_s = 15$ m (neutral stability). By choice of the orientation of the coordinate system, $V = 0$ in this layer.
- A modified Ekman boundary layer over 15 m $= h_s \leq z \leq \delta = 750$ m.
- A geostrophic layer above $z = \delta = 750$ m; a strong thermal wind leads to a strong vertical gradient in U above δ

The best fit solution in the above example is: $u_* = 0.45$ m^{-1}, $L = 5000$ m, $h_s = 15$ m, $\delta = 750$ m, $U_T = -0.008$ s^{-1}, $V_T = -0.0005$ s^{-1}, $U_{G0} = 1.97$ m s^{-1}, $V_{G0} = -1.6$ m s^{-1}. In this fitting the average fractional error in S is $\Delta^{\%} S = 6.3\%$, and the average error in wind direction is $\Delta^{\beta} = 2.9°$ (the span in β over the profile was 114°). According to our criteria, this is an “excellent fit”.

Figure 2 illustrates a representative range in the quality of two-layer model fits to GEM wind profiles. The first example (a) is another excellent fit ($u_* = 0.6$ m s^{-1},

⋆ In Section 4.4 we examine the influence on the fitted profile of the number of levels at which wind is prescribed.

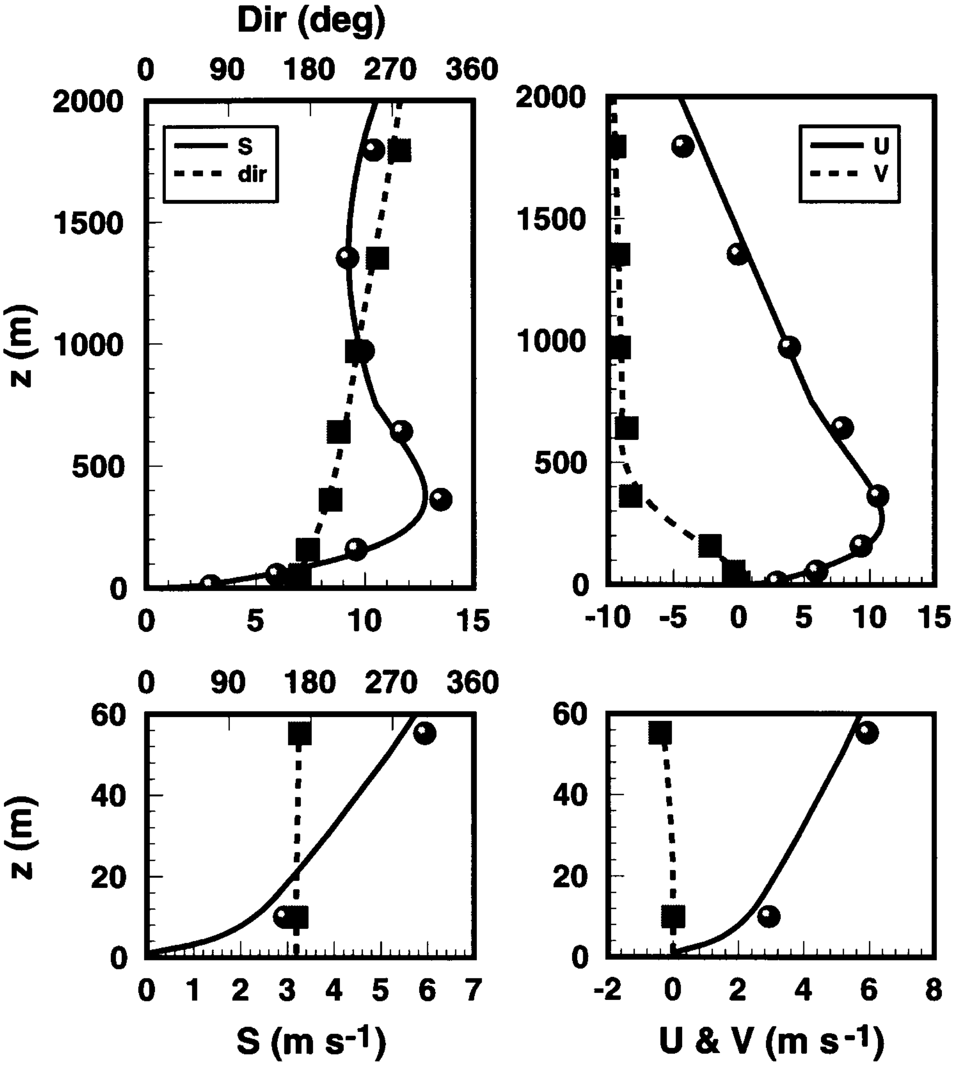

Figure 1. Fitted analytical wind profile (lines) versus output profile from the GEM numerical weather model (symbols). The lower panels are a 'blow-up' for the surface layer. Orientation of the coordinate system ensures $V = 0$ in the surface layer.

$h_s = 100$ m, $\delta = 500$ m, $L = 5000$ m) yielding $\Delta^{\%}S = 5.6\%$ and $\Delta^{\beta} = 4°$; in general the two-layer model gave excellent fits to simpler profile shapes like this one (either a monotonic increase with height in windspeed, or a low-level jet giving a single local speed maximum).

The second example (b) on Figure 2 shows a slightly more complex profile shape (both a local maximum and minimum S), and an intermediate quality of fit (for which $u_* = 0.06$ m s^{-1}, $h_s = 5$ m, $\delta = 100$ m, $L = 10$ m) with $\Delta^{\%}S = 17\%$ and $\Delta^{\beta} = 9.6°$. Example (c) is even more complicated, the GEM windspeed

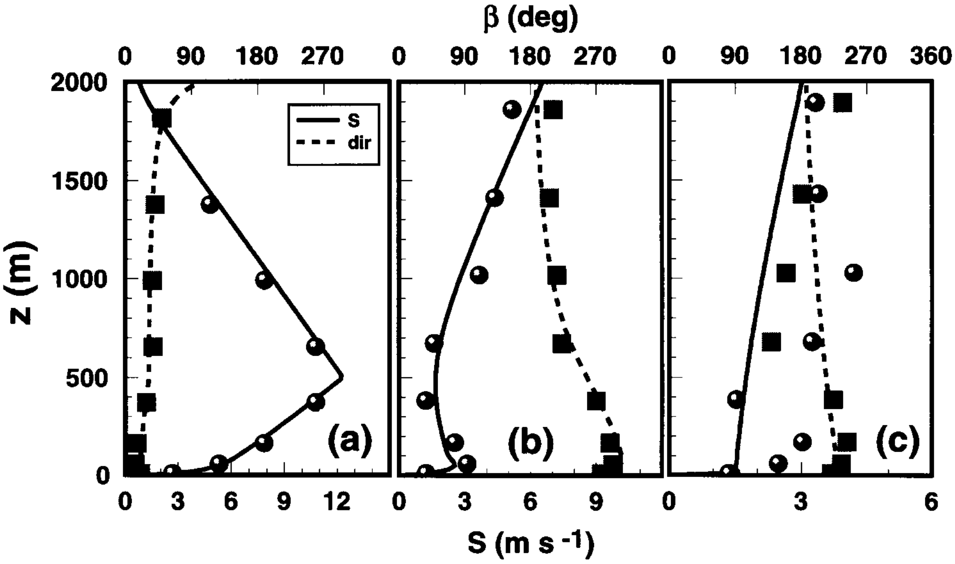

Figure 2. Fitted analytical profiles of wind speed and direction (lines) versus output profiles from the GEM numerical weather model (symbols).

TABLE II

Summary of the quality of conformance of the two-layer wind profile with measurements (criteria for the quality-designation are given in the text).

Data	# Profiles	Excellent	Intermediate	Poor
GEM	29	17	7	5
LROD	7	7	0	0
Wangara	21	4	13	4
All	57	28 (49%)	20 (35%)	9 (16%)

exhibiting two local maxima and a local minimum. The two-layer model does a very poor job of fitting this profile, with $\Delta^{\%}S = 30\%$ and $\Delta^{\beta} = 25°$ stemming from a best fit with ($u_* = 0.01$ m s^{-1}, $h_s = 10$ m, $\delta = 50$ m, $L = 1$ m). Note the very low windspeeds implied by the small u_* in these strongly stably stratified cases (b, c).

Across all 29 GEM cases we examined, the average profile errors were $\Delta^{\%}S = 11\%$ and $\Delta^{\beta} = 6°$. In all there were 17 ‘excellent’, 7 ‘intermediate’, and 5 ‘poor’ fits (see Table II). All of the poor fit cases had profiles having a local minimum in S (i.e., low level jet with a return to high winds aloft). Three of the five poor-fit cases had $u_* \leq 0.1$ m s^{-1}.

4.2. TWO-LAYER FITS TO THE WIND PROFILES OF LROD

The Long-Range Overwater Diffusion (LROD) experiment was designed to provide information on alongwind diffusion at intermediate to long range, and was conducted over the ocean near Hawaii in July, 1993 (Bowers et al., 1994). Wind profiles (S, β) were derived from balloon observations, supplemented during seven of the thirteen gas release trials by a single ship observation level, at $z = 10$ m. We fitted two-layer profiles to the measurements of these seven cases, again using eight observation heights.

The parameters z_0, u_*, L were *not* in this case considered as unknowns to be diagnosed, but rather we considered them known, for Bowers et al. had estimated them from empirical formulae based on windspeed and air-ocean temperature differences.* However when fitting the experimental profiles it became clear there was an inconsistency in the data: reported S (10 m) was not reconcilable with the reported (u_*, z_0, L). Bowers et al. had estimated z_0 using a formula given by Hosker (1974), which relates z_0 to S. Regarding z_0 as an empirical parameter, it makes sense for us to use a z_0 that agrees with the observed windspeed; our results apply with the reported z_0 reduced by 10.

Figure 3 gives the best and worst fits among the seven LROD profiles. The best fit (a) has $\Delta^{\%}S = 1.1\%$ and $\Delta^{\beta} = 0.3°$ with parameters ($u_* = 0.35$ m s^{-1}, $h_s = 125$ m, $\delta = 900$ m, $L = -10$ m). The worst fit (b) shows $\Delta^{\%}S = 5.3\%$ and $\Delta^{\beta} = 1.8°$ with parameters ($u_* = 0.40$ m s^{-1}, $h_s = 30$ m, $\delta = 800$ m, $L = -25$ m). Statistically, the profile fits of LROD were much better than those for the GEM data; all seven cases yielded "excellent" fits by the criteria defined earlier. As an overall average, the two-layer model fitted the LROD profiles with $\Delta^{\%}S = 3.3\%$ and $\Delta^{\beta} = 1.6°$.

The better model fit to LROD profiles relative to GEM profiles is due to the simpler LROD profile shapes. Conditions during LROD (daytime, over the tropical ocean) gave deep boundary layers under slightly unstable stratification. Windspeed and wind direction were much less variable with height than in the selected GEM data: there were no periods with local minima in the windspeed profile, a condition that had caused the poor fits to GEM data.

4.3. TWO-LAYER FITS TO THE WIND PROFILES OF THE WANGARA EXPERIMENT

The Wangara Experiment featured boundary-layer wind measurements taken over several weeks in New South Wales, Australia (Clarke et al., 1971). These consisted of tower observations (up to $z = 16$ m) and balloon observations (up to $z = 2000$ m). Three days from this experiment were selected for profile fitting (days 10 and 40 were essentially random choices; day 33 was chosen for its prom-

* This illustrates the great variety of possible scenarios, regarding available information, which any useful 'rational interpolation/extrapolation' wind model must accomodate.

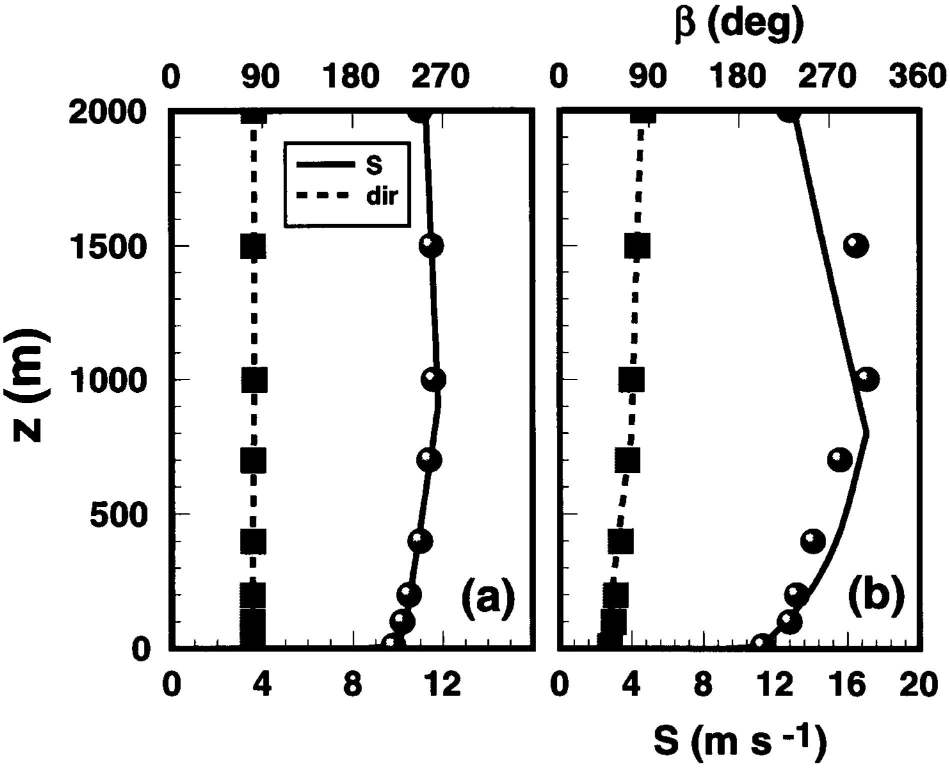

Figure 3. Fitted analytical profiles of wind speed and direction (lines) versus observations of LMOD (symbols).

inent role in analyses of the Wangara data and testing of ABL models). On each day the profiles at 0300, 0600, 0900, 1200, 1500, 1800, and 2100 local time were used. Each profile was defined from the observations at 23 observation heights* over the range 0.5 m $\leq z \leq$ 2000 m. We used the reported $z_0 = 0.0012$ m.

During the evening hours the boundary layer was characterised by ground level temperature inversions, with (presumably) low mixed-layer depths. Because a large proportion of the Wangara profile observations in those conditions would lie above the boundary layer (thus above the domain our model is intended to describe), during nighttime hours (0300, 0600, 2100) we ignored observations above $z = 500$ m. This is completely arbitrary, and it illustrates the tension between impartiality and naivety, in the matter of optimizing the fit of (any) wind model to observations in widely-varying circumstances.

Figures 4a–c illustrate the range of profile fits found for Wangara. The first (a; Day 40, 1500) is one of the best fits we have seen; with $\Delta^{\%} S = 7\%$ and $\Delta^{\beta} = 3.5°$ it is not the case with smallest average error, but in view of the complexity of the

* Data from each of these 23 levels were used in the optimization. Section 4.4 reports the sensitivity of the fitted profile to the number of levels of given data.

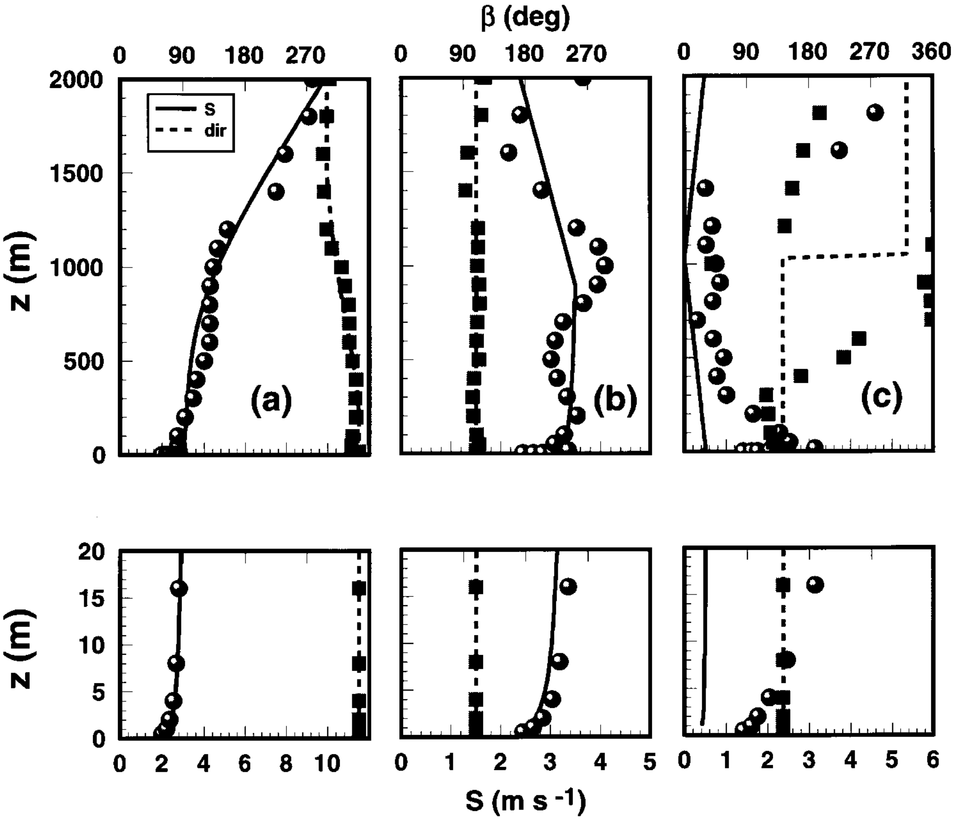

Figure 4. Fitted analytical profiles of wind speed and direction (lines) versus observations of the Wangara experiment (symbols).

profiles, this fit ($u_* = 0.14$ m s^{-1}, $h_s = 150$ m, $\delta = 1500$ m, $L = -10$ m) is very good. The 'wiggles' in the windspeed profile are common features of the Wangara data. Here the two-layer model smoothes out the speed profile, but captures its essence nicely.

The model does a poorer job in case b (Day 33, 1500), with $\Delta^{\%}S = 10\%$ and $\Delta^{\beta} = 3.8°$. The fitted scales are $u_* = 0.16$ m s^{-1}, $h_s = 500$ m, $\delta = 900$ m, $L = -5$ m, while Yamada (1976) gives best fit surface-layer (MO) scales $u_* = 0.155$ m s^{-1}, $L = -2.1$ m for this period.* The profile 'wiggles' here are more pronounced than in Figure 4a, with three pronounced local minima and three local maxima. The two-layer model cannot recreate these complexities, and instead smoothes them out; in essence, the model has relatively few degrees of freedom

* Please note that since our best-fit scales for the two-layer model must be optimal relative to observations in the surface layer *and above*, there is no requirement that they be "best fit" for the surface layer, alone.

and simply cannot reproduce a profile this complex. Given the propensity of data to contain errors, this parsimony of the two-layer model is, we think, a virtue.

Most Wangara profiles show a discontinuity between the tower and the balloon observations; this is seen, with some difficulty, in Figure 4b, where there appears to be a step decrease in windspeed above the tower (at $z = 16$ m). The discontinuity adds to the difficulty in fitting the Wangara profiles. The final profile (c, Day 10, 1800) represents a very poor model fit, with $\Delta^{\%} S = 78\%$ and $\Delta^{\beta} = 68°$ (fitted $u_* = 0.03$ m s^{-1}, $h_s = 25$ m, $\delta = 25$ m, $L = -1$ m). This profile illustrates a common characteristic of poor fit cases: very light winds. It was also one of the few cases with rain falling at the observation time.

A reviewer was curious as to the performance of this two-layer solution for CBL wind profiles of the idealized type, in which a well-mixed layer is topped by strong shear across a capping inversion. In such cases the two-layer profile may still conform satisfactorily to observations, because the geostrophic shear parameter is able to 'mimic' shear in the transition layer. This implies a falsely-continuing wind shear above the capping inversion, but as the two-layer model is meant to describe winds within the ABL, that is of no concern. Invoking a third layer (e.g., Lapworth, 1987) would have entailed four further degrees of freedom (upper and lower limits of the transition layer, and the shear in U, V), for dubious return.

When applied to the Wangara data (21 profiles examined) the two-layer model yielded overall-average errors of $\Delta^{\%} S = 17\%$ and $\Delta^{\beta} = 9°$. There were 4 'excellent', 13 'intermediate', and 4 'poor' fits. All of the poor-fit cases had profiles with a local minimum in windspeed (i.e., low-level jet with a return to high winds aloft), and three of four had $u_* < 0.1$ m s^{-1}. Two of the Wangara poor-fit cases were particularly bad, with errors in windspeed exceeding 35%; both occurred during rain.

We end this section by noting that the complexity of (some of the) Wangara wind profiles likely reflects an underlying *three-dimensionality* of the flow, for it is not captured by the time-dependent but horizontally-homogeneous, third-order closure model of André et al. (1978). Taking as an example the 1500 hrs windspeed profile of Wangara day 133 (case b of our Figure 4; please compare with their Figure 6a), the André et al. model, initialised with measured profiles at 0900 and driven by *measured* conditions at ground (u_*, L) and aloft (larger scale pressure gradient and thermal winds), merely produces a well-mixed layer (constant speed $S \approx 3$ m s^{-1}) from near ground to $z \approx 1300$ m, beneath a sharp shear layer with a speed minimum at $z \approx 1400$ m. The inference to be drawn is that for all its complexity, the higher-order closure model is no closer to matching the complex ('wiggly') measured windspeed profile, than is the simple two-layer model.

4.4. Model Profiles Resulting from Sparse Data

To illustrate the progressive 'degradation' of the fitted two-layer wind profile as the number of wind observation levels is reduced, we focused on the 1500 h profile for

Wangara day 33. Figure 5 shows the profiles that result when the full complement of 23 observation heights is used, and when the number of observation levels is systematically reduced (eliminating every-other observation level) to eleven, six, three, and finally just two levels. In every case the two-layer model returns a reasonable fit to the winds it is given, and differences between the fitted profiles correspond to plausible responses to the elimination of individual data points. The most significant changes in the fitted profile are above $z = 1$ km, in reaction to the elimination of the upper observations.

4.5. Accuracy of Model Fitting Parameters: u_*, L, δ

One may legitimately ask, why take the trouble to fit this particular 'scientific curve', rather than, say, an arbitrary best-fit polynomial? What is gained by using a 'wind model', given that fitting the latter involves optimizing numerous free parameters? And more especially, are the optimal values of those parameters (u_*, L, h_s, $\delta, \ldots$) legitimately to be regarded as the 'real' values to be associated with the measured wind data, or are they an inherently useless combination of arbitrary coefficients? Following this line of thought, we can argue for a clear advantage of fitting a 'wind model' rather than an arbitrary curve, if the magnitudes of the best-fit parameters turn out (at least statistically) to be plausible estimators of the 'real' underlying scales. And so, although we have seen that the two-layer model will usually return a fitted profile that realistically represents the (given) mean wind data, it is of interest to know how well it estimates the friction velocity, Obukhov length, and mixed-layer depth (the depth h_s of the surface layer, which is somewhat vaguely defined, and anyway does not enter quantitatively in the standard boundary-layer theory, is of less interest).

Such a comparison requires that we identify the 'true' values of these scales. The GEM model provides u_*, L, δ as model outputs. For the LROD experiment, u_*, L had been estimated (Bowers et al., 1994; Hosker, 1974) from empirical formula based on windspeed and air-ocean temperature differences, and δ had been determined from aircraft measurements of tracer concentration far downwind from the gas release. Surface-layer scales for Wangara runs have been given by Yamada (1976) and Melgarejo and Deardorff (1975), but as they are derived from profiles, they evidently are not to be regarded as entirely independent of the values obtained by fitting our two-layer model.* As for boundary-layer depth δ, for eight daytime periods we estimated it from the Wangara temperature profiles, assuming it coincides with the base of the lowest elevated temperature inversion.

* We are grateful to a referee for directing us to these previously-given scales for Wangara. We did not revise them to compensate for their having been based on (von Karman constant) $k_v = 0.35$ rather than our choice $k_v = 0.4$, for that would have required that we follow all steps of their computation (tedious or impossible, on the basis of the given information), or else substitute our own fitting procedure (which would have eliminated any last vestige of independence).

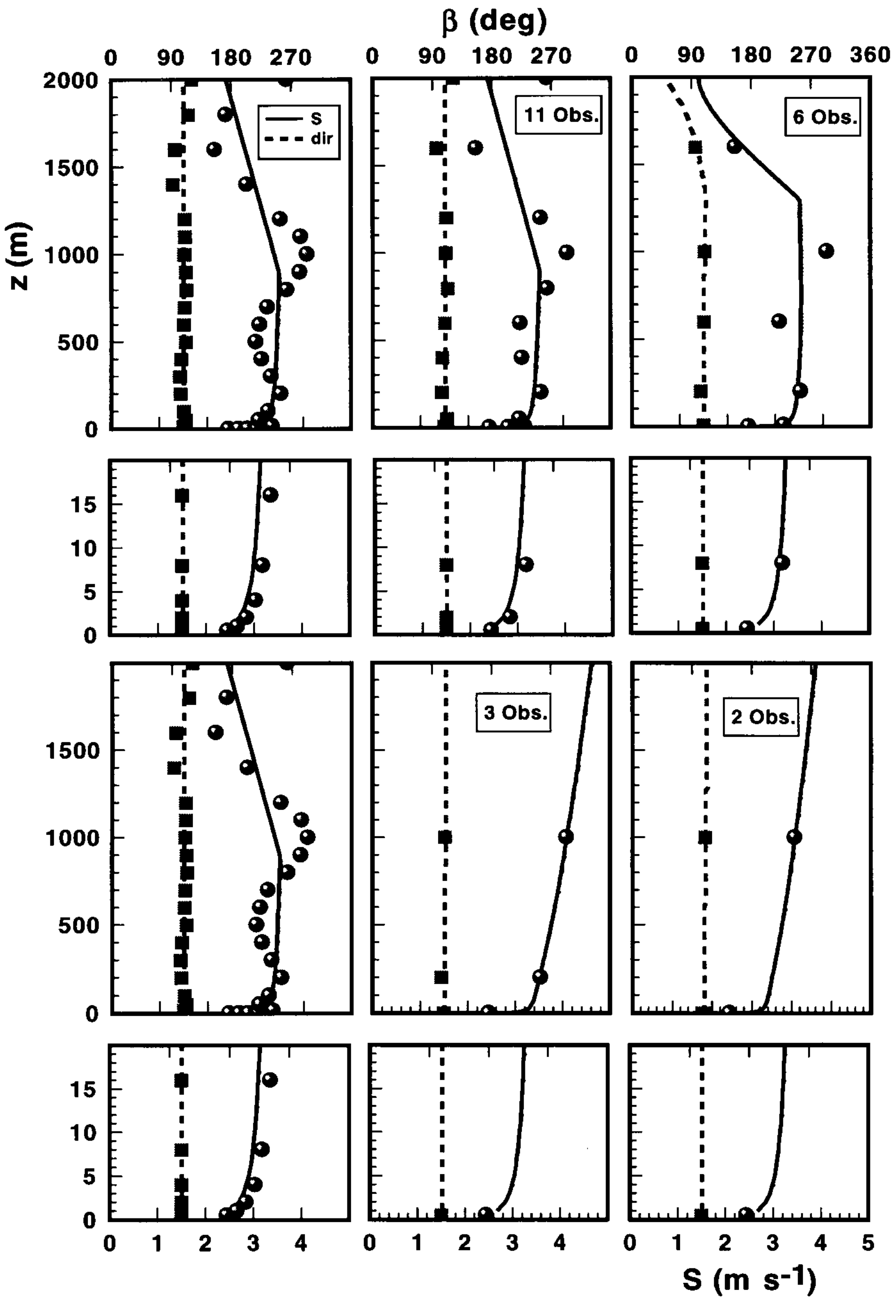

Figure 5. Fitted analytical profiles of wind speed and direction (lines) versus observations at 1500, Wangara day 33 (symbols). Left-hand panels show profile fitted to 23 data levels; other panels based on reduced number of levels of given winds.

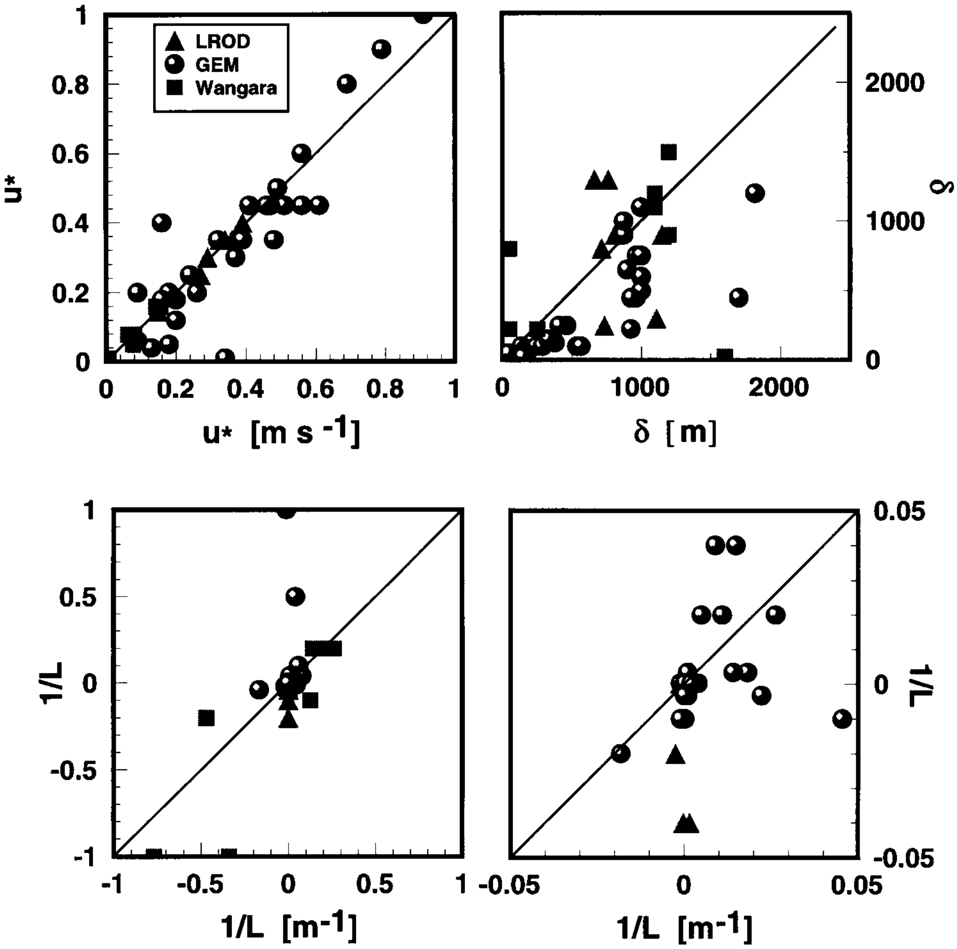

Figure 6. Comparison of best-fit (vertical axis) versus 'observed' (horizontal axis) values of wind parameters.

Figure 6 compares the best-fit model values of u_*, L, δ against the experimental values. The two-layer model diagnoses friction velocity well, and though it tends to overestimate the magnitude of L (i.e., diagnose more moderate stabilities than actually occurred), in most cases (34 of 43) it correctly classifies stability. As for boundary-layer depth δ, while there is *some* skill in the estimates (i.e., positive correlation), the model tends to underpredict (by an average of about 300 m).

5. Conclusions

Table II summarizes the performance of the two-layer model over the 57 wind profiles we examined. In general the Wangara data were fitted less well than either the LMOD data or the GEM profiles. This is understandable in view of the more complex profile shapes of Wangara, where more often the profiles had: (1) Localized and sharp windspeed extrema; and (2) a discontinuity (or inconsistency?) between the tower and balloon observations. We noted that poorer fits are strongly associated with light windspeeds, and it is telling that from their much more comprehensive suite of profiles examined, Garratt et al. (1982) eliminated from consideration profiles for which $u_* \leq 0.2$ m s^{-1}.

As regards the diagnosis of turbulence scales, in the majority of cases the model returns reasonable values for u_* (especially) and L, though greater errors surround δ, where the average error in the model's diagnosis was 200–300 m, i.e., roughly 30% of the true value. By these indications, fitting the 'scientific' two-layer wind profile to wind measurements, as opposed to some arbitrary curve, has the advantage that it results in plausible estimates of these other quantities that are also crucial in the description or modelling of boundary-layer processes.

Subjectivity cannot be eliminated from any judgement of 'success' for a model such as this, and we leave that judgement to the reader (one could have examined a million profiles: There would have been many good and many bad fits). In our opinion, no comparably parsimonious wind model would achieve solely 'excellent' fits, not only because (sometimes) observations are wrong, but also because the real atmosphere, by virtue of being 'real' (inhomogeneous, non-stationary, etc.), eludes a parsimonious model.

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Appendix A: Boundary and Matching Conditions

The variables of the idealized two-layer wind profile are required to obey the matrix equation $\mathbf{MS} = \mathbf{R}$. The coefficient matrix

$$\mathbf{M} = \begin{pmatrix} e^{\mu\delta}\cos\mu\delta & -e^{\mu\delta}\sin\mu\delta & e^{-\mu\delta}\cos\mu\delta & e^{-\mu\delta}\sin\mu\delta & 0 & 0 \\ e^{\mu\delta}\sin\mu\delta & e^{\mu\delta}\cos\mu\delta & -e^{-\mu\delta}\sin\mu\delta & e^{-\mu\delta}\cos\mu\delta & 0 & 0 \\ e^{\mu h_s}\cos\mu h_s & -e^{\mu h_s}\sin\mu h_s & e^{-\mu h_s}\cos\mu h_s & e^{-\mu h_s}\sin\mu h_s & 1 & 0 \\ e^{\mu h_s}\sin\mu h_s & e^{\mu h_s}\cos\mu h_s & -e^{-\mu h_s}\sin\mu h_s & e^{-\mu h_s}\cos\mu h_s & 0 & 1 \\ \mu e^{\mu h_s}[\cos\mu h_s - \sin\mu h_s] & -\mu e^{\mu h_s}[\cos\mu h_s + \sin\mu h_s] & -\mu e^{-\mu h_s}[\cos\mu h_s + \sin\mu h_s] & \mu e^{-\mu h_s}[\cos\mu h_s - \sin\mu h_s] & 0 & 0 \\ \mu e^{\mu h_s}[\sin\mu h_s + \cos\mu h_s] & \mu e^{\mu h_s}[\cos\mu h_s - \sin\mu h_s] & \mu e^{-\mu h_s}[\sin\mu h_s - \cos\mu h_s] & -\mu e^{-\mu h_s}[\sin\mu h_s + \cos\mu h_s] & 0 & 0 \end{pmatrix}$$

while the matrix on the right-hand side is

$$\mathbf{R} = [0, 0, U_{MO}(h_s) - U_T h_s, -V_T h_s, (\partial U_{MO}/\partial z)_{h_s} - U_T, -V_T]^T \tag{A1}$$

(superscript 'T' denotes the transpose) and the 'solution' matrix is

$$\mathbf{S} = [\alpha_R, \alpha_I, \beta_R, \beta_I, U_{G0}, V_{G0}]^T. \tag{A2}$$

In descending sequence through $\mathbf{M}$, these equations correspond to: the upper boundary conditions $U(\delta) = U_G$, $V(\delta) = V_G$; the matching conditions on velocity, $U(h_s) = U_{\text{MO}}(h_s)$, $V(h_s) = 0$; and the matching conditions on shear, $(\partial U/\partial z)_{h_s} = u_*/(k_v h_s)\, \phi(h_s/L)$, $(\partial V/\partial z)_{h_s} = 0$.

Given guess values for u_*, L, h_s, one determines K and thus μ: the (guess) coefficient matrix is specified. These values also determine the first four entries of $\mathbf{R}$, whose (guess) specification is completed by guessing the components U_T, V_T of the thermal wind shear. The (guess) solution matrix is easily found, in terms of the inverse $\mathbf{M}^{-1}$ of the coefficient matrix, as $\mathbf{S} = \mathbf{M}^{-1}\mathbf{R}$. One may then determine the guess (model) values corresponding to each of the given wind velocities, and define a mean square error associated with the guess parameters.

This seems to us a natural way to fit the trial wind profile, although there may be others. The status of U_{G0}, V_{G0} as *outputs* of the fit could be altered, if these were to be considered as specified.

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