Deposition of particles to a thin windbreak: The effect of a gap

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Received 1 November 2004; received in revised form 13 June 2005; accepted 14 June 2005

Abstract

It is shown that the ‘bleed velocity’ through the ‘fabric’ of a thin windbreak or shelterbelt is practically insensitive to the existence of nearby holes or gaps in the fabric. This provides the basis for a straightforward extension of an earlier formula for particle ‘scrubbing’ by a thin windbreak, to account for irregularities of the filtering vegetation or mesh.

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Keywords: Wind; Shelter; Windbreak; Particulates; Aerosols; Filtering; Deposition

1. Introduction

This note will extend the analytical theory of Raupach et al. (2001) for the filtering of a particulate-loaded airstream by a laterally-uniform, porous windbreak. Its focus therefore lies on the wind velocity through a thin, natural or artificial windbreak, ‘thin’ signifying that variation of the wind across the shelter \(X/2 \leq x \leq X/2\) may for practical purposes be neglected because \(X \ll H\), where \(H\) is the windbreak height. As an ‘internal’ property of the windbreak flow, albeit an important one because it sets the overall level of windbreak drag, the ‘bleed’ velocity at the windbreak is not normally of practical interest. However it also controls the filtering effect of a windbreak on a particle-laden airstream, for the particle deposition rate per unit crosswind distance \((y)\) is (Raupach et al.)

\[
D_b = \int_{z_0}^{H} U(0, z)(C_0 - C_1) \, dz, \tag{1}
\]

where \(C_0, C_1\) are the particle concentrations in the air upwind and downwind of the thin windbreak (which is centred at \(x = 0\), \(z_0\) is the surface roughness length and \(U(0, z)\) is the profile of the (mean) bleed velocity. Exploiting the (inexact) similarity between the transfer rates of momentum and particulate mass to the windbreak ‘fabric’, Raupach et al. reframed this expression for the deposition flux in terms of a ‘harmonic mean bleed velocity’

\[
U_b^2 = \frac{1}{H} \int_{z_0}^{H} U^2(0, z) \, dz \tag{2}
\]

in terms of which the windbreak drag force per unit crosswind length is

\[
F_b = \rho k_r U_b^2 H \tag{3}
\]

(\(\rho\) is the air density and \(k_r\) is the dimensionless resistance coefficient of the windbreak ‘fabric’, i.e. assemblage of leaves and branches, or porous mesh). The linkage between equations (1) and (3) is the aerodynamical basis

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1The resistance coefficient \(k_r\) of a mesh or fabric is here defined by \(\Delta P = k_r \rho U^2\), where \(\Delta P\) is the pressure drop across the material when it is mounted so as to impede a uniform, confined flow of speed \(U\) and density \(\rho\). Please note that many other authors (including Raupach et al.) define \(k_r\) by \(\Delta P = k_r \frac{1}{2} \rho U^2\), but (for consistency with the author’s earlier work) here the 1/2 will be omitted.
for the key point that “the total deposition of particles to a windbreak is determined by a trade-off between particle absorption and throughflow... the windbreak must be dense enough to absorb particles efficiently, but sparse enough to allow some particles to flow through and be trapped.” An interesting question is the extent to which this trade-off can be manipulated to achieve a more complete scrubbing of the airstream, by adjustment of the windbreak characteristics, e.g. its “outline” \((H, X)\), the depth \(H_1\) of gap or trunk space, and the internal structure as specified by the element surface area density \(a [m^{-1}]\) and drag coefficient \(c_e\), which determine the bulk optical porosity \(\tau\) and effective resistance coefficient. Grunert et al. (1984) wrote that “aerosol protection plantations must... have a looser and wider construction to promote throughflow and filtering than wind protection plantations.”

The formula given by Raupach et al. for efficiency of particle scrubbing is appropriate only for a “thin” windbreak, and consequently is unable to address the question of an optimal configuration \((X, a, c_e, \ldots)\). However here their ‘thin windbreak’ theory of particle scrubbing will be extended in a simple way to account for the influence of gaps in the windbreak, such as (e.g.) the case of a single line of trees with a substantial trunk space. A surprising fact that will be demonstrated is that for given \(H, k_r\), and given upwind flow (friction velocity \(u_s\)), the bleed velocity \(U(0, z)\) is indifferent to the existence and depth \(H_1\) of a gap. In Section (2) this will be proposed on the basis of a highly idealized approximate analytical solution of a linearized vorticity equation valid for the case \(k_r \ll 1\), and confirmed by realistic numerical solutions of the non-linear mean momentum equations. On the basis of this finding, in Section (3) the Raupach et al. result for particle deposition rate to the windbreak will be extended in terms of a re-defined harmonic mean bleed velocity.

2. Influence of a gap on the bleed flow

It may be helpful to emphasize at the outset that this paper is not at all concerned with the ‘shelter’ provided by a windbreak, i.e. the velocity reduction in its wake, but primarily with the velocity field at the windbreak, and (only to the extent it affects the bleed velocity) with the velocity further upwind: i.e., the domain of interest is \(x \ll 0\). This focus largely justifies (and simplifies the interpretation of) the analysis to follow, which concerns the region of the flow where an equilibrium surface layer is disturbed by the perturbation pressure field its own interaction with the windbreak generates. As far as the streamwise momentum budget is concerned, in this upwind region vertical advection and the Reynolds stress gradients are weak, specifically in relation to streamwise advection and in relation to the pressure gradient (e.g. Wang and Takle, 1997, Fig. 3; Wilson and Yee, 2003, Fig. 5). Thus these latter forces, in combination with the direct drag of the barrier, dominate the flow in the region of interest to us. The approximate analytical solution now to be derived will capitalize on the dominance of these forces.

Assume the mean wind blows normal to an infinitely-long windbreak, so that it suffices to consider a two-dimensional mean flow with streamwise and vertical components \((U, W)\). The windbreak material will be assumed to have bulk resistance coefficient \(k_r\) that does not vary across its section (for a natural windbreak with uniform leaf area density \(a\), the effective resistance coefficient \(k_r \sim c_e a X\)). To a first approximation the influence of a thin windbreak on the flow about it may be represented by a momentum sink \(k_r U^2\) in the streamwise mean momentum equation, localized at the windbreak, viz. (symbolically)

\[
\frac{\partial U}{\partial t} + \cdots = -k_r U^2 \delta(x - 0) Q(z),
\]

where \(\delta(x - 0)\) is the delta-function localizing the drag to the windbreak location \(x = 0\), and \(Q(z)\) localizes the drag on the height axis.\(^3\) To be more specific, appropriate steady-state mean momentum equations for neutral flow about a porous barrier are

\[
\frac{\partial}{\partial x} \left( U^2 + \overline{u^2} + P/\rho \right) + \frac{\partial}{\partial z} (UW + \overline{uw})
\]

\[
= -k_r U^2 \delta(x - 0) Q(z),
\]

\[
\frac{\partial}{\partial x} (UW + \overline{uw}) + \frac{\partial}{\partial z} (W^2 + \overline{w^2} + P/\rho) = 0,
\]

where \(P\) is the local disturbance in mean pressure caused by interaction of the wind with obstacles, and \(\overline{uw}\) etc.) are components of the Reynolds stress tensor. Accordingly the conservation equation for the mean vorticity \(\Omega = U_z - W_x\) is

\[
U \frac{\partial Q}{\partial x} + W \frac{\partial Q}{\partial z} = -k_r \delta(x - 0) \left( \frac{\partial U^2}{\partial z} + U^2 \frac{\partial Q}{\partial z} \right) + R,
\]

where \(R\) collects the terms arising from the Reynolds stresses, and will be neglected since it plays only a small role on the perturbation flow in the upwind region.\(^3\)

Now if we decompose the mean velocity relative to the

\(^3\)In the case of a uniform windbreak without gaps, \(Q(z) = s(z - H)\), where \(s(z - H)\) is a unit dimensionless step function at \(z = H\).

\(^3\)That this is so is evident from the demonstrated insensitivity of the computed flow in the region \(x/H \leq 2\) to the closure chosen for the Reynolds stresses (Wilson, 1985) and from the previously mentioned studies of the magnitudes of terms in the \(U\)-momentum equation.
Now neglect background shear so that

\[ U = U_0 + \Delta U = U_0 + k_r u, \]

\[ W = \Delta W = k_r w \]

and introduce a perturbation streamfunction \( \psi \) in terms of which the velocity and vorticity perturbations are

\[ u = -\psi_z, \]

\[ w = \psi_x, \]

\[ \omega = \nabla^2 \psi, \]

the resulting perturbation vorticity equation, to first order in the small parameter \( k_r \), is

\[ U_0 \frac{\partial \omega}{\partial x} + w \frac{\partial^2 U_0}{\partial z^2} = -\delta(x-0) \left( U_0 \frac{\partial Q}{\partial z} + Q(z)2U_0 \frac{\partial U_0}{\partial z} \right). \]  

(9)

Now neglect background shear so that \( U_0 = \text{const} \), and specify a uniform windbreak spanning \( H_1 \leq z \leq H \) by writing

\[ Q(z) = s(z - H_1) - s(z - H), \]  

(10)

\[ \frac{\partial Q}{\partial z} = \delta(z - H) - \delta(z - H_1). \]  

(11)

Henceforth considering all velocities to have been normalized on \( U_0 \) and all lengths on \( H \), Eq. (9) reduces to

\[ \nabla^2 \psi = \nabla^2 w = -\delta(x-0)[\delta(z-1) - \delta(z-\ell)], \]  

(12)

where \( \ell = H_1/H \). The Laplacian of the vertical velocity perturbation vanishes everywhere except at the upper and lower extremities of the windbreak. The Green’s function for the Laplacian in a two-dimensional unbounded space is

\[ G(x - x_s) = \frac{1}{2\pi} \ln \sqrt{(x - x_s)^2 + (z - z_s)^2} \]  

(13)

and so the vertical velocity perturbation is readily derived as

\[ w = \psi_x = \frac{1}{4\pi} \left[ \ln \frac{(x^2 + (z - \ell)^2)}{x^2 + (z + \ell)^2} + \ln \frac{(x^2 + (z + 1)^2)}{x^2 + (z - 1)^2} \right], \]  

(14)

where for each vorticity source an image has been added to ensure \( \psi = 0 \) along \( z = 0 \). Integrating w.r.t. \( x \) and differentiating w.r.t. \( z \) gives the alongwind velocity

\[ u = \frac{1}{2\pi} \left[ \tan^{-1} \frac{x}{z - 1} - \tan^{-1} \frac{x_0}{z - 1} - \tan^{-1} \frac{x}{z + 1} + \frac{1}{2\pi} \left[ \tan^{-1} \frac{x}{z + \ell} - \tan^{-1} \frac{x_0}{z + \ell} - \tan^{-1} \frac{x}{z - \ell} + \tan^{-1} \frac{x_0}{z - \ell} \right] \right], \]  

(15)

where the constant of integration \( x_0 \) is defined such that \( u(x_0, z) = 0 \) (for the solutions to be shown, \( x_0 = -100 \)).

Fig. 1(a,b) show this solution for the cases \( \ell = H_1/H = 0.2, 0.4 \). Transects through the windbreak fabric show deceleration, while transects under or over the windbreak show acceleration, the strongest acceleration occurring on the transect through a narrow gap. In interpreting these solutions one must recall that the influences of background shear and turbulent shear stress have been neglected, for only the three (locally) dominating terms have been retained (streamwise advection, pressure gradient and drag on the windbreak). Therefore the only agency for cross-stream “communication” of the flow disturbance is the pressure force, and the only asymmetry between transects at distances \( \pm 0.1H \) above and below the windbreak is the

Fig. 1. Analytical solutions for the fractional velocity perturbation \( u = (1/k_r)\Delta U/U_0 \) around a very porous windbreak, showing transects at several \( z/H \) as a function of gap depth \( \ell = H_1/H \).
an easy generalization of the Raupach et al. theory for particle filtration.

2.1. Non-linear numerical solution for the bleed flow

The above analytical solution captures the dominant influences on the bleed flow provided \( k_r \ll 1 \), but to verify that the main result holds in the full complexity of the problem (finite \( k_r \) implying non-linear advection; no-slip imposed on ground, implying background shear; influence of the Reynolds stresses retained) one may examine numerical solutions of the full steady-state mean momentum equations (5). The numerical method used here has been described at length by Wilson (1985, 2004) and, for the case of neutrally-stratified flow oriented perpendicular to a windbreak, solutions for the mean wind have been shown to be in good agreement with field measurements by Bradley and Mulhearn (1983; \( H/z_0 = 600 \), \( k_r = 2.0 \)) and others. Simulations have here been performed using the second-order turbulence closure of Rao et al. (1974) on a domain spanning \(-60 \leq x/H \leq 112, z/H \leq 47\) (‘standard domain’ of Wilson, 1985, 2004), with uniform resolution \( \Delta x/H = 1, \Delta z/H = 0.1 \).

Fig. 2(a,b) gives the results of simulations of the velocity profile at the windbreak for the case \( H = 1.2 \text{m}, z_0 = 0.002 \text{m} \) (\( H/z_0 = 600 \)). Fig. 2(a) shows that the bleed velocity decreases monotonically as the resistance coefficient is increased, and that a gap under the fence results in a jet where the peak speed exceeds the speed (at the same height) far upstream. The less predictable (and in the present context, more important) point is that Fig. 2(b), showing simulations with fixed \( k_r = 2 \) and three gap-depths \( H_1/H = (0.0, 0.25, 0.5) \), demonstrates that the bleed velocity profile (in the range \( H_1 \leq z \leq H \)) is indifferent to the existence and depth of the underlying gap. This is consistent with the simplistic analytical theory outlined above, and it raises an interesting question: if the bleed velocity is indifferent to a gap beneath the windbreak, might it also be indifferent to a ‘gap’ above the windbreak, that is, might the bleed velocity be invariant relative to the height of the windbreak? Fig. 3 indicates that to a good approximation indeed it is, at least in the case of variations in \( H \) covering a range such that \( H/z_0 \geq 300 \). Thus we have the surprising result that bleed velocity through a thin windbreak, which one might have expected to respond to many or all of the governing (‘external’) scales, apparently is set by (or responds to) only (a) the overall

\[ \frac{1}{k_r} \frac{\Delta U}{U_0} = \frac{1}{2} \cdot \frac{D}{x} \]  

The unequivocal simplicity of this asymptotic (small \( k_r \)) result suggests that in the case of a more general and (possibly) irregular form of windbreak gap, such as an opening on the crosswind (\( y \)) axis or an irregularity in the height or outline of the shelter, to a first approximation the bleed velocity should be indifferent to the existence and geometry of the gap: on any mean streamline passing through the windbreak fabric (as opposed to above or below the fabric or through any gap) the bleed velocity according to this simplified treatment is \( U_0(1 - k_r/2) \). To the extent that this proves true for practical values of \( k_r \) (i.e. order 1), it will permit

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5It is interesting that Taylor (1944; Eq. 3) obtained an equivalent result for the velocity perturbation immediately upstream of a porous plate exposed in an unbounded laminar flow, by treating the barrier as a source of fluid volume.

6To within a modest level of error or uncertainty that stems from the arbitrariness (and imperfection) of the turbulence closure and computational resolution.

7There must be some logical limit to this invariance, for as \( H \to 0 \) we end up without any windbreak through which the wind ‘bleeds’.
velocity (as indexed by friction velocity $u_*$) and (b) the resistance coefficient of the fabric. It is this result (whose validity is here supported by two lines of argument) that justifies a conjecture that bleed velocity will be invariant no matter what the form of the gaps in a windbreak—provided only that these be ‘macroscopic’ gaps, i.e. outright holes or free passages in the windbreak having a categorically larger size than the pores of the windbreak mesh.

In the case that our windbreak ‘gap’ is continuous on the crosswind ($y$-) axis, we may modify Eqs. (2) and (3) so that the harmonic mean bleed velocity is defined

$$U_b^2 = \frac{1}{H - H_1} \int_{H_1}^{H} U^2(0, z) \, dz$$

and the drag

$$F_b = \rho k_r (H - H_1) U_b^2.$$  (18)

Fig. 4 shows that while the drag $F_b$ decreases (as expected) with increasing gap width $H_1/H$, to a good first approximation $U_b$ is independent of $H_1/H$, as earlier shown. Interestingly these simulations showed that over a wide range in parameter space ($0 \leq H_1/H \leq 0.5$, $0.5 \leq k_r \leq 8$) the linear-mean bleed velocity

$$U_{b,\text{lin}} = \frac{1}{H - H_1} \int_{H_1}^{H} U(0, z) \, dz$$

differs negligibly from the harmonic mean bleed velocity. Thus the shape factor

$$n_s = \frac{U_{b,\text{lin}}}{U_b}$$

introduced by Raupach et al. can be set to unity with no loss of accuracy, reflecting the fact that the mean velocity at the windbreak is (roughly) height independent over the bleed-span $H_1 \leq z \leq H$, due to the strong feedback provided by the force $-k_r U^2$ which tends to flatten the bleed velocity profile.

In concluding this section it may be helpful to clarify that it is not being argued here that these non-linear solutions to the momentum equations reproduce exactly the bleed velocity profile given by the analytical solution\footnote{Though as an aside, for sufficiently small $k_r$, they probably would be very similar at $x \equiv 0$. A comparison of Figs. 2 and 3 of Wilson et al. (1990), which are (no-gap) windbreak velocity transects according to the (same) linearized analytic solution and according to the non-linear numerical simulation, would suggest otherwise. However subsequent work revealed that an}; this would be astonishing given their differing...
assumptions. The point is that in the region \( x \leq 0 \) the analytical solution is a plausible qualitative idealization, and suggests (or even ‘explains’) the pattern subsequently discerned in the full solution... namely that the bleed velocity profile is rather insensitive to the depth of a windbreak gap. There is no reason to suppose this qualitative harmony of the analytical and numerical solution rests on the particular parameter choices (e.g. \( H/\zeta_0 \)) made for the latter.

3. Particle entrapment by a thin windbreak

Raupach et al. considered the case of an approaching flow that is uniformly seeded with single-sized particles (diameter \( 1 \mu m \leq d_p \leq 100 \mu m \)), and normally-incident on a long, straight and vertically-uniform windbreak. They gave a relationship between the mean particle concentration \( C \) on the downwind side \((C_1)\) and the upwind loading \((C_0)\) by integrating across the windbreak a simplified version of the mass conservation equation, namely

\[
U \frac{\partial C}{\partial x} = -xg_p C, \tag{21}
\]

where \( g_p \) is the particle deposition velocity (deposition to the ground has been neglected relative to deposition onto vegetation, a good approximation if one is considering a rather narrow and dense windbreak, and turbulent transport of particles by the wind has been neglected relative to transport by the local mean wind \( U \)). Taking \( x \) and \( g_p/U \) as constant along a trajectory through the windbreak, the ratio of the particle- and momentum-fluxes to the windbreak is

\[
Db = \frac{(C_0 - C_1) \int_{H_1}^H U(0,z) \, dz}{C_0(H-H_1)}, \tag{23}
\]

where the presence of a gap has been accounted for, and (as previously noted) the shape factor \( n_i \approx 1 \). Evidently the ratio of the particle- and momentum-fluxes to the windbreak is

\[
\frac{Dp}{Ub} = \frac{1}{pk_r} \frac{Ub(C_0 - C_1)}{UbUb}, \tag{24}
\]

in which \( Ub \) masquerades as both the bulk conductance (of the two flow properties to the windbreak) and—in the case of momentum—as the entity conducted. The important point is that it follows from Fig. (4) that the normalized deposition flux

\[
\frac{Db}{U_0} = \frac{Ub}(1-\sigma) \tag{25}
\]

is invariant relative to the depth of any windbreak gap, and the empirical formula given by Raupach et al.

\[
\frac{Ub}{U_0} = \left( \frac{\Gamma_{b1}}{k_r + \Gamma_{b1}k_1} \right)^{1/2}, \tag{26}
\]

\( (\Gamma_{b1} = 1.07, k_1 = 1.5) \) remains valid irrespective of the introduction of a gap. Thus as a consequence merely of having re-defined the harmonic mean bleed velocity and the scale for the particle flux, the formula of Raupach et al. for the normalized deposition rate of particles to a thin windbreak carries over directly to cases where there are gaps in the windbreak.

4. Conclusion

Although the present study extends the theory of particle deposition to the specific case of a windbreak gap that is continuous on the crosswind axis, i.e. a trunk space or its equivalent, it seems warranted to conjecture that a generalization to arbitrarily irregular gaps (fully three-dimensional flow) would be valid; if so the bleed velocity would be substantially indifferent to the geometry of the gap(s) and it would merely be necessary to appropriately redefine the reference flux of particles, i.e. generalize Eq. (25) by altering the denominator.
$U_{0\text{H}} (H - H_1) C_0$ to exclude that part of the incident particle flux that impinges on the gap(s).

**Acknowledgements**

This work has been supported by a research grant from the Natural Sciences and Engineering Research Council of Canada (NSERC).

**References**


